# Class 12 Maths Chapter 9 Differential Equations Important Questions

Very Short Answer Type Questions (1 Mark)

1. Write the order and degree of the following differential equations.

(1) dydx+cosy=0.$\frac{dy}{dx} + \cos\ y = 0.$

Ans: dydx+cosy=0$\frac{dy}{dx} + \cos\ y = 0$

y′+cosx=0$y^{'} + \cos\ x = 0$

Highest order of derivative =1 $= 1$

∴ ∴ Order =1 $= 1$

Degree = Power of y′$y^{'}$

Degree =1 $= 1$

(ii)(dydx)2+3d2ydx2=4 $\left(\frac{dy}{dx}\right)^2 + 3\frac{d^2y}{dx^2} = 4$

Ans:

(dydx)2+3d2ydx2=4 $\left(\frac{dy}{dx}\right)^2 + 3\frac{d^2y}{dx^2} = 4$

Highest order of derivative = 2

Order = 2

Degree = Power of y

Degree = 1

(iii) ∂4y∂x4+sinx=(∂2y∂x2)5.$\frac{\partial^4 y}{\partial x^4} + \sin\ x = \left(\frac{\partial^2 y}{\partial x^2}\right)^5$ .

Ans:

∂4y∂x4+sinx=(∂2y∂x2)5$\frac{\partial^4 y}{\partial x^4} + \sin\ x = \left(\frac{\partial^2 y}{\partial x^2}\right)^5$

Highest order of derivative = 4

Order = 4

Degree = Power of y

Degree = 1

(iv) ∂5y∂x5+log(dydx)=0$\frac{\partial^5 y}{\partial x^5} + \log\ \left(\frac{dy}{dx}\right) = 0$

Ans:

∂5y∂x5+log(dydx)=0$\frac{\partial^5 y}{\partial x^5} + \log\ \left(\frac{dy}{dx}\right) = 0$

Highest order of derivative = 5

Order = 5

Degree = Power of y

Degree = not defined

(v)1+dydx−−−−−−√=(∂2y∂x2)13 $\sqrt{1+\frac{dy}{dx}} = \left(\frac{\partial^2 y}{\partial x^2}\right)^{\frac{1}{3}}$

Ans:

1+dydx−−−−−−√=(∂2y∂x2)13 $\sqrt{1+\frac{dy}{dx}} = \left(\frac{\partial^2 y}{\partial x^2}\right)^{\frac{1}{3}}$

Highest order of derivative = 2

Order = 2

Degree = Power of y

Degree = 2

(vi)[1+(dydx)2]32=k∂2y∂x2 $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{\partial^2 y}{\partial x^2}$

Ans:

[1+(dydx)2]32=k∂2y∂x2 $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{\partial^2 y}{\partial x^2}$

Squaring on both sides

[1+(dydx)2]3=(k∂2y∂x2)2 $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(k\frac{\partial^2 y}{\partial x^2}\right)2$

Highest order of derivative = 2

Order = 2

Degree = Power of y

Degree = 2

(vii)(∂3y∂x3)2+(∂2y∂x2)3=sinx $\left(\frac{\partial^3 y}{\partial x^3}\right)^2 + \left(\frac{\partial^2 y}{\partial x^2}\right)^3 = \sin\ x$

Ans:

(∂3y∂x3)2+(∂2y∂x2)3=sinx $\left(\frac{\partial^3 y}{\partial x^3}\right)^2 + \left(\frac{\partial^2 y}{\partial x^2}\right)^3 = \sin\ x$

Highest order of derivative = 3

Order = 3

Degree = Power of y

Degree = 2

(viii) dydx+tan(dydx)=0$\frac{dy}{dx} + \tan\ \left(\frac{dy}{dx}\right) = 0$

Ans:

dydx+tan(dydx)=0$\frac{dy}{dx} + \tan\left(\frac{dy}{dx}\right) = 0$

Highest order of derivative = 1

Order = 1

Degree = Power of y

Degree = Not defined

2 Write the general solution of following differential equations

(i)dydx=x5+x2−2x$\frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}$

Ans:

dydx=x5+x2−2x$\frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}$

Integrating on both side

∫dydxdx=∫x5dx+∫x2dx−∫2xdx $\int \frac{dy}{dx}dx = \int x^5 dx + \int x^2 dx - \int \frac{2}{x}dx$

y=x66+x33−2log|x|+c$y = \frac{x^6}{6} + \frac{x^3}{3} - 2\log|x| + c$

(ii) (ex+e−x)dy=(ex−e−x)dx $(e^x + e^{-x})\,dy = (e^x - e^{-x})\,dx$

Ans:

