Important Questions Class 12 Maths Chapter 4 Determinants

1 Mark Questions

1.Find values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ **.**

Ans. $(3 - x)^2 = 3 - 8$
$3 - x^2 = 3 - 8$
$-x^2 = -8$
$x = \pm\sqrt{8}$
$x = \pm 2\sqrt{2}$

2. A be a square matrix of order 3 $\times$ 3, there $|KA|$ is equal to

Ans. $|KA| = K^n |A|$
N=3
$|KA| = K^3 |A|$

3. Evaluate

$$\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$$

Ans.

$$\Delta = 0 \left[C_1 \ and \ C_3 \ identical \right]$$

4. Let $\begin{vmatrix} 4 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 4 & 1 \end{vmatrix}$ **find all the possible value of x and y if x and y are natural numbers.**

Ans. 4 – xy = 4 -8
xy = 8
of x = 1 x = 4 x = 8
y = 8 y =1 y = 1

5. Solve

$$\begin{vmatrix} x^2-x+1 & x+1 \\ x+1 & x+1 \end{vmatrix}$$

Ans. $(x^2 - x + 1)(x + 1) - (x + 1)(x - 1)$
$= x^3 - x^2 + x + x^2 - x + 1 - (x^2 - 1)$
$= x^3 + 1 - x^2 + 1$
$= x^3 - x^2 + x^2$

---

6. Find minors and cofactors of all the elements of the det. $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$

Ans.

$$M_{11} = 3, \qquad A_{11} = 3$$

$$M_{12} = 4, \qquad A_{12} = -4\left[\because Aij = (-1)^{i+J} MiJ\right]$$

$$M_{21} = -2, \qquad A_{21} = 2$$

$$M_{22} = 1, \qquad A_{22} = 1$$

---

7. Evaluate

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Ans.

$$= \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6\times17 & 6\times3 & 6\times6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0$$

[R_1 and R_3 are identical]

---

8. Show that

$$\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix} = 1$$

Ans.

$$= \sin 10 . \cos 80 + \cos 10 \sin 80$$

$$= \sin(10 + 90)$$
$$= [\because \sin A \cos B + \cos A . \sin B = \sin(A + B)]$$

$$= \sin 90$$
$$= 1$$

---

9. Find value of x, if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

Ans. $(2 – 20) = (2x^2 – 24)$
-18 = 2×2 – 24
$-2x^2 = -24 + 18$
$-2x^2 = 6$
$2x^2 = 6$
$x^2 = 3$
$x = \pm\sqrt{3}$

---

10. Find adj A for $A=\begin{bmatrix}2 & 3\\1 & 4\end{bmatrix}$

Ans. adJ A = $\begin{bmatrix}4 & -3\\-1 & 2\end{bmatrix}$

$$\left[\because \quad A=\begin{vmatrix}2 & 3\\1 & 4\end{vmatrix} \quad \begin{matrix}\textit{change sign} & \text{inter-change}\end{matrix}\right]$$

---

11. Without expanding, prove that

$$\Delta=\begin{vmatrix}x+y & y+z & z+x\\ z & x & y\\ 1 & 1 & 1\end{vmatrix}=0$$

Ans. $R_1 \to R_1+R_2$

$$\Delta=\begin{vmatrix}x+y+z & x+y+z & x+y+z\\ z & x & y\\ 1 & 1 & 1\end{vmatrix}$$

$$\Delta=(x+y+z)\begin{vmatrix}1 & 1 & 1\\ z & x & y\\ 1 & 1 & 1\end{vmatrix}$$

$$=0\begin{bmatrix}\because R_1 \text{ and } R_3\\ \text{area identical}\end{bmatrix}$$

---

12. If matrix

$$A=\begin{bmatrix}1 & -2 & 3\\ 1 & 2 & 1\\ x & 2 & -3\end{bmatrix}$$

is singular, find x.

