Class 12 Maths Chapter 6 Application of Derivatives Important Questions

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. The side of a square is increasing at the rate of 0.2 cm/sec$0.2cm\ /\ sec$. Find the rate of increase of the perimeter of the square.

Ans: It is given that the side of a square is increasing at the rate of 0.2 cm/sec$0.2\mathrm{cm}\ /\ \mathrm{sec}$.

Let us consider the edge of the given cube be x cm$x\mathrm{cm}$ at any instant.

According to the question,

The rate of side of the square increasing is,

dxdt=0.2 cm/sec…$\frac{\mathrm{dx}}{dt} = 0.2\mathrm{cm}\ /\ \mathrm{sec}$ ... ...(i)

Therefore the perimeter of the square at any time tt will be,

P=4x cm$P = 4x\mathrm{cm}$

By applying derivative with respect to time on both sides, we get

⇒dPdt=d(4x)dt $\Rightarrow \frac{dP}{dt} = \frac{d\ (\ 4x\ )}{dt}$

⇒dPdt=4dxdt $\Rightarrow \frac{dP}{dt} = 4\frac{dx}{dt}$

⇒dPdt=4×0.2=0.8 cm/sec $\Rightarrow \frac{dP}{dt} = 4 \times 0.2 = 0.8\mathrm{cm}/\mathrm{sec}$

Hence from equation (i). The rate at which the perimeter of the square will increase is 0.8cm/sec$0.8\mathrm{cm}/\mathrm{sec}$.

2. The radius of the circle is increasing at the rate of 0.7 cm/sec$0.7cm\ /\ sec$. What is the rate of increase of its circumference?

Ans: It is given that the radius of a circle is increasing at the rate of 0.7 cm/sec$0.7\mathrm{cm}\ /\ \mathrm{sec}$.

Let us consider that the radius of the given circle be r cmcm at any instant.

According to the question,

The rate of radius of a circle is increasing as,

drdt=0.7 cm/sec$\frac{\text{dr}}{dt} = 0.7\text{cm} / \sec$ ...(i)

Now the circumference of the circle at any time tt will be,

C=2πcm$\text{C} = 2\pi\text{cm}$

By applying derivative with respect to time on both sides, we get

⇒dCdt=d(2πr)dt $\Rightarrow \frac{dC}{dt} = \frac{d\ (\ 2\pi r\ )}{dt}$

⇒dCdt=2πdrdt $\Rightarrow \frac{dC}{dt} = 2\pi\frac{dr}{dt}$

⇒dCdt=2π×0.7=1.4πcm/sec $\Rightarrow \frac{dC}{dt} = 2\pi \times 0.7 = 1.4\pi\text{cm} / \sec$

From the equation (i). We can conclude that the rate at which the circumference of the circle will be increasing is 1.4πcm/sec$1.4\pi cm / \sec$

3. If the radius of a soap bubble is increasing at the rate of 12 cm/sec$\frac{1}{2}cm / sec$. At what rate its volume is increasing when the radius is 1 cm$1cm$.

Ans: It is given that the radius of an air bubble is increasing at the rate of 0.5 cm/sec $0.5\text{cm} / \sec$.

Let us consider that the radius of the given air bubble be r cmcm and let VV be the volume of the air bubble at any instant.

According to the question,

The rate at which the radius of the bubble is increasing is,

drdt=0.5 cm/sec$\frac{dr}{dt} = 0.5\text{cm}/\sec$ ... (i)

The volume of the bubble, i.e., volume of sphere is V=43πr3$V = \frac{4}{3}\pi r^3$

By applying derivative with respect to time on both sides,

⇒dVdt=d(43πr3)dt ⇒ $\frac{dV}{dt} = \frac{d\left(\frac{4}{3}\pi r^3\right)}{dt}$

⇒dVdt=43πd(r3)dt ⇒ $\frac{dV}{dt} = \frac{4}{3}\pi \frac{d\left(r^3\right)}{dt}$

⇒dVdt=43π×3r2drdt ⇒ $\frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$

⇒dVdt=4πr2×0.5...(ii) ⇒ $\frac{dV}{dt} = 4\pi r^2 \times 0.5 \ldots (ii)$

When the radius is 1 cm$1\mathrm{cm}$,

The above equation becomes

⇒dVdt=4π×(1)2×0.5 ⇒ $\frac{\mathrm{dV}}{\mathrm{dt}} = 4\pi \times (1)^2 \times 0.5$

