Important Questions Class 12 Maths Chapter 7 Integrals

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1 Mark Questions

1. $\int \frac{x^3-1}{x^2}dx$

Ans.

$$\int \frac{x^3-1}{x^2}dx = \int\left(\frac{x^3}{x^2}-\frac{1}{x^2}\right)dx$$

$$= \int\left(x - x^{-2}\right)dx$$

$= \frac{x^2}{2} - \frac{x^{-1}}{-1} + c$

$= \frac{x^2}{2} + \frac{1}{x} + c$

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2. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$

Ans. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx = \int \frac{1}{\cos^2 x} \times \frac{\sin^2 x}{1}dx$$

$$\int \tan^2 x\, dx = \int\left(\sec^2 x - 1\right)dx$$

$= \tan x - x + c$

3.

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

Ans.

$$\int \frac{x^2(x-1) + 1(x-1)}{(x-1)} dx$$

$$= \int \frac{(x-1)(x^2+1)}{(x-1)} dx$$

$$= \frac{x^3}{3} + x + c$$

4.

$$\int \frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} dx$$

Ans. Put $\tan \sqrt{x} = t$

$$\sec^2 \sqrt{x}. \frac{1}{2\sqrt{x}} dx = dt$$

$$\frac{\sec \sqrt{x}}{\sqrt{x}} dx = 2dt$$

$$= 2\int t^4 dt = 2\frac{t^5}{5} + c$$

$$= \frac{2}{5} \tan^5 \sqrt{x} + c$$

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5. Prove

$$\int \sec x dx = \log|\sec x + \tan x| + c$$

Ans.

$$\int \sec x \, dx = \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)} dx$$

$$Put \ \sec x + \tan x = t$$
$$(\sec x . \tan x . + \sec^2 x) dx = dt$$

$$\sec x(\tan x + \sec x) dx = dt$$
$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$
$$= \log|\sec x + \tan x| + c$$

6.

$$\int \cos^3 x . e^{\log \sin x} dx$$

Ans. $\because e^{\log \theta} = \theta$

$$\therefore e^{\log \sin x} = \sin x$$
$$= \int \cos^3 x . \sin x \, dx$$

$$put \ \cos x = t$$
$$-\sin x \, dx = dt$$
$$\sin x \, dx = -dt$$
$$= \int -t^3 dt$$

$$= -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$$

7.

$$\int f'(ax+b)[f(ax+b)]^x dx$$

Ans. Put $f(ax+b) = t$

$$f'(ax+b).a\ dx = dt$$
$$= \int \frac{1}{a} t^n dt = \frac{1}{a} . \frac{t^{n+1}}{n+1} + c$$

$$= \frac{1}{a} . \frac{[f(ax+b)]^{n+1}}{n+1} + c$$

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8. $\int_o^1 xe^x dx$

Ans.

$$\int \underset{I}{x} \underset{II}{e^x} dx = xe^x - \int 1.e^x dx$$

$$= xe^x - e^x + c$$
$$\int_o^1 xe^x dx = \left[xe^x - e^x \right]_o^1$$

$$= (1.e^1 - e^1) - (0.e^0 - e^0) \quad \left[\because e^0 = 1 \right]$$
$$= 0 - (0 - 1)$$

=1

9.

$$\int_{-1}^{1} \sin^5 x . \cos^4 x \, dx$$

Ans. Let

$$f(x) = \sin^5 x . \cos^4 x$$

$$f(-x) = \sin^5(-x) . \cos^4(-x)$$
$$= -\sin^5 x . \cos^4 x$$
$$= -f(x)$$

f is odd function

$$\therefore \int_{-1}^{1} \sin^5 x . \cos^4 x \, dx = 0$$

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10. $\int \frac{dx}{x + x \log x}$

Ans.

$$\int \frac{dx}{x + x \log x} = \int \frac{dx}{x(1 + \log x)}$$

$$put \ 1 + \log x = t$$
$$0 + \frac{1}{x} dx = dt$$

$$= \int \frac{dt}{t}$$
$$= \log |t| + c$$
$$= \log |1 + \log x| + c$$

11.

$$\int \frac{10x^9 + 10^x . \log e10}{x^{10} + 10^x} dx$$

Ans. Put $x^{10} + 10^x = t$

$$(10x^9 + 10^x . \log_e 10\ ) dx = dt$$
$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$
$$= \log(x^{10} + 10^x) + c$$

12.