Ans. For singular |A| = 0

1(-6 -2) + 2(-3 -x) + 3 (2 -2x) = 0

-8 – 6 – 2x + 6 – 6x = 0

-8x = + 8

x = -1

---

13. Show that, using properties if det.

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1-x^3\right)^2$$

Ans.

$$C_1 \to C_1 + C_2 + C_3$$

$$= \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$$

$$= \left(1+x+x^2\right)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$$

$$R_1 \to R_1 - R_3 \qquad R_2 \to R_2 - R_3$$

$$= \left(1+x+x^2\right)\begin{vmatrix} 0 & x-x^2 & x^2-1 \\ 0 & 1-x^2 & x-1 \\ 1 & x^2 & 1 \end{vmatrix}$$

$$=\left(1+x+x^2\right)\begin{vmatrix}0 & x(1-x) & -(1-x)(1+x)\\0 & (1-x)(1+x) & -(1-x)\\1 & x^2 & 1\end{vmatrix}$$

Taking (1 – x) common from R_1 and R_2

$$=\left(1+x+x^2\right)(1-x)^2\begin{vmatrix}0 & x & -(1+x)\\0 & 1+x & -1\\1 & x^2 & 1\end{vmatrix}$$

Expending along C^1

$$\begin{aligned}&=(1+x+x^2)(1-x)^2[-x+(1+x)^2]\\&=(1+x+x^2)(1-x)^2\left(-x+1+x^2+2x\right)\end{aligned}$$

$$\begin{aligned}&=(1-x)(1+x+x^2)(1-x)(1+x+x^2)\\&=\left(1-x^3\right)^2\end{aligned}$$

---

14. If

$$\begin{vmatrix}x & 2\\18 & x\end{vmatrix}=\begin{vmatrix}6 & 2\\18 & 6\end{vmatrix},$$

than x is equal to

Ans. x^2 – 36 = 36 – 36

x^2 = 36

$x=\pm\sqrt{6}$

---

15. $A=\begin{bmatrix}1 & 2\\1 & 4\end{bmatrix}$ **is singular or not**

Ans. $|A|=\begin{vmatrix}1 & 1\\4 & 8\end{vmatrix}$

= 8 – 8

= 0

Hence A is singular

---

16. Without expanding, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Ans.

$$\frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$\frac{abc}{abc} = \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \begin{bmatrix} C_1 \leftrightarrow C_3 \\ C_2 \leftrightarrow C_3 \end{bmatrix}$$

Hence Prove

17.

$$A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix},$$

Verify that det A = det (A')

Ans.

$$|A| = 2(0-20) + 3(-42-4) + 5(30-0)$$

$= -28$

Hence prove.

---

18. If then show that
Ans.

Hence Prove

---

19. A be a non – singular square matrix of order 3 3. Then is equal to
Ans.
N=3

---

20. If A is an invertible matrix of order 2, then det is equal (A^{-1}) to
Ans. A is invertible AA^{-1} =
det (AA^{-1}) = det (I)
det A.(det A^{-1}) = det ( )
det A^{-1} =

4 Marks Questions

---

1. Show that, using properties of determinants.

OR

Ans. Multiplying R_1 R_2 and R_3 by a, b, c respectively

Taking a, b, c, common from c_1, c_2, and c_3

Expending along R_1

OR {solve it}
{hint : }
Taking common 3 (a+b) from C_1

2.
Ans.

$$R_1 \to xR_1, \quad R_2 \to yR_2, \quad R_3 \to ZR_3$$

$$\Delta = \frac{1}{xyz}\begin{vmatrix} x(y+z)^2 & x^2y & x^2z \\ xy^2 & y(x+z)^2 & y^2z \\ xz^2 & yz^2 & z(x+y)^2 \end{vmatrix}$$

$$\Delta = \frac{xyz}{xyz}\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (x+z)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix}$$

$$C_2 \to C_2 - C_1, \quad C_3 \to C_3 - C_4$$

$$\Delta = \begin{vmatrix} (y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2 \\ y^2 & (x+z)^2-y^2 & 0 \\ z^2 & 0 & (x+y)^2-z^2 \end{vmatrix}$$