⇒dVdt=2πcm3/sec ⇒ $\frac{\mathrm{dV}}{\mathrm{dt}} = 2\pi \mathrm{cm}^3/\mathrm{sec}$

Hence the volume of air bubble is increasing at the rate of 2πcm3/sec$2\pi \mathrm{cm}^3/\mathrm{sec}$.

4. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/sec $4cm / sec$. At the instant when the radius of the circular wave is

10 cm

$$10cm$$

, how fast is the enclosed area increasing?
Ans: It is given that when a stone is dropped into a quiet lake and waves are formed which moves in circles at a speed of 4 cm/sec$4\mathrm{cm} / \mathrm{sec}$.

Let us consider that, r be the radius of the circle and AA be the area of the circle.

When a stone is dropped into the lake, waves are formed which move in a circle at speed of 4 cm/sec$4\mathrm{cm} / \mathrm{sec}$.

Thus, we can say that the radius of the circle increases at a rate of,

drdt=4 cm/sec$\frac{\mathrm{dr}}{dt} = 4\mathrm{cm/sec}$

Area of the circle is πr2πr^2, therefore

⇒dAdt=d(πr2)dt ⇒ $\frac{\mathrm{dA}}{\mathrm{dt}} = \frac{\mathrm{d}(\pi \mathrm{r}^2)}{\mathrm{dt}}$

⇒dAdt=πd(r2)dt ⇒ $\frac{\mathrm{dA}}{\mathrm{dt}} = \pi\frac{\mathrm{d}(\mathrm{r}^2)}{\mathrm{dt}}$

⇒dAdt=π×2rdrdt ⇒ $\frac{\mathrm{dA}}{\mathrm{dt}} = \pi \times 2\mathrm{r}\frac{\mathrm{dr}}{\mathrm{dt}}$

⇒dAdt=2πr×4……. (ii) ⇒ $\frac{\mathrm{dA}}{\mathrm{dt}} = 2\pi \mathrm{r} \times 4 \ldots\ \ldots\ .\ (\mathrm{ii})$

Hence, when the radius of the circular wave is 10 cm$10\mathrm{cm}$, the above equation becomes

⇒dAdt=2π×10×4 ⇒ $\frac{\mathrm{dA}}{dt} = 2\pi \times 10 \times 4$

⇒dAdt=80πcm2/sec ⇒ $\frac{\mathrm{dA}}{dt} = 80\pi \mathrm{cm}^2/\mathrm{sec}$

Thus, the enclosed area is increasing at the rate of 80πcm2/sec$80\pi \mathrm{cm}^2/\mathrm{sec}$.

5. The total revenue in Rupees received from the sale of xx units of a product is given by, R(x)=13x2+26x+15$R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when x=7$x = 7$.

Ans: Marginal revenue is the rate of change of total revenue with respect to the number of units sold.

Let us consider 'MR' to be the marginal revenue, therefore

MR=dRdx$MR = \frac{dR}{dx}$

It is given that,

Total revenue, i.e., R(x)=13x2+26x+15$R(x) = 13x^2 + 26x + 15\ \ldots(1)$

We need to find marginal revenue when x=7$x = 7$

i.e., MR when x=7$x = 7$

⇒MR=d(R(x))dx ⇒ $\mathrm{MR} = \frac{d\,(\,R\,(\,x\,)\,\,)}{dx}$

⇒MR=d(13x2+26x+15)dx ⇒ $\mathrm{MR} = \frac{d\,(\,13x^2 + 26x + 15\,)}{dx}$

⇒MR=d(13x2)dx+d(26x)dx+d(15)dx ⇒ $\mathrm{MR} = \frac{d\,(\,13x^2\,)}{dx} + \frac{d\,(\,26x\,)}{dx} + \frac{d\,(\,15\,)}{dx}$