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

Ans.

$$= \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$
$$= \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx = \int \sec^2 x\, dx$$
$$= \tan x + c$$

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13. $\int_{-\frac{\pi}{4}}^{\pi/4} \sin^2 x\, dx$

Ans. Let $f(x) = \sin^2 x$

$$f(-x) = \sin^2(-x)$$
$$= \sin^2 x$$

$= f(x)$

$\therefore$ function is even

$$\therefore \int_{-\pi/4}^{\pi/4} \sin^2 x\, dx = 2\int_0^{\pi/4} \sin^2 x\, dx$$
$$= \int 2\left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/4}$$
$$= \frac{\pi}{4} - \frac{1}{2}$$

14.

$$\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$$

Ans.

$$\int \left(\frac{x^5 - x^4}{x^3 - x^2}\right) dx \quad \left[\because e^{\log\theta} = \theta\right.$$

$$= \int \frac{x^4 \cancel{(x-1)}}{x^2 \cancel{(x-1)}} dx$$

$= \int x^2 dx$

$= \frac{x^3}{3} + c$

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15. $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx$

Ans.

$$= \int \frac{\cancel{e^x}\left(e^x - e^{-x}\right)}{\cancel{e^x}\left(e^x + e^{-x}\right)} dx$$

$$put\ e^x + e^{-x} = t$$
$$\left(e^x - e^{-x}\right) dx = dt$$

$$= \int \frac{dt}{t} = \log|t| + c$$
$$= \log\left(e^x + e^{-x}\right) + c$$

16.

$$\int \frac{\sqrt{\tan x}}{\sin x . \cos x} dx$$

Ans.

$$= \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x} . \cos^2 x} dx$$

$$= \int \frac{\sqrt{\tan x}}{\tan} . \sec^2 x\ dx$$
$$put\ \tan x = t$$

$$\sec^2 dx = dt$$
$$= \int \frac{\sqrt{t}}{t} dt$$

$$= \int \left(t^{\frac{1}{2}} . t^{-1} \right) dt$$

$$= \int t^{-1/2} dt$$

$$= \frac{t^{1/2}}{1/2} + c = 2\sqrt{\tan x} + c$$

17.

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Ans. Put $x + \log x = t$

$$\left(1 + \frac{1}{x}\right) dx = dt$$

$$\left(\frac{x+1}{x}\right) dx = dt$$

$$= \int t^2 dt$$

$$= \frac{t^3}{3} + c$$

$$= \frac{(x+\log x)^3}{3} + c$$

18.

$$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$$

Ans.

$$= \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{(\cos x - \cos \alpha)} dx$$

$$= \int \frac{2(\cos x + \cos\alpha)\,\cancel{(\cos x + \cos\alpha)}}{\cancel{(\cos x - \cos\alpha)}}\,dx$$

$$= 2(\sin x + \cos\alpha . x) + c$$

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19 $\int \frac{dx}{x^2 - 16}$

Ans.

$$= \int \frac{dx}{x^2 - (4)^2} = \frac{1}{8}\log\left|\frac{x-4}{x+4}\right| + c$$

20.

$$\int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$$

Ans. $f(x) = \tan^{-1}x$

$$f'(x) = \frac{1}{1+x^2}$$

$$\int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx = e^x\tan^{-1}x + c$$

$$\left[\because \int e^x f(x) + f'(x) = e^x f(x) + c\right.$$

20.

$$\int_o^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$$

Ans.

$$I = \int_o^1 \tan^{-1}\left(\frac{x+x-1}{1-x(x-1)}\right)dx$$

$$I=\int_{o}^{1}\left[\tan^{-1}(x)+\tan^{-1}(x-1)\right]dx------(1)$$

$$I=\int_{o}^{1}\left[\tan^{-1}(1-x)+\tan^{-1}(\not{1}-x-\not{1})\right]dx \qquad [\because P_4$$

$$I=\int_{o}^{1}\left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx------(2)$$

$$(1)+(2)$$

$$2I=o$$
$$I=o$$

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21. If

$$f(a+b-x)=f(x)$$

then

$$\int_{a}^{b}(x)f(x)dx=?$$

Ans.

$$I=\int_{a}^{b}(x).f(x)dx------(1)$$

$$I=\int_{a}^{b}(a+b-x).f(a+b-x)dx$$

$$I=\int_{a}^{b}(a+b-x).f(x)dx \quad [\because f(a+b-x)=f(x)$$

$$=\int_{a}^{b}\left[(a+b).f(x)-x\,f(x)\right]dx$$

$$=\int_{a}^{b}(a+b)f(x)\,dx-\int_{a}^{b}x\,f(x)\,dx$$

$$I=(a+b)\int_a^b f(x)\,dx-I$$
$$I=\frac{a+b}{2}\int_a^b f(x)\,dx$$

22.

$$\int_o^{\pi/2}\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$$

Ans.

$$I=\int_o^{\pi/2}\left(\frac{4+3\sin x}{4+3\cos x}\right)dx-----(1)$$

$$I=\int_o^{\pi/2}\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx[by P_4$$

$$I=\int_o^{\pi/2}\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$$

$$=-\int_o^{\pi/2}\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$$
$$I=-I$$

$$2I=0$$
$$I=0$$

23.

$$\int_{-\pi/2}^{\pi/2}\left(x^3+x\cos x+\tan^5 x+1\right)dx$$

Ans.