Taking (x + y + z) common from c_2 and C_3

$$\Delta = (x+y+z)^2\begin{vmatrix} (y+z)^2 & x-y-z & x-y-z \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

$$R_1 \rightarrow R_1 - (R_2 + R_3)$$

$$\Delta = (x+y+z)^2 \begin{vmatrix} 2yz & -2z & -2y \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

$$C_2 \rightarrow C_2 + \frac{1}{y}C_1 \quad and \quad C_3 \rightarrow C_3 + \frac{1}{z}C_1$$

$$\Delta = (x+y+z)^2 \begin{vmatrix} 2yz & 0 & 0 \\ y^2 & x+z & \frac{y^2}{z} \\ z^2 & \frac{z^2}{y} & x+y \end{vmatrix}$$

Expending along R_1

$$= (x+y+z)^3(2xyz)$$

---

3. Find the equation of line joining (3, 1) and (9, 3) using determinants.
Ans. Let (x, y) be any point on the line containing (3, 1) and (9, 3)

$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

x-3y=0

---

4. If

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

then verify that $(AB)^{-1} = B^{-1} A^{-1}$
Ans.

$$AB = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$$
$$|AB| = -11 \neq 0$$

$$(AB)^{-1} = \frac{1}{11} adj(AB)$$
$$= \frac{-1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$
$$|A| = -11 \neq 0, \quad |B| = 1 \neq 0$$

$$A^{-1} = \frac{-1}{11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$$
$$B^{-1} = \frac{1}{1}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{-1}{11}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{-11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$

Hence prove.

---

5. Using cofactors of elements of third column, evaluate

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

Ans.

$$\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$

$$= yz(z-y) + zx(x-z) + xy(y-x)$$
$$= yz^2 - y^2z + zx^2 - z^2x + xy^2 - x^2y$$

$$= zx^2 - x^2y + xy^2 - z^2x + yz^2 - y^2z$$
$$= x^2(z-y) + x(y^2 - z^2) + yz(z-y)$$

$$= (z-y)[x^2 + x(z+y) + yz]$$
$$= (z-y)[x^2 - xz - xy + yz]$$

$$= (z - y[x(x-y) - z(x-y)]$$
$$= (z-y)[(x-y)(x-z)]$$

$$= (z-y)(x-y)(x-z)$$

---

6. If

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

find A^{-1}, using A^{-1} solve the system of equations
2x – 3y + 5z = 11
3x + 2y – 4z = -5
x + y -2z = -3
Ans.

$$|A| = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$(A) = -1 \neq 0$$
$$A^{-1} exists$$

$$A^{-1} = \frac{1}{|A|}(adjA)$$

$$= \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equation can be written is Ax = B, X = A^{-1}B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 5 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x = 1$$
$$y = 2$$
$$z = 3$$

---

7. Show that, using properties of determinants.

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

Ans. $R_1 \to R_1 + b.R_3$

$$L.H.S = \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Taking common (1 + a^2 + b^2) from R_1

$$= 1+a^2+b^2 \begin{vmatrix} 1 & 0 & -b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$R_1 \to R_1 - a.R_3$

$$= 1+a^2+b^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1+a^2+b^2 & a\left(1+a^2+b^2\right) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Taking (1 + a^2 + b^2) common from R_2

$$= 1+a^2+b^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Expending entry R_1

$$= (1+a^2+b^2)^2[1(1-a^2-b^2+2a^2)-b(-2b)]$$
$$= (1+a^2+b^2)^2[1+a^2-b^2+2b^2]$$

$$= (1+a^2+b^2)^2(1+a^2+b^2)$$
$$= (1+a^2+b^2)^3$$

8.

$$\Delta = \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = \left(1-x^2\right)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

Ans.

$$L.H.S = R_1 \to R_1 - xR_2$$

$$\Delta = \begin{vmatrix} a(1-x^2) & c(1-x^2) & p(1-x^2) \\ ax+b & cx+d & bx+q \\ u & v & w \end{vmatrix}$$

$$\Delta = (1-x^2)\begin{vmatrix} a & c & p \\ ax+b & cx+d & bx+q \\ u & v & w \end{vmatrix}$$

$R_2 \to R_2 - xR_1$

$$\Delta = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

9.