⇒MR=13d(x2)dx+26d(x)dx+0 ⇒ $\mathrm{MR} = 13\frac{d\,(\,x^2\,)}{dx} + 26\frac{d\,(\,x\,)}{dx} + 0$

⇒MR=13×2x+26 ⇒ $\mathrm{MR} = 13 \times 2x + 26$

⇒MR=26x+26 ⇒ $\mathrm{MR} = 26x + 26$

⇒MR=26(x+1) ⇒ $MR = 26\,(\,x + 1\,)$

Taking x=7$x = 7$, we get

⇒MR=26(7+1) ⇒ $MR = 26\,(\,7 + 1\,)$

⇒MR=26×8 ⇒ $MR = 26 \times 8$

⇒MR=208 ⇒ $MR = 208$

Therefore, the required marginal revenue is Rs208$\mathrm{Rs}208$.

6. Find the maximum and minimum values of function f(x)=sin2x+5$f\,(\,x\,)\, = sin2x + 5$.

Ans: Given function is,

f(x)=sin2x+5$f\,(\,x\,)\, = \sin\,\,2x + 5$

We know that,

−1≤sinθ≤1,∀θ∈R $-\,1 \le \sin\,\,\theta \le 1$, $\forall\theta \in R$

−1≤sin2x≤1 $-\,1 \le \sin\,\,2x \le 1$

Adding 5 on both sides,

−1+5≤sin2x+5≤1+5 $-1+5 \leq \sin\ 2x+5 \leq 1+5$

4≤sin2x+5≤6 $4 \leq \sin\ 2x+5 \leq 6$

Therefore,

Max value of f(x)=sin2x+5 $f(x) = \sin\ 2x+5$ will be 6 and,

Min value of f(x)=sin2x+5 $f(x) = \sin\ 2x+5$ will be 4.

7. Find the maximum and minimum values (if any) of the function

f(x)=−|x−1|+7∀x∈R $f(x) = -|x-1| + 7 \forall x \in R$

Ans: Given equation is f(x)=−|x+1|+3 $f(x) = -|x+1| + 3$

|x+1|>0 $|x+1| > 0$

⇒−|x+1|<0 $\Rightarrow -|x+1| < 0$

Maximum value of g(x)= $g(\mathrm{x}) =$ maximum value of −|x+1|+7 $-|\mathrm{x}+1| + 7$

⇒0+7=7 $\Rightarrow 0+7=7$

Maximum value of f(x)=3 $f(x) = 3$

There is no minimum value of f(x) $f(x)$.

8. Find the value of a a for which the function f(x)=x2−2ax+6,x>0 $f(x) = x^2 - 2ax + 6, x > 0$ is strictly increasing.

Ans: Given function is f(x)=x2−2ax+6,x>0 $f(x) = x^2 - 2ax + 6, x > 0$

It will be strictly increasing when f′(x)>0 $f'(x) > 0$.

f′(x)=2x−2a>0 $f'(x) = 2x - 2a > 0$

⇒2(x−a)>0 $\Rightarrow 2(x-a) > 0$

⇒x−a>0 $\Rightarrow x - a > 0$

⇒a<x ⇒ $a < x$

But x>0$x > 0$

Therefore, the maximum possible value of aa is 0 and all other values of a will be less than 0.

Hence, we get a≤0$a \leq 0$.

9. Write the interval for which the function f(x)=cosx,0£x£2π$f(x) = cosx, 0£x£2\pi$ is decreasing.

Ans: The given function is f(x)=cosx,0≤x≤2π$f(x) = \cos x, 0 \leq x \leq 2\pi$.

It will be a strictly decreasing function when f′(x)<0$f'(x) < 0$.

Differentiating w.r.t. xx, we get

f′(x)=−sinx$f'(x) = -\sin x$

Now,$\mathrm{Now},$

f′(x)<0$f'(x) < 0$

⇒−sinx<0 ⇒ $-\sin x < 0$

⇒sinx>0i.e.,(0,π) ⇒ $\sin x > 0 \mathrm{i.e.}, (0, \pi)$

Hence, the given function is decreasing in (0,π) $(0, \pi)$.