$$I=\int_{-\pi/2}^{\pi/2}\left(x^3+x\cos x+\tan^5 x\right)dx+\int_{-\pi/2}^{\pi/2}1\ dx$$

$$let\ f(x)=x^3+x\cos x+\tan^5 x$$
$$f(-x)=-x^3-x\cos x-\tan^5 x$$

$$=-(x^3+x\cos x+\tan^5 x)$$
$$=-f(x)$$

Hence odd function

$$I=o+[x]_{-\pi/2}^{\pi/2}$$
$$=\frac{\pi}{2}+\frac{\pi}{2}=\pi$$

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24. Show that

$$\int_o^a f(x).g(x)dx=2\int_o^a f(x)dx$$

Ans.

$$I=\int_o^a f(x).g(x)\ dx$$

$$=\int_o^a f(a-x).g(a-x)\ dx\ \ [byP_4$$

$$=\int_o^a f(x).[4-g(x)dx$$

Fromgiven

$$=\int_o^a 4f(x)dx-\int_o^a f(x).g(x)\ dx$$

$$I = 4\int_o^a f(x)\,dx - I$$

$$I = 2\int_o^a f(x)\,dx$$

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4 Mark Questions

1. $\int_{-1}^{2}\left|x^3 - x\right|dx$

Ans.

$$\int_{-1}^{2}|x^3 - x|\,dx = \int_{-1}^{o}\left(x^3 - x\right) + \int_{o}^{1} -\left(x^3 - x\right)dx + \int_{1}^{2}\left(x^3 - x\right)dx$$

$$= \int_{-1}^{o}\left(x^3 - x\right) + \int_{o}^{1}\left(x - x^3\right)dx + \int_{1}^{2}\left(x^3 - x\right)dx$$

$$= \frac{11}{4}$$

2.

$$\int_o^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$$

Ans.

$$I = \int_o^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx \text{------}(1)$$

$$I = \int_o^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)}\,dx$$

$$= \int_o^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x}\,dx$$

$$= \int_o^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx - \int_o^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$$

$$I = \int_o^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx - I$$

$$2I = \pi \int_o^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx$$

$$= \pi \int_o^{\pi} \frac{\sin x}{1 + \sin x} dx$$

$$= \pi \int_o^{\pi} \frac{1 + \sin x - 1}{1 + \sin x} dx$$

$$= \pi \int_o^{\pi} \left[\frac{1 + \sin x}{1 + \sin x} - \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} \right] dx$$

$$= \pi \int_o^{\pi} 1\, dx - \pi \int_o^{\pi} \frac{1 - \sin x}{\cos^2 x} dx$$

$$= \pi [x]_o^{\pi} - \pi \int_o^{\pi} \left(\sec^2 x - \tan x \sec x \right) dx$$

$$= \pi [\pi - o] - \pi [\tan x - \sec x]_o^{\pi}$$

$$= \pi^2 - \pi \left[(\tan \pi - \sec \pi) - (\tan o - \sec o) \right]$$

$$= \pi^2 - \pi \left[(0 - (-1)) - (o - 1) \right]$$

$$2I = \pi^2 - \pi 2$$
$$= \pi^2 - 2\pi$$

$$= \pi(\pi - 2)$$
$$I = \frac{\pi}{2}(\pi - 2)$$

3.

$$\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x.\cos^2 x} dx$$

Ans.

$$\int \frac{(\sin^4 x)^2 - (\cos^4 x)^2}{1 - 2\sin^2 x.\cos^2 x} dx$$

$$\int \frac{(\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{1 - 2\sin^2 x.\cos^2 x} dx$$

$$\int \frac{(1 - 2\sin^2 x.\cos^2 x)(\sin^2 x - \cos^2 x)}{(1 - 2\sin^2 x.\cos^2 x)} dx$$

$$\int -(\sin^2 x - \cos^2 x)$$

$$\int -\cos 2x dx$$
$$-\frac{\sin 2x}{2} + c$$

4.

$$\int_o^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$$

Ans.

$$I=\int_o^{\pi} \frac{x\sin x}{1+\cos^2 x}dx ------(1)$$

$$=\int_o^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx \quad [byP_4$$

$$=\int_o^{\pi} \frac{(\pi-x).\sin x}{1+\cos^2 x}dx$$

$$I=\int_o^{\pi} \frac{\pi.\sin x}{1+\cos^2 x}dx - I$$

$$2I=\pi\int_o^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

$put\ \cos x = t$

$\sin x\, dx = -dt$

$when\ x=0$

$t \to 1$

$when\ x=\pi$

$t \to -1$

$$=\int_1^{-1} \frac{dt}{1+t^2}$$

$$=\pi\left[\tan^{-1} t\right]_1^{-1}$$

$$=\frac{\pi^2}{4}$$

5.