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

Verify that

$$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$$

Ans.

$$a_{11} = 2, \qquad a_{12} = -3, \qquad a_{13} = 5$$

$$A_{31} = -12, \qquad A_{32} = 22, \qquad A_{33} = 18$$

$$L.H.S = a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$$

=2 (-12) + (-3) (22) +5 (18)
= 0 Hence prove.

10.If $A=\begin{bmatrix}3 & -4\\ -1 & 2\end{bmatrix}$ **, find matrix B such that AB = I**

Ans. $|A|=2\neq 0$

Therefore A^{-1} exists

AB = I

A^{-1} AB = A^{-1}I

B = A^{-1}

$$adj\ A=\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(adj A)$$

$$=\frac{1}{2}\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ \frac{1}{2} & \frac{3}{2}\end{bmatrix}$$

$$Hence\ \ B=\begin{bmatrix}1 & 2\\ \frac{1}{2} & \frac{3}{2}\end{bmatrix}$$

---

11. Using matrices solve the following system of equation

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$

$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$

$$\frac{6}{x}+\frac{9}{y}+\frac{-20}{z}=2$$

Ans. Let

$$\frac{1}{x}=4, \quad \frac{1}{y}=v, \quad \frac{1}{z}=w$$

24 + 3v + 10v = 4
44 – 64 + 5w = 1
64 + 9v – 20w = 2

$$A=\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix} \quad y=\begin{bmatrix}y\\ v\\ w\end{bmatrix} \quad B=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$$|A|=1200\neq 0$$

$$aiJA=\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

$$y=A^{-1}B$$

$$=\frac{1}{1200}\begin{bmatrix}600\\ 400\\ 240\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{5}\end{bmatrix}$$

$$\begin{bmatrix} y \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$u = \frac{1}{2}, \qquad v = \frac{1}{3}, \qquad w = \frac{1}{5}$$
$$\frac{1}{x} = \frac{1}{2}, \qquad \frac{1}{y} = \frac{1}{3}, \qquad \frac{1}{z} = \frac{1}{5}$$

$$x = 2, \qquad y = 3 \qquad z = 5$$

12.Given

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} and\ B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

find AB and use this result in solving the following system of equation.

$$\begin{aligned} x - y + z &= 4 \\ x - 2y - 2z &= 9 \\ 2x + y + 3z &= 1 \end{aligned}$$

OR
Use product

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

To solve the system of equations.

x – y + 2z = 1
2y – 3z = 1
3x – 2y + 4z = 2
Ans. $x - y + z = 4$
$x - 2y - 2z = 9$
$2x + y + 3z = 1$
Let

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$AX = C$

$$AB = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$AB = 8I$

$$A^{-1} = \frac{1}{8} B \begin{bmatrix} \because A^{-1}AB = 8A^{-1}I \\ B = 8A^{-1} \end{bmatrix}$$

$AX = C$
$X = A^{-1}C$

$$= \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$x = 3, \quad y = -2, \quad z = -1$$

OR

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

x = 0 y = 5 z = 3

13. If a, b, c is in A.P, and then finds the value of

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

Ans. $R_1 \to R_1 + R_3$

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+2a+2c \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+4b \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \quad [2b = a + c]$$

$R_1 \to R_1 - 2R_2$

$$= \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$= 0$

14. $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$,

Find the no. a and b such that $A^2 + aA + bI = 0$ Hence find A^{-1}

Ans. $A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

$$A^2 + aA + bI = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix}$$

$$ATQ \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

a = -4, b =1
$A^2 - 4A + I = 0$
$A^2 - 4A = -I$
$AAA^{-1} - 4AA^{-1} = -IA^{-1}$
$A - 4I = -A^{-1}$
$A^{-1} = 4I - A$
= $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

---

15. Find the area of Δ whose vertices are (3, 8) (-4, 2) and (5, 1)
Ans.

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left[3(2-1)-8(-4-5)+1(-4-10)\right]$$

$$=\frac{1}{2}\left[3+72-14\right]=\frac{61}{2}$$

---

16. Evaluate

$$\Delta = \begin{bmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{bmatrix}$$

Ans.