10. What is the interval on which the function f(x)=logxx,xI^(0,¥)$f(x) = \frac{logx}{x}, x\hat{I}(0, ¥)$ is increasing?

Ans: The given function is f(x)=logxx,x∈(0,∞)$f(x) = \frac{\log x}{x}, x \in (0, \infty)$.

It will be a strictly increasing function when f′(x)>0$f'(x) > 0$.

f(x)=logxx$f(x) = \frac{\log x}{x}$

Therefore,

$f'(x) = \frac{1}{x^2} - \frac{\log x}{x^2}$

$f'(x) = \frac{1 - \log x}{x^2}$

$\because f'(x) > 0$

$\Rightarrow \frac{1 - \log x^2}{x^2} > 0$

$\Rightarrow 1 - \log x > 0$

$\Rightarrow 1 > \log x$

$\Rightarrow e > x$

Therefore, $f(x)$ is increasing in the interval $(0, e)$.

11. For which values of x, the functions $y = x^4 - \frac{4}{3}x^3$ is increasing?

Ans: The given function is $y = x^4 - \frac{4}{3}x^3$

It will be a strictly increasing function when $f'(x) > 0$.

$f'(x) > 0$ and,

$f'(x) = 4x^3 - 4x^2$

$f'(x) = 4x^2(x - 1)$

$4x^2(x - 1) > 0$

Now,

$\frac{dy}{dx} = 0 \Rightarrow x = 0, x = 1$

Since f′(x)<0∀x∈(−∞,0)∪(0,1)$f'(x) < 0 \forall x \in (-\infty, 0) \cup (0, 1)$ and ff is continuous in (−∞,0]$(-\infty, 0]$ and [0,1]$[0, 1]$. Therefore ff is decreasing in (−∞,1]$(-\infty, 1]$ and ff is increasing in [1,∞)$[1, \infty)$.

Here ff is strictly decreasing in (−∞,0)∪(0,1)$(-\infty, 0) \cup (0, 1)$ and is strictly increasing in (1,∞)$(1, \infty)$.

12. Write the interval for which the function f(x)=1x$f(x) = \frac{1}{x}$ is strictly decreasing.

Ans: The given equation is f(x)=1x$f(x) = \frac{1}{x}$.

It will be a strictly decreasing function when f′(x)<0$f'(x) < 0$.

f(x)=x+1x$f(x) = x + \frac{1}{x}$

⇒f′(x)=1−1x2 ⇒ $f'(x) = 1 - \frac{1}{x^2}$

⇒f′(x)=x2−1x2 ⇒ $f'(x) = \frac{x^2 - 1}{x^2}$

⇒f′(x)=0 ⇒ $f'(x) = 0$

⇒x2−1x2=0 ⇒ $\frac{x^2 - 1}{x^2} = 0$

⇒x2−1=0 ⇒ $x^2 - 1 = 0$

⇒x=±1 ⇒ $x = \pm 1$

The intervals are (−∞,−1),(−1,1),(1,∞) $(-\infty, -1), (-1, 1), (1, \infty)$

f′(0)<0$f'(0) < 0$

∴ $\therefore$ Strictly decreasing in (−1,1) $(-1, 1)$

13. Find the sub-interval of the interval (0,π/2) $(0, \pi / 2)$ in which the function f(x)=sin3x $f(x) = sin3x$ is increasing.