$$\int_{o}^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

Ans. Let

$$I = \int_{o}^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

$$put \ \sin x - \cos x = t$$
$$(\cos x + \sin x) dx = dt$$

$$\Rightarrow (\sin x - \cos)^2 = t^2$$
$$1 - \sin 2x = t^2$$

$$1 - t^2 = \sin 2x$$
$$= \int_{-1}^{o} \frac{dx}{9 + 16(1 - t^2)}$$

$$\int_{-1}^{o} \frac{dt}{9 + 16 - 16t^2}$$

$$= \int_{-1}^{o} \frac{dt}{25 - 16t^2} = \int_{-1}^{o} \frac{dx}{16\left[\frac{25}{16} - t^2\right]}$$

$$= \frac{1}{16} \int_{-1}^{o} \frac{dx}{\left(\frac{5}{4}\right)^2 - t^2}$$

$$= \frac{1}{16} \times \frac{1}{2 \times \frac{5}{4}} \log \left[\frac{\frac{5}{4} + t}{\frac{5}{4} - t} \right]_{-1}^{0}$$

$$= \frac{1}{40} \log 9.$$

6.

$$\int \frac{5x}{(x+1)(x^2+9)} dx$$

Ans. Let

$$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+9}$$

$$5x = A(x^2+9) + (Bx+c)(x+1)$$

On comparing coeff. Of x^2 and x and constant.

$$0 = A + B$$
$$5 = C + B$$
$$O = 9A + C$$

$$\Rightarrow \begin{array}{l} A - C = -5 \\ \underline{9A + C = O} \\ 10A = -5 \\ A = -\frac{1}{2} \end{array}$$

$$B = 1/2$$
$$C = \frac{9}{2}$$

$$\int \frac{5x}{(x+1)(x^2+9)} = \int \frac{-1/2}{(x+1)} + \int \frac{\frac{1}{2}x+\frac{9}{2}}{x^2+9}dx$$

$$= -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{x}{x^2+9}dx + \frac{9}{2}\int \frac{dx}{x^2+9}$$
$$= -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{2x}{x^2+9}dx + \frac{9}{2}\int \frac{dx}{x^2+3^2}$$

$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{9}{2}.\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$$
$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + c$$

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7. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

Ans.

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\frac{\sin x}{\cos x}}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \text{-------}(1)$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right.$$
$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \text{-------}(2)$$

$$(1)+(2)$$
$$2I=\int_{\pi/6}^{\pi/3}1dx$$

$$=[x]_{\pi/6}^{\pi/3}$$
$$=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$$

$$I=\frac{\pi}{12}$$

8.

$$\int_0^{\pi/4}\frac{\sin x\cos x}{\cos^4 x+\sin^4 x}dx$$

Ans.

$$I=\int_0^{\pi/4}\frac{\sin x.\cos x}{\cos^4 x+\sin^4 x}dx$$

Dividing N and D by $\cos^4$x

$$=\int_0^{\pi/4}\frac{\frac{\sin x.\cos x}{\cos^4 x}}{\frac{\cos^4 x}{\cos^4 x}+\frac{\sin^4 x}{\cos^4 x}}dx$$

$$=\int_0^{\pi/4}\frac{\tan x.\sec^2 x}{1+\tan^4 x}dx$$
$$=\int_0^{\pi/4}\frac{\tan x.\sec^2 x}{1+(\tan^2 x)^2}dx$$

$$put\ \ \tan^2 x=t$$
$$2\tan x.\sec^2 x\,dx=dt$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{1+t^2}$$
$$= \frac{1}{2}\left[\tan^{-1} t\right]_0^1$$

$$= \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$$

9.

$$\int \frac{dx}{\cos(x+a)\cos(x+b)}$$

Ans.

$$I = \int \frac{dx}{\cos(x+a).\cos(x+b)}$$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin[(x+a)-(x+b)]}{\cos(x+a).\cos(x+b)}$$

$$= \frac{1}{\sin(a-b)}\int\left[\frac{\sin(x+a).\cos(x+b)-\cos(x+a).\sin(x+b)}{\cos(x+a).\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\int\left[\tan(x+a)-\tan(x+b)\right]dx$$

$$= \frac{1}{\sin(a-b)}\left[\log\sec(x+a)-\log\sec(x+b)\right]+c$$
$$= \frac{1}{\sin(a-b)}\left[\log\frac{\sec(x+a)}{\sec(x+b)}\right]+c$$

10.

$$\int_{o}^{\pi/4} \log(1+\tan x)\,dx$$

Ans.

$$I = \int_{o}^{\pi/4} \log(1+\tan x)\,dx ------(1)$$

$$= \int_{o}^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad [by\ \text{p4}$$

$$= \int_{o}^{\pi/4} \log\left[1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}.\tan x}\right]dx$$

$$= \int_{o}^{\pi/4} \log\left[1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$= \int_{o}^{\pi/4} \log\left[\frac{1+\cancel{\tan x}+1-\cancel{\tan x}}{1+\tan x}\right]dx$$

$$= \int_{o}^{\pi/4} \log\left[\frac{2}{1+\tan x}\right]dx$$

$$= \int_{o}^{\pi/4} \log 2dx - \int_{o}^{\pi/4} \log(1+\tan x)dx$$

$$= \log 2[x]_{o}^{\pi/4} - I$$

$$2I = \log 2\left[\frac{\pi}{4}-o\right]$$

$$I = \log 2.\frac{\pi}{8}$$

11.

$$\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$$

Ans.

