Important Questions Class 12 Maths Chapter 8 Application of Integrals

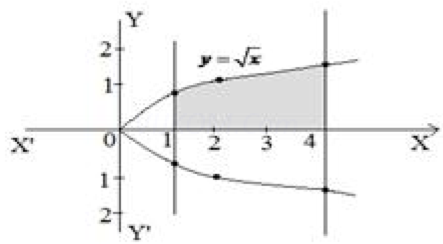
4 Mark Questions

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and x – axis.

Ans. $y^2 = x$ is the equation of parabola and $x = 1$, $x = 4$ and x – axis

Req. area = $\int_1^4 \sqrt{x}dx$

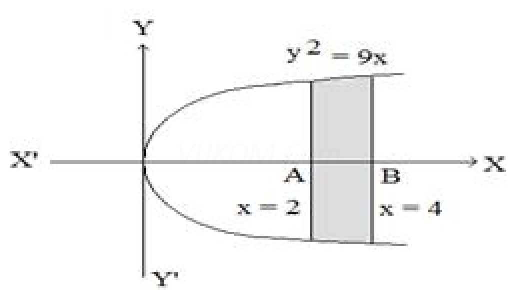
$= \frac{14}{3} sq\ unit$



2. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x – axis in the first quadrant.

Ans. $y^2 = 9x$, $x = 2$, $x = 4$, x – axis in the first quadrant.

$$= \int_2^4 \sqrt{9x}dx = \left(16 - 4\sqrt{2}\right) sq\ unit$$

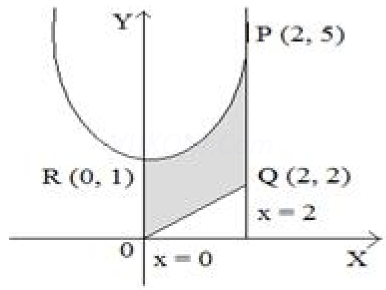


3. Find the area of the region bounded by the parabola $y = x^2 + 1$ and the lines $y = x$, $x = 0$ and $x = 2$.

Ans. $y = x^2 + 1$

$y = x$, $x = 0$, $x = 2$

$$= \int_o^2 (x^2 + 1)\,dx - \int_o^2 x\,dx$$



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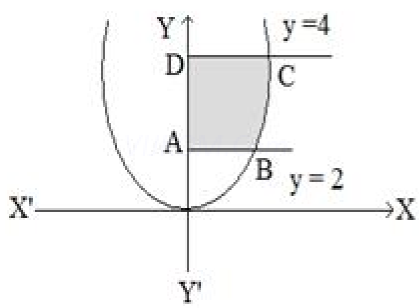
4. Find area of the region bounded x^2 = 4y, y = 2, y = 4 and the y – axis in the first quadrant.

Ans. x^2 = 4y, y = 2, y = 4 y – axis in the first quadrant

$$= 2\int_2^4 \sqrt{y}\,dy$$

$$= \left(\frac{32 - 8\sqrt{2}}{3}\right) sq\ unit$$



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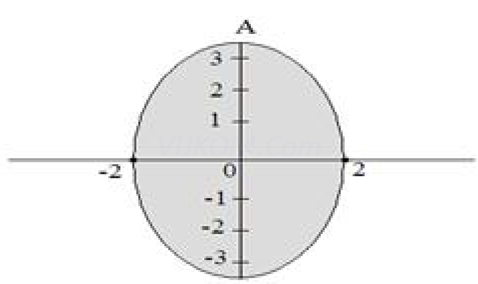
5. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.



Ans. $\frac{x^2}{4}+\frac{y^2}{9}=1$

$$= 4\int_o^2 \frac{3}{2}\sqrt{4-x^2}\,dx$$
$$= 4\int_o^2 \frac{3}{2}\sqrt{2^2-x^2}\,dx$$
$$= 6\pi sq\ unit$$

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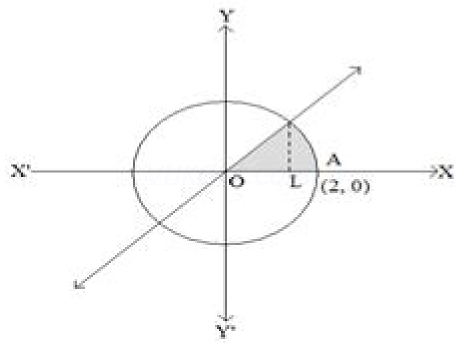
6. Find the area of the region in the first quadrant enclosed by x – axis and $x=\sqrt{3}y$ by the circle $x^2 + y^2 = 4$.



Ans.
x – axis
$x=\sqrt{3}y$
$x^2+y^2=4$
in first quadrant.

$$=\frac{1}{\sqrt{3}}\int_o^{\sqrt{3}} x\,dx+\int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$$
$$=\frac{\pi}{3}sq\ unit$$

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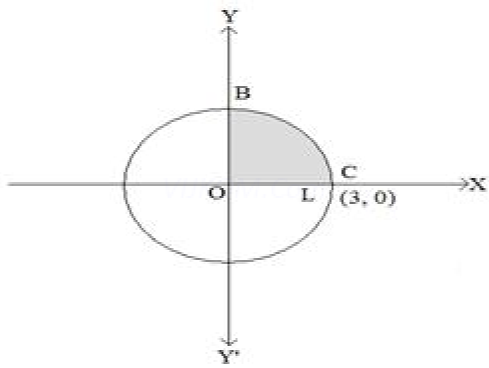
7. Draw the graph of the curve $y=\sqrt{9-x^2}$ and find the area bounded by this curve and the coordinate axis.
Ans.



$$= \int_{o}^{3} \sqrt{9 - x^2}\, dx$$

$$= \int_{o}^{3} \sqrt{3^2 - x^2}\, dx$$

$$= \frac{9\pi}{4} sq\ unit$$

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8. The area between x = y^2 and x = 4 is divided into equal parts by the line x = a, find the value of a.