Ans: The given function is f(x)=sin3x$f(x) = \sin 3x$

On differentiating the above function with respect to xx, we get,

f′(x)=3cos3x$f'(x) = 3\cos 3x$

f(x)$f(x)$ will be increasing, when f′(x)>0$f'(x) > 0$

Given that x∈(0,π2)$x \in (0, \frac{\pi}{2})$

⇒3x∈(0,3π2) ⇒ $3x \in (0, \frac{3\pi}{2})$

Cosine function is positive in the first quadrant and negative in the second quadrant.

case 1:

When 3x∈(0,π2)$3x \in (0, \frac{\pi}{2})$

⇒cos3x>0 ⇒ $\cos 3x > 0$

⇒3cos3x>0 ⇒ $3\cos 3x > 0$

⇒f′(x)>0 for 0<3x<π2 ⇒ $f'(x) > 0 \text{ for } 0 < 3x < \frac{\pi}{2}$

⇒f′(x)>0 for 0<x<π6 ⇒ $f'(x) > 0 \text{ for } 0 < x < \frac{\pi}{6}$

∴f(x) $\therefore f(x)$ is increasing in the interval (0,π6) $(0, \frac{\pi}{6})$

case 2:

When 3x∈(π2,3π2)$3x \in (\frac{\pi}{2}, \frac{3\pi}{2})$

⇒cos3x<0 ⇒ $\cos 3x < 0$

⇒3cos3x<0 ⇒ $3\cos 3x < 0$

⇒f′(x)<0 ⇒ $f'(x) < 0$ for π2<3x<3π2 $\frac{\pi}{2} < 3x < \frac{3\pi}{2}$

⇒f′(x)<0 ⇒ $f'(x) < 0$ for π6<x<π2 $\frac{\pi}{6} < x < \frac{\pi}{2}$

∴f(x) $\therefore f(x)$ is decreasing in the interval (π6,π2) $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

14. Without using derivatives, find the maximum and minimum value of y=|3sinx+1| $y = |3sinx + 1|$.

Ans: The given function is y=|3sinx+1| $y = |3\sin x + 1|$

Maximum and minimum values of sinx={−1,1} $\sin x = \{-1, 1\}$ respectively.

Therefore, the value of the given function will be maximum and minimum at only these points.

Taking sinx=−1 $\sin x = -1$

y=|3×(−1)+1|=>2 $y = |3 \times (-1) + 1| => 2$

Now, put sinx=1 $\sin x = 1$

y=|3×1+1|=>4 $y = |3 \times 1 + 1| => 4$

The maximum and minimum values of the given function are 4 and 2 respectively.

15. If f(x)=ax+cosx $f(x) = ax + cosx$ is strictly increasing on R R, find a a.

Ans: It is given that the function f(x)=ax+cosx $f(x) = ax + \cos x$ is strictly increasing on R R

Here function, f(x)=ax+cosx $f(x) = ax + \cos x$

Differentiating f(x) $f(x)$ with respect to x x we get,

f′(x)=a+(−sinx)=a−sinx $f'(x) = a + (-\sin x) = a - \sin x$

for strictly increasing, f′(x)>0 $f'(x) > 0$

Therefore,

a−sinx>0$a - \sin\ x > 0$ it will be correct for all real value of xx only when a∈(−1,1) $a \in (-1, 1)$

Hence the value of a belongs to (−1,1) $(-1, 1)$.

16. Write the interval in which the function f(x)=x9+3x7+64$f(x) = x^9 + 3x^7 + 64$ is increasing.

Ans: The given function is f(x)=x9+3x7+64$f(x) = x^9 + 3x^7 + 64$.

For it to be a increasing function f′(x)>0$f'(x) > 0$

On differentiating both sides with respect to x, we get

f(x)=x9+3x7+64$\mathrm{f}(\mathrm{x}) = \mathrm{x}^9 + 3\mathrm{x}^7 + 64$

⇒f′(x)=9x8+21x6 $\Rightarrow \mathrm{f}'(\mathrm{x}) = 9\mathrm{x}^8 + 21\mathrm{x}^6$

⇒f′(x)=3x6(3x2+7) $\Rightarrow f'(x) = 3\mathrm{x}^6(3\mathrm{x}^2 + 7)$

∵ $\because$ function is increasing.

3x6(3x2+7)>0$3x^6(3x^2 + 7) > 0$

⇒ $\Rightarrow$ function is increasing on R$\mathbb{R}$.

17. What is the slope of the tangent to the curve f(x)=x3−5x+3$f(x) = x^3 - 5x + 3$ at the point whose xx co-ordinate is 2?