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$$

$$put \ \sin x - \cos x = t$$

$$(\cos x + \sin x)dx = dt$$
$$(\sin x - \cos x)^2 = t^2$$

$when \ x = \pi/6$

$\sin 2x = 1 - t^2$

$$t \to \frac{1}{2} - \frac{\sqrt{3}}{2}$$
$$when \ x \to \pi/3$$
$$t \to \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$\int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$
$$= \left[\sin^{-1} t\right]_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) - \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$$
$$= \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

$$= 2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

12.

$$\int_1^4 [|x-1|+|x-2|+|x-3|]\,dx$$

Ans.

$$I = \int_1^4 [|x-1|+|x-2|+|x-3|]dx$$

$$= \int_1^4 |x-1|\,dx = \int_1^4 (x-1)dx = \frac{9}{2}$$
$$= \int_1^4 |x-2|\,dx = \int_1^2 -(x-2)dx + \int_2^4 (x-2)dx = \frac{5}{2}$$

$$\int_1^4 (x-3) = \int_1^3 -(x-3)dx + \int_3^4 (x-3)dx = \frac{5}{2}$$
$$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$$

13.

$$\int_o^{\pi/2} (2\log\sin x - \log\sin 2x)\,dx$$

Ans.

$$I = \int_o^{\pi/2} [2\log\sin x - (\log 2\sin x.\cos .)]dx$$

$$I=\int_o^{\pi/2}\left(2\underline{\log\sin x}-\log 2-\underline{\log\sin x}-\log\cos x\right)dx$$
$$I=\int_o^{\pi/2}\left(\log\sin x-\log 2-\log\cos x\right)dx$$

$$I=\int_o^{\pi/2}\log\sin x\,dx-\int_o^{\pi/2}\log 2\,dx-\int_o^{\pi/2}\log\cos x dx$$

$$I=\int_o^{\pi/2}\log\sin x\,dx-\int_o^{\pi/2}\log 2\,dx-\int_o^{\pi/2}\log\cos\left(\frac{\pi}{2}-x\right)dx$$
$$I=\cancel{\int_o^{\pi/2}\log\sin x}\,dx-\log 2\int_o^{\pi/2}1\,dx-\cancel{\int_o^{\pi/2}\log\sin x}\,dx$$

$$=-\log 2[x]_o^{\pi/2}$$
$$=-\log 2\left(\frac{\pi}{2}-o\right)=-\frac{\pi}{2}.\log 2$$

14.

$$\int\frac{2+\sin 2x}{1+\cos x}e^x dx$$

Ans.

$$I=\int\left(\frac{2+2\sin x.\cos x}{2\cos^2 x}\right)e^x dx$$

$$\int\left(\frac{\cancel{2}}{\cancel{2}\cos^2 x}+\frac{\cancel{2}\sin x.\cancel{\cos x}}{\cancel{2}\cos^2 x}\right)e^x dx$$
$$=\int\left(\sec^2 x+\tan x\right)e^x dx$$

$$let\, f(x)=\tan x$$
$$f'(x)=\sec^2 x$$

∵ We know that

$$\int e^x[f(x)+f'(x)]dx = e^x f(x)+c$$
$$\therefore \int (\sec 2x+\tan x)e^x dx$$
$$= e^x.\tan x + c$$

15.

$$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$

Ans.

$$I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$

$$put x = \cos\theta$$
$$dx = -\sin\theta d\theta$$

$$= \int \tan^{-1}\sqrt{\left(\frac{1-\cos\theta}{1+\cos\theta}\right)} \times -\sin\theta d\theta$$
$$= \int \tan^{-1}\sqrt{\left(\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)} \times -\sin\theta d\theta$$

$$= \int \tan^{-1}\left(\tan\frac{\theta}{2}\right)(-\sin\theta)d\theta$$
$$= -\int \frac{\theta}{2}\sin\theta d\theta = \frac{-1}{2}\int \underset{I}{\theta}.\underset{II}{\sin}\theta d\theta$$

$$= \frac{-1}{2}\left[\theta.(-\cos\theta) - \int 1\times(-\cos\theta)d\theta\right]$$

$$=\frac{-1}{2}\left[-\theta.\cos\theta+\sin\theta\right]+c$$

$$=\frac{-1}{2}\left[-\theta.\cos\theta-\sqrt{1-\cos^2\theta}\right]+c$$

$$=\frac{-1}{2}\left[-x.\cos^{-1}x-\sqrt{1-x^2}\right]+c$$

6 Marks Questions

1.

$$\int\frac{dx}{\sqrt{\sin^3 x\sin(x+\alpha)}}$$

Ans.