Ans. x = y2

x = 4

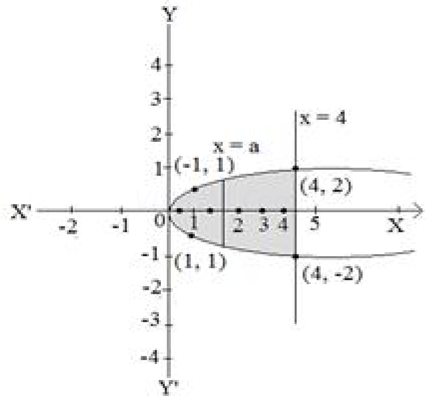
x = a

ATQ

$$2\int_{o}^{a} \sqrt{x}\, dx = 2\int_{2}^{4} \sqrt{x}\, dx$$

$$\left[\frac{x^{\frac{3}{2}}}{3/2}\right]_{o}^{1} = \left[\frac{x^{3/2}}{3/2}\right]_{2}^{4}$$

$$a = (4)^{2/3} sq\ unit$$



9. Find the area of the region bounded by the parabola y = x^2 and y = |x|.

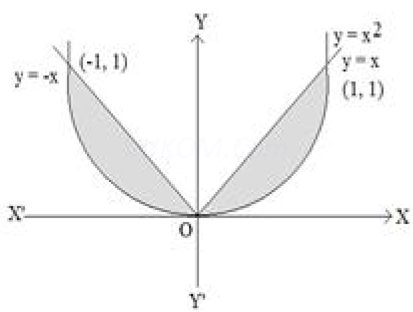
Ans. y = x^2

$y = |x|$

$\Rightarrow y = x$

$$y = -x$$

$$= 2\int_o^1 \left(x - x^2\right) dx$$

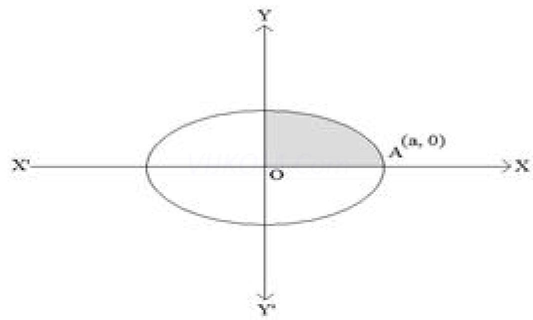


10. Find the area of ellipse $\frac{x^2}{a^2}\frac{y^2}{b^2} = 1$.

Ans. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$= 4\int_o^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

$$= \pi ab \ sq \ unit$$



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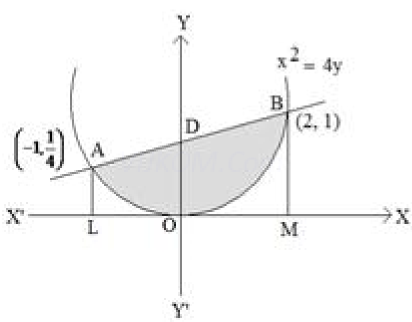
11. Find the area bounded by the curve x^2 = 4y and the line x = 4y – 2.
Ans. $x^2 = 4y$
x = 4y – 2
Req. area =

$$\int_{-1}^{2}\frac{1}{4}(x+2)\,dx-\frac{1}{4}\int_{-1}^{2}x^2\,dx$$

$$=\frac{9}{8}sq\ unit$$



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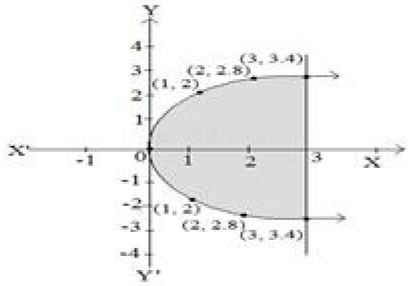
12. Find the area of the region bounded by the curve y^2 = 4x and the line x = 3.
Ans. $y^2 = 4x$
x = 3

$$=2\int_{o}^{3}\sqrt{4x}\,dx$$
$$=8\sqrt{3}sq\ unit$$



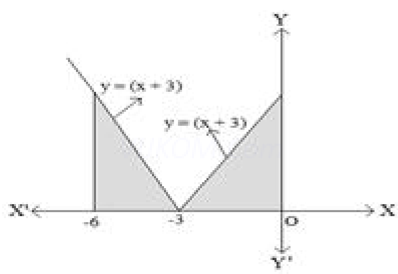
13. Find the area between the curve y = |x + 3|, the x – axis and the lines x = -6 and x = 0.