$$\Delta = 0\begin{vmatrix} 0 & \sin\beta \\ -\sin\beta & 0 \end{vmatrix} - \sin\alpha\begin{vmatrix} -\sin\alpha & \sin\beta \\ \cos\alpha & 0 \end{vmatrix} - \cos\alpha\begin{vmatrix} -\sin\alpha & 0 \\ \cos\alpha & -\sin\beta \end{vmatrix}$$

$= 0$

---

17. Solve by matrix method
x – y + z = 4
2x + y – 3 z = 0
x + y + z = 2
Ans.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$= 10 \neq 0$

$$AdJ\ \ A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\left(adJ\ A\right)$$

$$=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$$

System of equation can be written is

$X=A^{-1}B$

$$=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix},$$

$$x=2,\quad y=-1,\quad z=1$$

---

18. Show that using properties of det.

$$\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$=abc+bc+ca+ab$$

Ans. Taking a, b, c common from R_1, R_2 and R_3

$$=abc\begin{vmatrix}\frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a}\\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b}\\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1\end{vmatrix}$$

$$R_1 \to R_1 + R_2 + R_3$$

$$= abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$$C_1 \to C_1 - C_3, \quad C_2 \to C_2 - C_3$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & \frac{1}{b} \\ -1 & -1 & \frac{1}{c}+1 \end{vmatrix}$$

Expending along R_1

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)[1]$$

$$= abc + bc + ac + bc$$

---

19. If x, y, z are different and

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

then show that 1 + xyz = 0 ans.

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

Ans:

$$\Delta = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \Delta = \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + |xyz| \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 0 & y\text{-}x & y^2-x^2 \\ 0 & z\text{-}x & z^2-x^2 \end{vmatrix} \begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$$

$$=(1+xyz)(y-x)(z-x)\begin{vmatrix}1 & x & x^2\\ 0 & 1 & y+x\\ 0 & 1 & z+x\end{vmatrix}$$

$$=(1+xyz)(y-x)(z-x)(z-y)$$
$$\Delta = 0(given)$$

x, y, z all are different

$$x-y\neq 0, \quad y-z\neq 0, \quad z-x\neq 0$$
$$\therefore 1+xyz=0$$

---

20. Find the equation of the line joining A (1, 30 and B (0, 0) using det. Find K if D (K, 0) is a point such then area of Δ ABC is 3 square unit
Ans. Let P (x, y) be any point on AB. Then area of Δ ABP is zero

$$\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 1 & 3 & 1\\ x & y & 1\end{vmatrix}=0$$

$y=3x$
Area Δ ABD =3 square unit

$$\frac{1}{2}\begin{vmatrix}1 & 3 & 1\\ 0 & 0 & 1\\ K & 0 & 1\end{vmatrix}=\pm 3$$
$$k=\pm 2$$

---

21. Show that the matrix $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$. Using this equation, find A^{-1}
Ans.

$$A^2=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}.\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$= 4\begin{bmatrix} 7 & 12 \\ 1 & 7 \end{bmatrix}$$

$$A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 1 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= 0$$

$$A^2 - 4A + I = 0$$
$$A^2 - 4A = -I$$

$$AAA^{-1} - 4AA^{-1} = -IA^{-1}$$
$$AI - 4I = -IA^{-1}\left[\because AA^{-1} = I\right]$$

$$A^{-1} = 4I - A$$
$$= \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

---

22. Solve by matrix method.
3x – 2y + 3z = 8
2x + y – z = 1
4x – 3y + 2z = 4
Ans. The system of equation be written in the form AX = B, whose

$$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$|A| = -17 \neq 0$

$$A^{-1} = \frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$X = A^{-1}B$

$$= \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x = 1, \quad y = 2, \quad z = 3$$

---

23. The sum of three no. is 6. If we multiply third no. by 3 and add second no. to it, we get II. By adding first and third no. we get double of the second no. represent it algebraically and find the no. using matrix method.