$$\int\frac{dx}{\sqrt{\sin^3 x.\sin(x+\alpha)}}$$

$$\int\frac{dx}{\sqrt{\sin^4 x.\frac{\sin(x+\alpha)}{\sin x}}}$$

$$\int\frac{dx}{\sin^2 x\sqrt{\frac{\sin(x+\alpha)}{\sin x}}}=\int\frac{\cos ec^2 dx}{\sqrt{\frac{\sin(x+\alpha)}{\sin x}}}$$

$$=\int\frac{\cos ec^2 dx}{\sqrt{\frac{\sin x.\cos\alpha+\cos x.\sin\alpha}{\sin x}}}$$

$$= \int \frac{\cos ec^2 dx}{\sqrt{\cos\alpha + \cot x . \sin\alpha}}$$

$$put \;\; \cos\alpha + \cot x . \sin\alpha = t$$
$$0 - \cos ec^2 x . \sin\alpha dx = dt$$

$$= \int -\frac{dt}{\sqrt{t}} = -\frac{1}{\sin\alpha} . \frac{t^{1/2}}{1/2} + c$$
$$= \frac{-2}{\sin\alpha}\sqrt{t} + c$$

2.

$$\int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$$

Ans.

$$I = \int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$$

$$I = \int \left(\frac{1}{\sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1}\right) dx$$
$$I = \int \frac{1 + \tan x}{\sqrt{\tan x}} dx$$

$put \;\; \sqrt{\tan x} = t$
$\tan x = t^2$

$$\sec^2 x \, dx = 2t \, dt$$
$$dx = \frac{2t \, dt}{\sec^2 x}$$

$$= \frac{2t\ dt}{1+\tan^2 x}$$

$$= \frac{2t}{1+t^4}$$

$$= \int \frac{1+t^2}{t} \times \frac{2t}{1+t^4}\,dt$$

$$= 2\int \frac{t^2+1}{t^4+1}dt = 2\int \frac{t^2\left(1+\frac{1}{t^2}\right)}{t^2\left(1+\frac{1}{t^2}\right)}dt$$

$$2\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}-2+2}dt$$

$$2\int \frac{1+1/t^2}{\left(t-\frac{1}{t}\right)^2+\left(\sqrt{2}\right)^2}dt$$

$$put\ t-\frac{1}{t}=u$$

$$\left(1-\frac{1}{t^2}\right)dt = du$$

$$= 2\int \frac{du}{(u)^2+\left(\sqrt{2}\right)^2}$$

$$= 2.\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+c$$

$$= \sqrt{2}\tan^{-1}\frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}}+c$$

$$= \sqrt{2}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right)+c$$

$$= \sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+c$$

3.

$$\int\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$$

Ans.

$$\int\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$$

$$\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}=\frac{\pi}{2}$$

$$\cos^{-1}\sqrt{x}=\frac{\pi}{2}-\sin^{-1}\sqrt{x}$$

$$=\int\frac{\sin^{-1}\sqrt{x}-\left(\frac{\pi}{2}-\sin^{-1}\sqrt{x}\right)}{\pi/2}dx$$

$$=\int\frac{2\sin^{-1}\sqrt{x}-\frac{\pi}{2}}{\pi/2}dx$$

$$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\,dx - \int 1\,dx$$
$$= \frac{4}{\pi}I - x + c \quad (i)$$

$$I = \int \sin^{-1}\sqrt{x}\,dx$$
$$put\ \sin^{-1}\sqrt{x} = t$$

$$\sin t = \sqrt{x}$$
$$\sin^2 t = x$$

$$2\sin t\cos t = dx$$
$$= \int t.2\sin t\cos t\,dt$$

$$= \int \underset{1}{t}.\underset{11}{\sin} 2t\,dt$$
$$= -t\frac{\cos 2t}{2} - \int 1.\left(-\frac{\cos 2t}{2}\right)dt$$

$$= \frac{-t\cos 2t}{2} + \frac{1}{2}.\frac{\sin 2t}{2} + c$$
$$= -t.\frac{(1-2\sin^2 t)}{2} + \frac{1}{4}.\frac{\sin 2t}{2} + c$$

$$= \frac{-t(1-2\sin 2t)}{2} + \frac{1}{2}.\sin t\sqrt{1-\sin^2 t}$$

$$= \frac{-\sin^{-1}\sqrt{x}(1-2x)}{2} + \frac{1}{2}.\sqrt{x}\sqrt{1-x} + c$$
$$= \frac{\sin^{-1}\sqrt{x}(2x-1)}{2} + \frac{1}{2}\sqrt{x-x^2} + c$$

From (i)

$$\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx=\frac{4}{\pi}\left(\frac{\sin^{-1}\sqrt{x}\,(2x-1)}{2}+\frac{1}{2}\sqrt{x-x^2}\right)-x+c$$

4.

$$\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$$

Ans.