Ans: The given equation of the curve is f=x3−5x+3$f = x^3 - 5x + 3$ …(1)

When x=2$x = 2$,

y=23−5.2+3$y = 2^3 - 5.2 + 3$

y=8−10+3$y = 8 - 10 + 3$

y=1$y = 1$

Therefore, the point on the curve is (2,1) $(2, 1)$.

Differentiating equation (1) with respect to x, we get

dydx=3x2−5$\frac{dy}{dx} = 3x^2 - 5$

Slope of tangent dydx$\frac{dy}{dx}$

Since x=2$x = 2$,

⇒3.22−5 ⇒ $3.2^2 - 5$

⇒12−5 ⇒ $12 - 5$

⇒7 ⇒ 7

Hence the slope of tangent is 7.

18. At what point on the curve

y=x2

$$y = x^2$$

does the tangent make an angle of 45∘45° with positive direction of the xx-axis?
Ans: The given equation of the curve is y=x2$y = x^2$

Differentiating the above with respect to xx,

⇒dydx=2x2−1 ⇒ $\frac{dy}{dx} = 2x^{2-1}$

⇒dydx=2x...(1) ⇒ $\frac{dy}{dx} = 2x \ldots (1)$

So,

dydx=$\frac{dy}{dx} =$ The slope of tangent =tanθ $= \tan\theta$

The tangent makes an angle of 45∘45° with xx-axis dydx=tan45∘=1…(2)$\frac{dy}{dx} = \tan\ 45^{\circ} = 1\ \ldots\ (2)$

Because the tan45∘=1$\tan\ 45^{\circ} = 1$

From the equation (1) $ (2), we get

⇒2x=1 ⇒ $2x = 1$

⇒x=12 ⇒ $x = \frac{1}{2}$

Substitute x=12$x = \frac{1}{2}$ in y=x2$y = x^{2}$

⇒y=(12)2 ⇒ $y = \left(\frac{1}{2}\right)^{2}$

⇒y=14 ⇒ $y = \frac{1}{4}$

Hence, the required point is (12,14) $\left(\frac{1}{2}, \frac{1}{4}\right)$.

19. Find the point on the curve y=3x2−12x+9$y = 3x^{2} - 12x + 9$ at which the tangent is parallel to xx-axis.

Ans: The given equation of the curve is y=3x2−12x+9$y = 3x^{2} - 12x + 9$.

Differentiating the above equation with respect to x, we get

dydx=6x−12$\frac{dy}{dx} = 6x - 12$

m=6x−12$m = 6x - 12$

dydx=$\frac{dy}{dx} =$ The slope of tangent =tanθ $= \tan\ \theta$

If the tangent is parallel to x-axis.

m=0$m = 0$

⇒6x−12=0 ⇒ $6x - 12 = 0$

⇒x=2 ⇒ $x = 2$

When x=2$x = 2$, then

y=3.22−12.2+9$y = 3.2^2 - 12.2 + 9$

y=12−24+9$y = 12 - 24 + 9$

y=−3$y = -3$

Hence, the required point (x,y)=(2,−3) $(x, y) = (2, -3)$.

20. What is the slope of the normal to the curve y=5x2−4sinx$y = 5x^2 - 4sinx$ at x=0$x = 0$.

Ans: The given equation of the curve is y=5x2−4sinx$y = 5x^2 - 4\sin\ x$.

Differentiating the above equation with respect to x, we get

dydx=10x−4cosx$\frac{dy}{dx} = 10x - 4\cos\ x$

dydx=$\frac{dy}{dx} =$ The slope of tangent =tanθ $= \tan\ \theta$

Thus, slope of tangent at x=0$x = 0$ is,

⇒10×0−4cos0 ⇒ $10 \times 0 - 4\cos\ 0$

⇒0−4=4 ⇒ $0 - 4 = 4$

Hence, slope of normal at the same point is,

∵m1×m2=−1 $\because m_1 \times m_2 = -1$

⇒4×m2=−1 ⇒ $4 \times m_2 = -1$

⇒m2=−14 ⇒ $m_2 = \frac{-1}{4}$