Ans. $y = |x+3|$

$x - aixs$

$x = -6,\ x = 0$

$$\int_{-6}^{0} |x+3| dx = \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{0} (x+3)\,dx$$
$$= 9 sq\ unit.$$



14. Find the Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x = 2.

Ans. $x^2 + y^2 = 4$

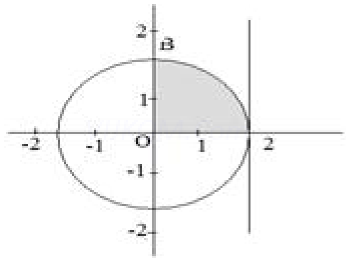
$x = 0$

$x = 2$

Area

$$= \int_{0}^{\pi/2} \sqrt{4 - x^2}\,dx$$

$$= \int_{0}^{\pi/2} \sqrt{2^2 - x^2}\,dx$$
$$= \pi sq\ unit$$



15. **Find the Area of the region bounded by the curve $y^2 = 4x$, y – axis and the line y = 3.**

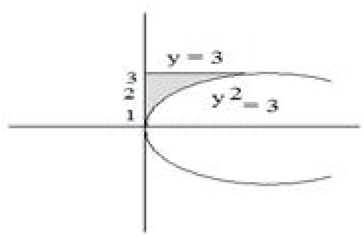
Ans. $y^2 = 4x$

$y - axis$

$y = 3$

$$Area = \int_o^3 \frac{y^2}{4} dy$$

$$= \frac{9}{4} sq.unit$$

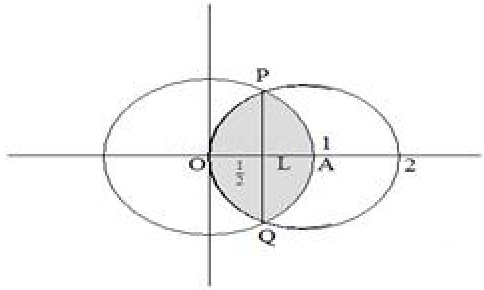


16. **Find the area bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.**

Ans.

$$(x-1)^2 + y^2 = 1 ------(1)$$

$$x^2 + y^2 = 1 --------(2)$$



On solving (1) and (2)

$x=\frac{1}{2}, y=\frac{\sqrt{3}}{2}$

$$Area=2\left[\int_{o}^{1/2}\sqrt{1-(x-1)^{2}}dx+\int_{1/2}^{1}\sqrt{1-x^{2}}dx\right]$$
$$=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)sq.\ unit$$

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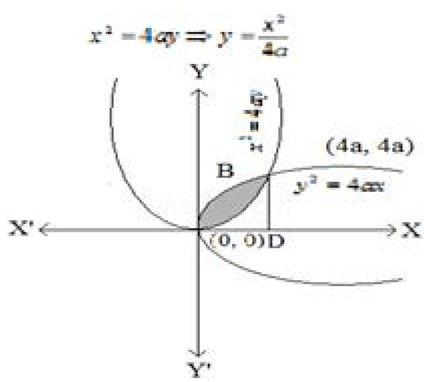
17. Find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, a > 0.

Ans. $y^2=4ax$

$x^2=4ay$

on solving

$$x=4a,\ y=4a$$
$$Area=\int_{o}^{4a}\sqrt{ax}dx-\int_{o}^{4a}\frac{x^{2}}{4a}dx$$
$$=\frac{16a^{2}}{3}sq\ unit.$$



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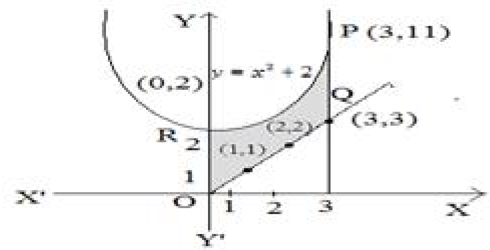
18. Find the area of the region bounded by the curves y = x^2 + 2, y = x, x = 0 and x = 3.

Ans. $y = x^2 + 2$



$$y = x$$
$$x = 0$$
$$x = 3$$

$$Area = \int_o^3 (x^2 + 2)dx - \int_o^3 xdx$$
$$= \frac{21}{2} sq\ unit.$$

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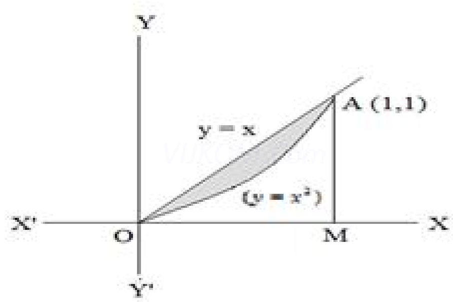
19. Find the area of the region

$$\{(x, y) : x^2 \le y \le x\}.$$

Ans. $y = x^2$



$$y = x$$
$$\Rightarrow x = 0, y = 0$$
$$x = 1, y = 1$$

$$Area = \int_o^1 x dx - \int_o^1 x^2 dx$$
$$= \frac{1}{6} sq.\ unit$$

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20. Find the area bounded by the curves

$$\left\{(x, y) : x^2 + y^2 \le 2ax, y^2 > ax, a > 0, x > 0, y > 0\right\}.$$

Ans.

$$x^2 + y^2 = 2ax \Rightarrow (x - a)^2 + y^2 = a^2$$

$y^2 = ax$
$\Rightarrow x = a, y = a$

$$x = 0, y = 0$$
$$Area = \int_o^a \sqrt{2ax - x^2}\,dx - \int_o^a \sqrt{ax}\,dx$$
$$= \frac{a^2}{12}(3\pi - 8) sq.\ unit$$