$$=\int \underset{I}{\log(\log x)}\times \underset{II}{1}dx+\int \frac{1}{(\log x)^2}dx$$

$$=\log(\log x).x-\int \frac{1}{\log x}.\frac{1}{x}.x\,dx+\int \frac{1}{(\log x)^2}dx$$

$$=\log(\log x).x-\frac{x}{\log x}-\int \underset{I}{\frac{1}{\log x}}\times \underset{II}{1}\,dx+\int \frac{1}{(\log x)^2}dx$$

$$=\log(\log x).x-\frac{x}{\log x}-\int \frac{dx}{(\log x)^2}+\int \frac{dx}{(\log x)^2}$$

$$=x.\log(\log x)-\frac{x}{\log x}+c$$

5.

$$\int_o^{\pi/2}\frac{\cos^2 x}{\cos^2 x+4\sin^2 x}dx$$

Ans.

$$I=\int_o^{\pi/2}\frac{\cos^2 x}{\cos^2 x+4\sin^2 x}dx$$

$$= \int_o^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\left(1 - \cos^2 x\right)} dx$$

$$= \int_o^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4 - 4\cos^2 x} dx$$

$$= \int_o^{\pi/2} \frac{\cos^2 x}{4 - 3\cos^2 x} dx$$

$$= \frac{-1}{3} \int_o^{\pi/2} \frac{-3\cos^2 x}{4 - 3\cos^2 x} dx$$

$$= \frac{-1}{3} \int_o^{\pi/2} \frac{4 - 3\cos^2 x - 4}{4 - 3\cos^2 x} dx$$

$$= \frac{-1}{3} \int_o^{\pi/2} \left(1 - \frac{4}{4 - 3\cos^2 x}\right) dx$$

$$= \frac{-1}{3} \int_o^{\pi/2} 1 dx + \frac{4}{3} \int_o^{\pi/2} \frac{dx}{4 - 3\cos^2 x}$$

$$= \frac{-1}{3} [x]_o^{\pi/2} + \frac{4}{3} \int_o^{\pi/2} \frac{dx/\cos^2 x}{\frac{4}{\cos^2 x} - \frac{3\cos^2 x}{\cos^2 x}}$$

$$= \frac{-1}{3}\left[\frac{\pi}{2} - o\right] + \frac{4}{3} \int_o^{\pi/2} \frac{\sec^2 x\, dx}{4\sec^2 x - 3}$$

$$= \frac{-\pi}{6} + \frac{4}{3} \int_o^{\pi/2} \frac{\sec^2 x}{4(1 + \tan^2 x) - 3} dx$$

$put \ \tan x = t$

$\sec^2 x \, dx = dt$

$$= \frac{-\pi}{6} + \int_0^\infty \frac{dt}{4(1+t^2)-3}$$

$$= \frac{-\pi}{6} + \frac{1}{4} \cdot \frac{4}{3} \int_0^\infty \frac{dt}{t^2 + \frac{1}{4}}$$

$$= \frac{-\pi}{6} + \frac{1}{3} \cdot \frac{1}{2} \left[\tan^{-1} 2t \right]_o^\infty$$

$$= \frac{-\pi}{6} + \frac{2}{3} \left(\tan^{-1} \infty - \tan^{-1} o \right)$$

$$= \frac{-\pi}{6} + \frac{2}{3} \left(\frac{\pi}{2} - o \right)$$

$$= \frac{-\pi}{6} + \frac{\pi}{3}$$

$$= \pi/6$$

6.

$$\int_o^{\pi/2} \log \sin x \, dx$$

Ans.

$$I = \int_o^{\pi/2} \log \sin x \, dx ----------(1)$$

$$I = \int_o^{\pi/2} \log \sin \left(\frac{\pi}{2} - x \right) dx \qquad \left[by \text{ p4} \right.$$

$$I = \int_o^{\pi/2} \log \cos x \, dx ----------(2)$$

$$(1) + (2)$$

$$2I = \int_o^{\pi/2} (\log \sin x + \log \cos x) dx$$

$$= \int_{o}^{\pi/2} \left(\underline{\log \sin x . \cos x + \log 2} - \log 2 \right) dx$$

$$= \int_{o}^{\pi/2} \log \sin 2x \, dx - \int_{o}^{\pi/2} \log 2 dx$$

$$put \;\; 2x = t$$

$$dx = \frac{dt}{2}$$

$$2I = \frac{1}{2} \int_{o}^{\pi} \log \sin t \; dt - \frac{\pi}{2} \log 2$$

$$= \frac{\not 2}{\not 2} \int_{o}^{\pi/2} \log \sin t \; dt - \frac{\pi}{2} \log 2 \qquad \left[byP_6 \right.$$

$$= \int_{o}^{\pi/2} \log \sin x \; dx - \frac{\pi}{2} \log 2 \qquad \left[byP_o \right.$$

$$2I = I - \frac{\pi}{2} \log 2$$

$$I = -\frac{\pi}{2} \log 2$$

7.

$$\int_{o}^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

Ans.