(ex+e−x)dy−(ex−e−x)dx=0 $(e^x + e^{-x})\,dy - (e^x - e^{-x})\,dx = 0$

(ex+e−x)dy−(ex−e−x)dx=0 $(e^x + e^{-x})\,dy - (e^x - e^{-x})\,dx = 0$

(ex+e−x)dy=(ex−e−x)dx $(e^x + e^{-x})\,dy = (e^x - e^{-x})\,dx$

dydx=ex−e−xex+e−xdx$\frac{dy}{dx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}\mathrm{dx}$

dy=ex−e−xex+e−xdx$dy = \frac{e^x - e^{-x}}{e^x + e^{-x}}\mathrm{dx}$

Integrating both sides.

∫dy=∫ex−e−xex+e−xdx $\int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}\mathrm{dx}$

y=∫ex−e−xex+e−xdx$y = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}\mathrm{dx}$

t=ex+e−x $\mathrm{t} = e^x + e^{-x}$

dtdx=(ex−e−x)$\frac{dt}{dx} = (e^x - e^{-x})$

dx=dtex−e−x$\mathrm{dx} = \frac{dt}{e^x - e^{-x}}$

Putting value of t$\mathrm{t}$ and dt$\mathrm{dt}$ in (1)

∫dy=∫ex−e−xtdtex−e−x. $\int dy = \int \frac{e^{x} - e^{-x}}{t} \frac{dt}{e^{x} - e^{-x}}$ .

∫dy=∫dtt $\int dy = \int \frac{dt}{t}$

y=log|t|+c$y = \log |t| + c$

Putting back t=ex−e−x$t = e^{x} - e^{-x}$

y=log(ex−e−x)+C$y = \log (e^{x} - e^{-x}) + C$

(iii) dydx=x3+ex+xe$\frac{dy}{dx} = x^{3} + e^{x} + x^{e}$

Ans:

dydx=x3+ex+xe$\frac{dy}{dx} = x^{3} + e^{x} + x^{e}$

dy=(x3+ex+xe)dx$dy = (x^{3} + e^{x} + x^{e})\, dx$

Integrating on both sides

∫dy=∫(x3+ex+xe)dx $\int dy = \int (x^{3} + e^{x} + x^{e})\, dx$

y=x44+ex+∫xedx$y = \frac{x^{4}}{4} + e^{x} + \int x^{e} dx$

y=x44+ex+xe+1e+1+c$y = \frac{x^{4}}{4} + e^{x} + \frac{x^{e+1}}{e+1} + c$

(iv)dydx=5x+y$\frac{dy}{dx} = 5^{x+y}$

Ans:

dydx=5x+y$\frac{dy}{dx} = 5^{x+y}$

dydx=5x.5y$\frac{dy}{dx} = 5^{x}.5^{y}$

15ydy=5xdx$\frac{1}{5^{y}} dy = 5^{x} dx$

Integrating on Both sides

∫15ydy=∫5xdx $\int \frac{1}{5^{y}} dy = \int 5^{x} dx$

5−y=5x+c$5^{-y} = 5^{x} + c$

5x+5−y=c$5^{x} + 5^{-y} = c$

(v) dydx=1−cos2x1+cos2y$\frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2y}$

Ans:

dydx=1−cos2x1+cos2y$\frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2y}$

We know that

cos2x=2cos2x−1$\cos 2x = 2\cos^{2} x - 1$

Putting x=x2 $x = \frac{x}{2}$

cos2x2=2cos2x2−1 $\cos\ \frac{2x}{2} = 2\cos^2\frac{x}{2} - 1$

cosx=2cos2x2−1 $\cos\ x = 2\cos^2\frac{x}{2} - 1$

1+cosx=2cos2x2 $1 + \cos\ x = 2\cos^2\frac{x}{2}$

We know

cos2x=1−2sin2x $\cos\ 2x = 1 - 2\sin^2 x$

Putting x=x2 $x = \frac{x}{2}$

cos22x2=1−2sin2x2 $\cos^2\frac{2x}{2} = 1 - 2\sin^2\frac{x}{2}$

cosx=1−2sin2x2 $\cos\ x = 1 - 2\sin^2\frac{x}{2}$

1−cosx=2sin2x2 $1 - \cos\ x = 2\sin^2\frac{x}{2}$

dydx=1−(1−2sin2x)1+1−2sin2y $\frac{dy}{dx} = \frac{1 - \ (1 - 2\sin^2 x)}{1 + 1 - 2\sin^2 y}$