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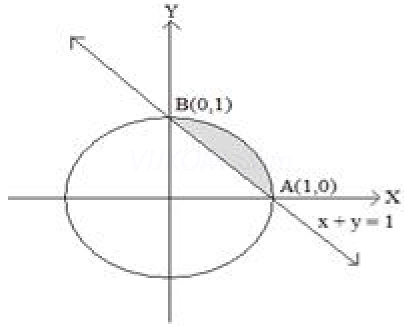
21. Find the area of the region:

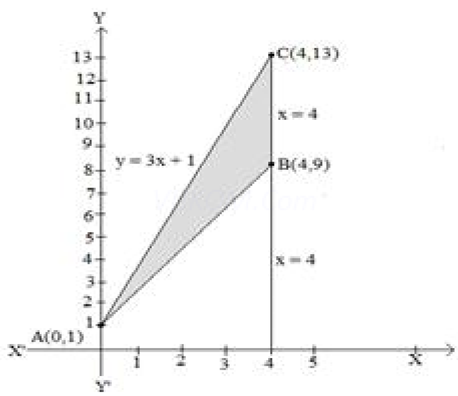
$$\left\{(x, y) : x^2 + y^2 \le 1 \le x + y\right\}.$$

Ans. $x^2 + y^2 = 1$

$$x + y = 1$$
$$Area = \int_o^1 \sqrt{1 - x^2}\,dx - \int_o^1 (1 - x)\,dx$$
$$= \frac{\pi}{4} - \frac{1}{2} sq\ unit$$



22. Using integration find the area of the triangular region whose side have the equations y = 2x
+ 1, y = 3x + 1, and x = 4.



Ans.

$y = 2x + 1$

$y = 3x + 1$

$x = 4$

On solving

$$A(0,1),\ B(4,9),\ C(4,13)$$

$$Area = \int_o^4 (3x+1)dx - \int_o^4 (2x+1)dx$$

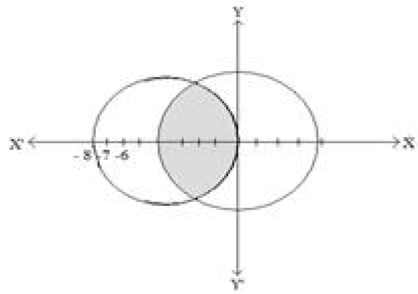
$$= 8sq\ unit$$

23. Calculate the area of the region enclosed between eh circles:
$x^2 + y^2 = 16$ and $(x + 4)^2 + y^2 = 16$.

Ans. $x^2 + y^2 = 16$

$$(x+4)^2 + y^2 = 16$$

Intersecting at x = -2



$$Area = 4\int_{-4}^{-2}\sqrt{16-x^2}\,dx$$
$$= \left(-8\sqrt{3}+\frac{32\pi}{3}\right) sq\ unit$$

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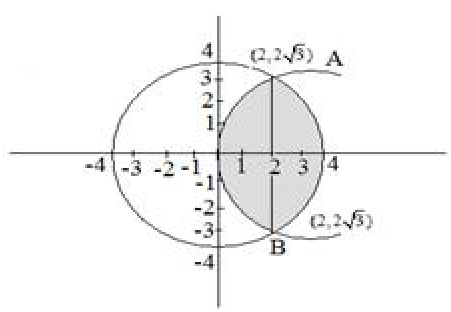
24. Find the area of the circle x^2 = y^2 = 15 exterior to the parabola y^2 = 6x
Ans.

$$Area = 2\int_0^2 \sqrt{6x}\quad dx + 2\int_2^4 \sqrt{16-x^2}$$
$$= \frac{4}{3}\left[8\pi - \sqrt{3}\right]$$



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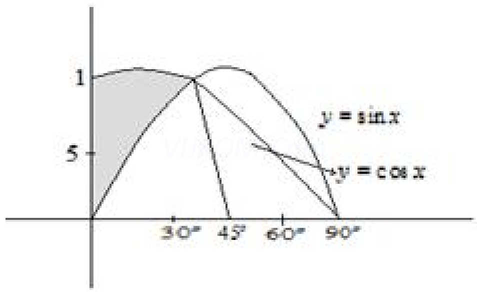
25. Find the area bounded by the y – axis, y = cosx and y = sinx, $0 \le x \le \frac{\pi}{2}$

Ans.

$$Area = \int_0^{\pi/4} (\cos x - \sin x)\ dx$$

$$= \sqrt{2} - 1 \ sq\ unit$$



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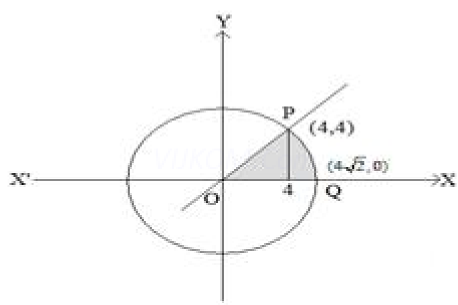
26. Using integration, find the area of the region in the first quadrant enclosed by the x – axis, the line y = x and the circle $x^2 + y^2 = 32$.
Ans.

$$Area = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\ dx$$

$$= 4\pi sq\ unit$$



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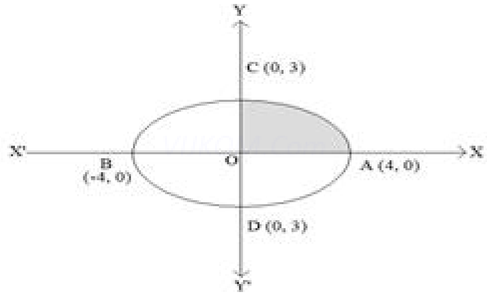
6 Marks Questions

1. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ **.**

Ans.