$$I = \int_{o}^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} (1)$$

$$I = \int_{o}^{\pi} \frac{(\pi - x)}{a^2 \cos^2 (\pi - x) + b^2 \sin^2 (\pi - x)} dx$$

$$I = \int_{o}^{\pi} \frac{(\pi - x)}{a^2 \cos^2 x + b^2 \sin^2 x} dx$$

$$I=\int_o^{\pi}\frac{\pi\,dx}{a^2\cos^2 x+b^2\sin^2 x}-\int_o^{\pi}\frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$$

$$I=\pi\int_o^{\pi}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}-I$$

$$2I=\int_o^{\pi}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$$

$$I=\frac{\pi}{2}\int_o^{\pi}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$$

$$=\frac{\pi}{2}\cdot 2\int_o^{\pi/2}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}\qquad\left[\because p_6^{by}\right.$$

Dividing N and D by $\cos^2$x

$$=\pi\int_o^{\pi/2}\frac{\sec^2 x\,dx}{a^2+b^2\tan^2 x}$$

$$put\ b\tan x=t$$

$$b\sec^2 x\,dx=dt$$

$$=\pi\int_o^{\infty}\frac{dt/b}{a^2+t^2}$$

$$=\pi.\frac{1}{a}\left[\tan^{-1}\frac{t}{a}\right]_o^{\infty}\times\frac{1}{b}$$

$$=\frac{\pi}{ab}\left(\frac{\pi}{2}\right)$$

$$=\frac{\pi^2}{2ab}$$

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8. Find its sum of limit $\int_o^4 \left(x+e^{2x}\right) dx$

Ans. $f(x) = x + e^{2x}$

$$a = 0,\ b = 4,\ h = \frac{b-a}{n}$$

$$\int_o^4 \left(x+e^{2x}\right)dx = \lim_{h\to o} h\left[f(o) + f(o+h) + f(o+2h) + ---- + f(o+(n-1)h)\right]$$

$$= \lim_{h\to o} h\left[(e^o) + (h+e^{2h}) + (2h+e^{4h}) + ------ + ((n-1)h + e^{2(n-1)h})\right]$$

$$= \lim_{h\to o} h\left[h\left[1+2+-----+(n-1)\right] + (1+e^{2h}+e^{4h}+----+(e^{2(n-1)h}\right]$$

$$= \lim_{h\to o} h\left[h\frac{n(n-1)}{2} + 1.\frac{\left(e^{2nh}-1\right)}{\frac{e^{2h}-1}{2h}\times 2h}\right]$$

$$= \lim_{n\to o} h\left[h\frac{n(n-1)}{2} + \frac{e^{2nh}-1}{2h}\right]$$

$$= \lim_{n\to\infty} \frac{4}{n}\left[\frac{4}{n}.\frac{n(n-1)}{2} + \frac{e^{2n\frac{4}{n}}-1}{2\times\frac{4}{n}}\right]$$

$$= \lim_{n\to\infty}\left[\frac{16}{n}\, n.n\,\frac{\left(1-\frac{1}{n}\right)}{2} + \frac{4}{n}\,\frac{e^8-1}{\frac{8}{n}}\right]$$

$$= 16 \times \frac{1}{2} + \frac{e^8 - 1}{2}$$

$$= 8 + \frac{e^8 - 1}{2}$$

$$= \frac{16 + e^8 - 1}{2}$$

$$= \frac{e^8 + 15}{2}$$

9.

$$\int_{-1}^{3/2} | x \sin(\pi x) | \, dx$$

Ans.

$$\int_{-1}^{3/2} | x.\sin(\pi x) | \, dx = \int_{-1}^{1} \underset{I}{x} \underset{II}{\sin} \pi x \, dx + \int_{-1}^{3/2} -x \sin \pi x \, dx$$

$$= \left[\frac{-x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_{-1}^{1} - \left[\frac{-x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_{1}^{3/2}$$

$$= \frac{2}{\pi} - \left[-\frac{1}{\pi^2} - \frac{1}{\pi} \right]$$

$$= \frac{3}{\pi} + \frac{1}{\pi^2}$$

10.

$$\int \frac{dx}{3x^2 + 13x - 10}$$

Ans.

$$= \int \frac{dx}{3\left[x^2 + \frac{13}{3}x - \frac{10}{3}\right]}$$

$$= \frac{1}{3}\int \frac{dx}{\left(x + \frac{13}{6}\right)^2 - \left(\frac{17}{6}\right)^2}$$

$put\ x + \frac{13}{6} = t$

$$dx = dt$$

$$= \frac{1}{3}\int \frac{dt}{t^2 - \left(\frac{17}{6}\right)^2}$$

$$= \frac{1}{\cancel{3} \times \cancel{2} \times \frac{17}{\cancel{6}}} \log\left|\frac{t - \frac{17}{6}}{t + \frac{17}{6}}\right| + c$$

$$= \frac{1}{17}\log\left|\frac{6x - 4}{6x + 30}\right| + c$$

$$= \frac{1}{17}\log\left|\frac{3x - 2}{x + 5}\right| + c$$