Ans. I = x II = y II = z

x + y + z = 6

y + 3z = 11

x + z = 2y

This system can be written as AX = B whose

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$|A| = 9 \neq 0$

$$A_{11} = 7, \quad A_{12} = 3, \quad A_{13} = -1$$
$$A_{21} = -3, \quad A_{22} = 0, \quad A_{23} = 3$$
$$A_{31} = 2, \quad A_{32} = -3, \quad A_{33} = 1$$

$$aiJ\ A = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} aiJ\ A = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x = 1, \quad y = 2, \quad z = 3$$

24.

$$\begin{vmatrix} \alpha & \alpha^2 & \beta-\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$$

Ans.

$$R_1 \to R_1 - R_3, \quad R_2 \to R_2 - R_3$$

$$L.H.S = \begin{vmatrix} \alpha-\gamma & \alpha^2-\gamma^2 & \beta+\gamma-\alpha-\beta \\ \beta-\gamma & \beta^2-\gamma^2 & \gamma+\alpha-\alpha-\beta \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$$= \begin{vmatrix} \alpha-\gamma & (\alpha+\gamma) & (\gamma-\alpha) \\ \beta-\gamma & (\beta-\gamma)(\beta+\gamma) & \gamma-\beta \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$$= (\alpha-\gamma)(\beta-\gamma)\begin{vmatrix} 1 & \alpha+\gamma & -1 \\ 1 & \beta+\gamma & 1 \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$= (\alpha-\gamma)(\beta-\gamma)\begin{vmatrix} 0 & \alpha-\beta & 0 \\ 1 & \beta+\gamma & -1 \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

Expending along R_1

$$= (\alpha-\gamma)(\beta-\gamma)\left[-(\alpha-\beta)(\alpha+\beta+\gamma)\right]$$
$$= (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$$

---

25. Find values of K if area of triangle is 35 square. Unit and vertices are (2, -6), (5, 4), (K, 4)

Ans.

$$area\ \Delta = \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ K & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(4-4)+6(5-K)+1(20-4K)\right]$$

$$\begin{aligned} &= \frac{1}{2}[50-10K] \\ &= 25-5K \end{aligned}$$

$$\begin{aligned} A+\theta \quad 25-5K &= 35 \\ K &= 12 \end{aligned}$$

---

26. Using cofactors of elements of second row, evaluate

$$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Ans.

$$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$

$$\begin{aligned} &= -2(9-16)+0(15-8)+1(10-3) \\ &= 14+0-7 \\ &= 7 \end{aligned}$$

---

27. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}

Ans.

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$A^2-5A+7I$

$$=\begin{bmatrix} 8 & 5 \\ -5 & 2 \end{bmatrix}-5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}+7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+50 & 3-10+7 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Prove.
$A^2 - 5A + 7I = 0$ (given)
$A^2 - 5A = -7I$
$A^2A^{-1} - 5AA^{-1} = -7IA^{-1}$
$AAA^{-1} - 5AA^{-1} = -7IA^{-1}$
$A - 5I = -7A^{-1}$ $\left[AA^{-1}=I\right]$
$7A^{-1} = 5I - A$

$$=5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$=\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$$A^{-1}=\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

---

28. The cost of 4kg onion, 3kg wheat and 2kg rice is Rs. 60. The cost of 2kg onion, 4kg wheat and 6kg rice is Rs. 90. The cost of 6kg onion 2kg wheat and 3kg rice is Rs. 70. Find the cost of each item per kg by matrix method.
Ans. cost of 1kg onion = x
cost of 1kg wheat = y
cost of 1kg rise = z
4x + 3y + 2z = 60
2x + 4y + 6z = 90

6x + 2y + 3z = 70

$$A=\begin{bmatrix}4 & 3 & 2\\ 2 & 4 & 6\\ 6 & 2 & 3\end{bmatrix} B=\begin{bmatrix}60\\ 90\\ 70\end{bmatrix} X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$|A|=\begin{vmatrix}4 & 3 & 2\\ 2 & 4 & 6\\ 6 & 2 & 3\end{vmatrix}=50\neq 0$$

$$aiJ\ A=\begin{bmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\left(aiJ\ A\right)=\frac{1}{80}\begin{bmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{bmatrix}$$

$X=A^{-1}B$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5\\ 8\\ 8\end{bmatrix}$$

$$x=5,\quad y=8,\quad z=8$$