$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$$\frac{y^2}{9}=1-\frac{x^2}{16}$$

$$y^2=\frac{9}{16}\left(16-x^2\right)$$

$$y=\frac{3}{4}\sqrt{16-x^2}$$

Required area

$$=4\int_o^4 \frac{3}{4}\sqrt{16-x^2}\,dx$$

$$=3\left[\frac{x}{4}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_o^4$$

$$\left[\because \sqrt{a^2-x^2}=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$=3\left[\left(o+8\sin^{-1}(1)\right)-(0)\right]$$

$$= 3\left[8\sin^{-1}\left(\sin\frac{\pi}{2}\right)\right]$$
$$= 3.8.\frac{\pi}{2}$$
$$= 12\pi sq\ unit$$

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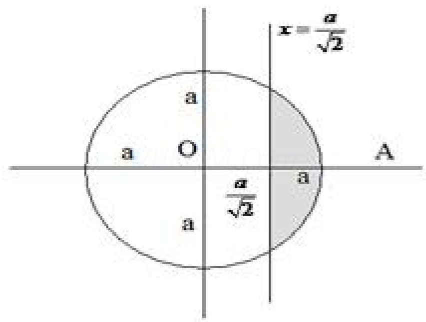
2. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

Ans.



$x^2 + y^2 = a^2$

$$x = \frac{a}{\sqrt{2}}$$
$$= 2\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$$
$$= \frac{\pi a^2}{4} - \frac{a}{2} sq\ unit.$$

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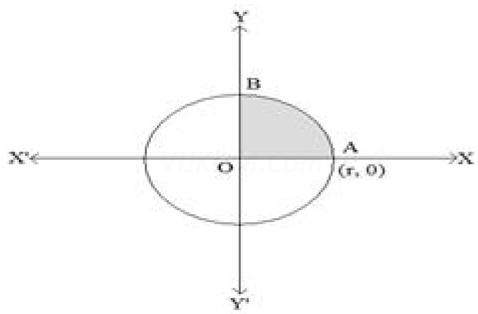
3. Prove the area of a circle of radius r is πr^2 square units.

Ans. $x^2 + y^2 = r^2$

$$= 4\int_{o}^{r} \sqrt{r^2 - x^2}\,dx$$
$$put\ x = r\sin\theta$$

$$dx = r\cos\theta\, d\theta$$
$$= 4\int_o^{\pi/2} r\cos\theta\, d\theta.\, r\cos\theta$$

$$= 4\int_o^{\pi/2} r^2\cos^2\theta\, d\theta$$
$$= 4r^2\int_o^{\pi/2}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$
$$= \pi r^2 sq\ unit$$



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4. Find the area enclosed between the curve y = x^3 and the line y = x.

Ans. $y = x^3,\ y = x$

$$\Rightarrow x^3 = x$$
$$x = 0,\ x = -1,\ x = 1$$

$$= 2\int_o^1\left(x - x^3\right)dx$$
$$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_o^1$$

$$= 2\left[\left(\frac{1}{2} - \frac{1}{4}\right) - (0)\right]$$
$$= 2\left(\frac{2-1}{4}\right) = \frac{1}{2} sq\ unit.$$

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5. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $y^2 = 4x$.
Ans.

$$4x^2 + 4y^2 = 9 ------(1)$$

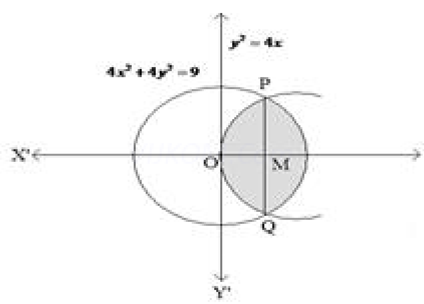
$$y^2 = 4x ---------(2)$$

On solving (1) and (2)

$$y = 1/2$$

$$= 2\left(\int_{o}^{1/2} 2\sqrt{y}dy + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - y^2}dy \right)$$

$$= \left[\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) \right] sq.\ unit.$$



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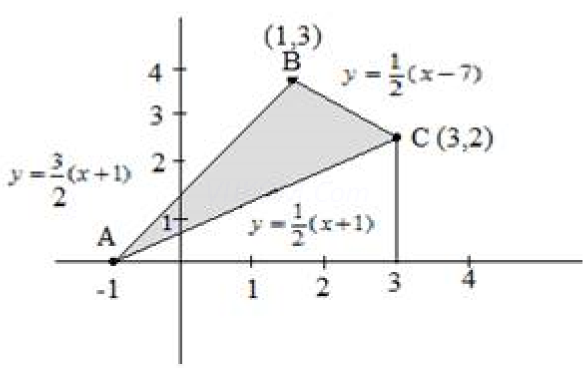
6. Using integration, find the area of region bounded by the triangle whose vertices are
(-1, 0),(1, 3) and (3, 2).
Ans.



A (-1, 0) B (1, 3) C (3, 2)
Equation of AB

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 0 = \frac{3 - 0}{1 + 1}(x + 1)$$

$y = \frac{3}{2}(x+1)$
Similarly
Equation of BC $y = \frac{-1}{2}(x-7)$
Equation of AC $= \frac{1}{2}(x+1)$

$$Area\ \Delta ABC = \int_{-1}^{1} \frac{3}{2}(x+1)\,dx + \int_{1}^{3} \frac{1}{2}(x-7)\,dx$$
$$- \int_{-1}^{3} \frac{1}{2}(x+1)\,dx$$
$$= 4 sq.\ unit$$

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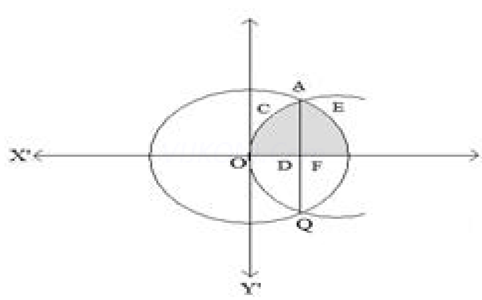
7. Draw a rough sketch of the region

$$\{(x, y) : y^2 \le 3x, 3x^2 + 3y^2 = 16\}$$

and find the area enclosed by the region using method of integration.
Ans. $y^2 = 3x$



$3x^2 + 3y^2 = 16$
On solving

$$x = \frac{-9+\sqrt{273}}{6} = p$$

$$Area = 2\left[\int_0^p \sqrt{3x}dx + \int_p^{4\sqrt{3}} \sqrt{\frac{16-3x^2}{3}}dx\right]$$

$$= \frac{4}{\sqrt{3}}(p)^{3/2} + \frac{4\pi}{3} - \frac{p}{2}\sqrt{\frac{16}{3} - p^2}$$
$$-\frac{8}{3}\sin^{-1}\left(\frac{p}{4/\sqrt{3}}\right)$$

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8. Using integration, find the area of the region given below:

$$\{(x,y): 0 \le y \le x^2+1, 0 \le y \le x+1, 0 \le x \le 2\}$$

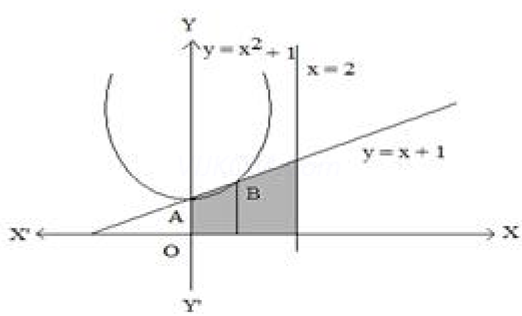
.

Ans. $y = x^2 + 1$

$y = x + 1$

$x = 2$

$$Area = \int_o^1 (x^2+1)dx + \int_1^2 (x+1)dx$$
$$= \frac{23}{6} sq\ unit$$



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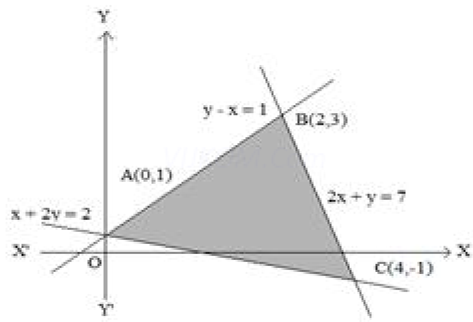
9. Compute the area bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7.

Ans.

$$x + 2y = 2 - - - - - (1)$$

$$y - x = 1 - - - - - - (2)$$
$$2x + y = 7 - - - - - (3)$$

$$Area = \int_o^2 \left[(1 + x) - \left(\frac{2 - x}{2} \right) \right] dx + \int_2^4 \left[(7 - 2x) - \left(\frac{2 - x}{2} \right) \right] dx$$
$$= 6 sq.\ unit$$



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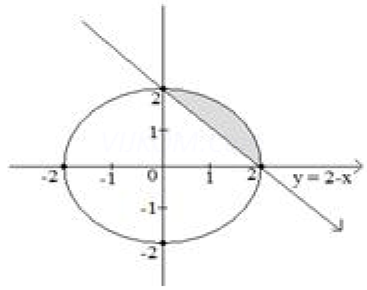
10. Find Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the lines x + y = 2.
Ans.

$$x^2 + y^2 = 4 - - - - - (1)$$

$$x + y = 2 - - - - - - (2)$$

Finding smaller area. On solving (1) and (2)

$$x = 0, 2$$
$$Area = \int_o^2 \sqrt{4 - x^2}\, dx - \int_o^2 (2 - x)\, dx$$
$$= (\pi - 2)\ sq\ unit$$



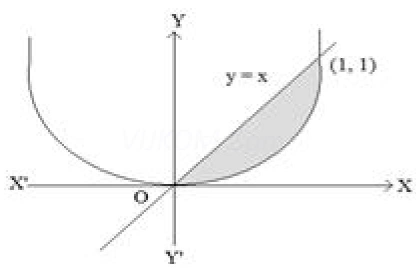
11. Find the area between the curves y = x and y = x^2.

Ans. $y = x$

$y = x^2$

On solving x = 0, 1

$$Area = \int_0^1 \left(x - x^2\right) dx$$
$$= \frac{1}{6} sq\ unit.$$



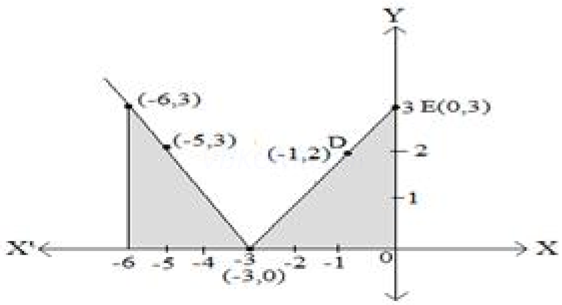
12. Sketch the graph of y = |x + 3| and evaluate $\int_{-6}^{0} |x+3| dx.$

Ans. $y = |x+3|$

$\Rightarrow y = (x+3)$

$y = -(x+3)$

$$\int_{-6}^{0} |x+3| dx = ?$$
$$Area = \int_{-6}^{-3} (x+3) dx + \int_{-3}^{0} (x+3) dx$$
$$= 9 sq.\ unit.$$

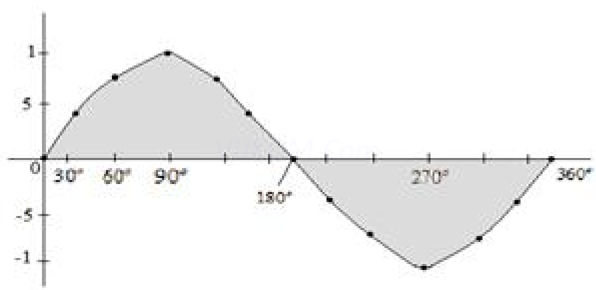


13. **Find the area bounded by the curve y = sinx between x = 0 and x = 2 π**

Ans. $y = \sin x$

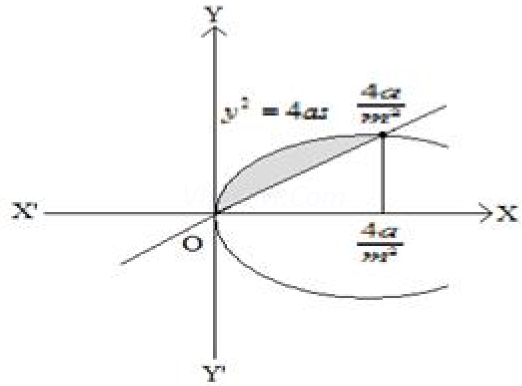
$$x = 0,\ x = 2\pi$$
$$Area = 2\int_o^\pi \sin x dx$$

$$= -2\left[\cos x\right]_o^\pi$$
$$= -2\left[\cos \pi - \cos \theta\right]$$
$$= -2\left[-1-1\right] = 4 sq\ unit$$



14. **Find the area enclosed by the parabola y^2 = 4ax and the line y = mx.**

Ans. $y^2 = 4ax$



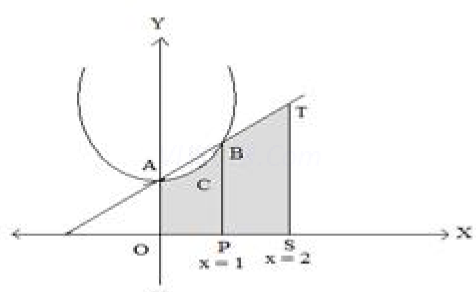
$$y = mx$$
$$x = \frac{4a}{m^2}$$

$$Area = \int_o^{4a/m^2} \sqrt{4ax}dx - \int_o^{\frac{4a}{m^2}} mx\ dx$$
$$= \frac{8a^2}{3m^2} sq\ unit.$$

---

15. Find the area of the region

$$\left\{(x, y) : 0 \le y \le (x^2 + 1), 0 \le y \le (x + 1), 0 \le x \le 2\right\}.$$



Ans. $y = x^2 + 1$

$y = x + 1$

$x \le 2$

$$Area = \int_o^1 (x^2+1)\,dx + \int_o^1 (x+1)\,dx$$
$$= \frac{23}{6}\,sq\ unit$$

---

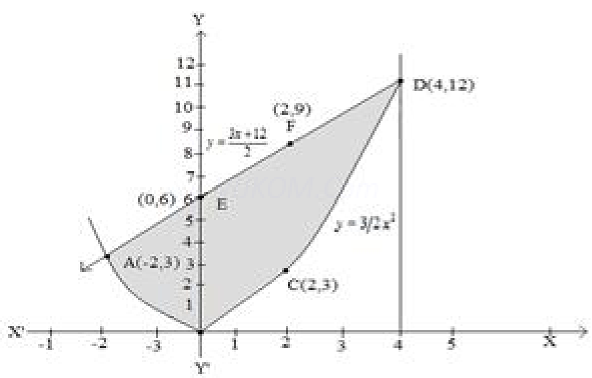
16. Find the area enclosed by the parabola 4y = 3x² and the line 2y = 3x + 12.

Ans. $4y = 3x^2$

$2y = 3x + 12$

$x = -2, 4$

$$Area = \int_{-2}^{4} \frac{3x+12}{2}\,dx - \int_{-2}^{4} \frac{3}{4}x^2\,dx$$
$$= 27\,sq\ unit.$$



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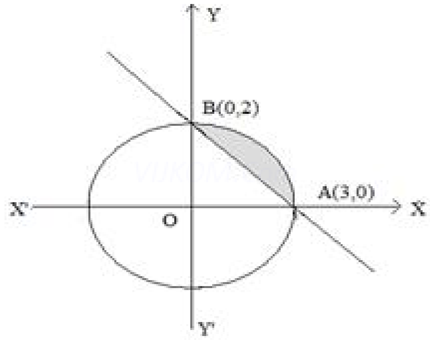
17. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1.$

Ans. $\frac{x^2}{9}+\frac{y^2}{4}=1$



$$\frac{x}{3}+\frac{y}{2}=1$$

$$\Rightarrow \frac{x^2}{(3)^2}\frac{y^2}{(2)^2}=1$$

is the equation of ellipse and

$\frac{x}{3}+\frac{y}{2}=1$ is the equation of intercept form

$$Area=\frac{2}{3}\int_o^3\sqrt{9-x^2}dx-\int_o^3\left(\frac{6-2x}{3}\right)dx$$
$$=\frac{3}{2}(\pi-2)squnit.$$

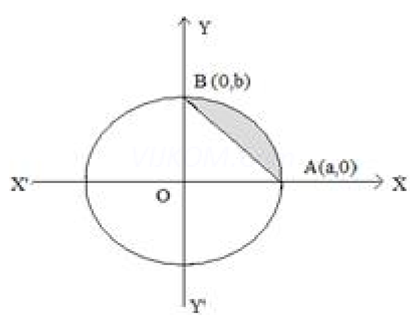
---

18. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$.

Ans.

$$y=\frac{b}{a}\sqrt{a^2-x^2}$$

$$Area=\int_o^a\frac{b}{a}\sqrt{a^2-x^2}dx-\int_o^a\frac{b}{a}(a-x)$$
$$=\frac{ab}{4}(\pi-2)\,sq\ unit$$



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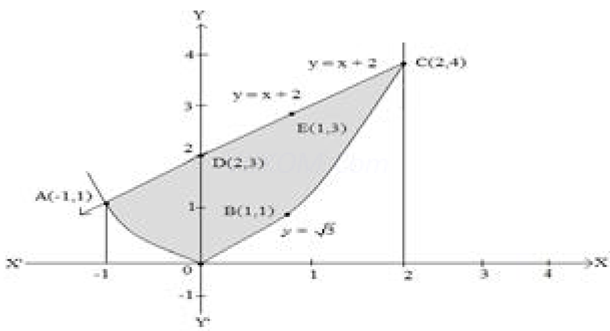
19. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x– axis.

Ans.

$$Area = \int_{-1}^{2}(x+2)\,dx - \int_{-1}^{2} x^2 dx$$

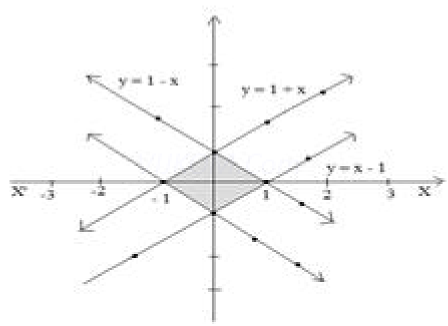
$$= \frac{9}{2} sq.\ unit$$



20. Using method of integration, find the area bounded by the curve |x| + |y| = 1.

Ans. $|x|+|y|=1$

$$\Rightarrow x+y=1$$
$$-x+y=1$$
$$x-y=1$$
$$-x-y=1$$



$$Area = 4\int_{o}^{1}(1-x)\,dx$$
$$= 2 sq\ unit$$

21. Find area bounded by curves

$$\{(x, y) : y \geq x^2 \text{ and } y = |x|\}.$$

Ans. $y = |x|$

$$Area = 2\int_o^1 (x - x^2)\, dx$$

$= \frac{1}{3}\, sq\ unit.$

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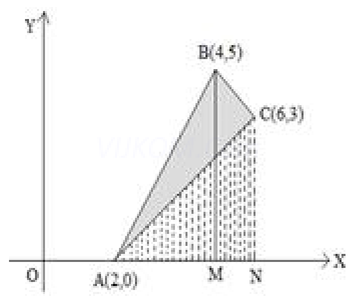
22. Using method of integration find the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3).



Ans.

$$Area = \frac{5}{2}\int_2^4 (x-2)\, dx + \int_4^6 -(x-9)dx - \frac{3}{4}\int_2^6 (x-2)\, dx$$

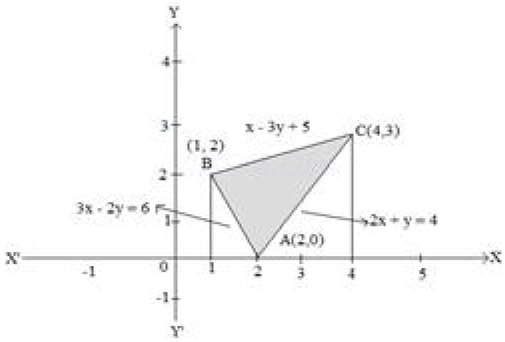
$= 7\, sq\ unit$

---

23. Using method of integration, find the area of the region bounded by lines:
2x + y = 4, 3x – 2y = 6
and x – 3y + 5 = 0.
Ans.

$$Area = \int_1^4 \frac{x+5}{3} dx + \int_1^2 -(2x-4) + \int_2^4 \frac{3x-6}{2}$$

$= \frac{7}{2} sq\ unit.$



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24. Find the area of two regions

$$\left\{(x,y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\right\}.$$

Ans.

$$y^2 = 4x,\ 4x^2 + 4y^2 = 9$$

$$Area = 2\left[\int_o^{1/2} 2\sqrt{x}dx + \int_{1/2}^{3/2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2}\,dx\right]$$

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left[\frac{1}{3}\right]$$