dydx=sin2x(1−sin2y) $\frac{dy}{dx} = \frac{\sin^2 x}{(1 - \sin^2 y)}$

(1−sin2y)dy=sin2xdx $(1 - \sin^2 y)\ dy = \sin^2 x dx$

Integrating on both sides

∫(1−sin2y)dy=∫sin2xdx $\int\ (1 - \sin^2 y)\ dy = \ \int \sin^2 x dx$

y−y2+sin2y4=x2−sin2x4+c $y - \frac{y}{2} + \frac{\sin\ 2y}{4} = \frac{x}{2} - \frac{\sin\ 2x}{4} + c$

y2+sin2y4=x2−sin2x4+c $\frac{y}{2} + \frac{\sin\ 2y}{4} = \frac{x}{2} - \frac{\sin\ 2x}{4} + c$

By equating

2y+sin2y=2x−sin2y+c $2y + \sin\ 2y = 2x - \sin\ 2y + c$

2y−2x+sin2y−sin2x=c2(y−x)+sin2y−sin2x=c $2y - 2x + \sin\ 2y - \sin\ 2x = c 2\ (y - x)\ + \sin\ 2y - \sin\ 2x = c$

(vi)dydx=1−2y3x+1 $\frac{dy}{dx} = \frac{1 - 2y}{3x + 1}$

Ans:

dydx=1−2y3x+1 $\frac{dy}{dx} = \frac{1 - 2y}{3x + 1}$

2y+sin2y=2x−sin2y+c $2y + \sin\ 2y = 2x - \sin\ 2y + c$

2y−2x+sin2y−sin2x=c2(y−x)+sin2y−sin2x=c $2y - 2x + \sin\ 2y - \sin\ 2x = c 2\ (y - x)\ + \sin\ 2y - \sin\ 2x = c$

Integrating on both sides

∫(11−2y)dy=∫(13x+1)dx $\int\ \left(\frac{1}{1 - 2y}\right) dy = \ \int\ \left(\frac{1}{3x + 1}\right) dx$

−12log(1−2y)=13log(3x+1) $-\frac{1}{2}\log\ (1-2y)\ =\frac{1}{3}\log\ (3x+1)$

2log(3x+1)+3log(1−2y)=c$2\log\ (3x+1)\ +3\log\ (1-2y)\ =c$

3. Write integrating factor of the following differential equations.

(I) dydx+y=cosx−sinx$\frac{dy}{dx}+y=\cos\ x-\sin\ x$

Ans:

dydx+y=cosx−sinx$\frac{dy}{dx}+y=\cos\ x-\sin\ x$

dy/dx+y=cosx−sinx$dy\ /\ dx+y=\cos\ x-\sin\ x$ is a linear differential equation of the type dydx+py=Q$\frac{dy}{dx}+py=Q$

Here I.F.=e∫1.dx=ex$I\ .\ F\ .\ =e^{\int 1\ .\ dx}=e^{x}$

Its solution is given by ⇒yex=∫ex(cosx−sinx)dx ⇒ $ye^{x}=\ \int e^{x}\ (\ \cos\ x-\sin\ x\ )\ dx$

⇒yex=∫excosxdx−∫exsinxdx ⇒ $ye^{x}=\ \int e^{x}\cos\ xdx-\ \int e^{x}\sin\ xdx$

Integrate by parts ⇒yex=excosx−∫−sinxexdx−∫exsindx ⇒ $ye^{x}=e^{x}\cos\ x-\ \int\ -\sin\ xe^{x}dx-\ \int e^{x}\sin\ dx$

∴yex=excosx+C ∴ $ye^{x}=e^{x}\cos\ x+C$

⇒y=cosx+Ce−x ⇒ $y=\cos\ x+Ce^{-x}$

(II) dydx+ysec2x=secx+tanx$\frac{dy}{dx}+y\sec^{2}x=\sec\ x+\tan\ x$

Ans:

dydx+ysec2x=secx+tanx$\frac{dy}{dx}+y\sec^{2}x=\sec\ x+\tan\ x$

dydx+Py=Q$\frac{dy}{dx}+Py=Q$.

∫etanx(dydx+ysec′x)dx=∫etanx⋅tanxsecx $\int e^{\tan\ x}\ (\frac{dy}{dx}+y\sec' x\ )\ dx=\ \int e^{\tan\ x}\cdot\tan\ x\sec\ x$

yetanx=∫etanx(tanxsec1xdx)$ye^{\tan\ x}=\ \int e^{\tan\ x}\ (\ \tan\ x\sec^{1}xdx\ )$

I=∫etanxtanxsec1x⋅dx$I=\ \int e^{\tan\ x}\tan\ x\sec^{1}x\cdot dx$

t=tanx$t=\tan\ x$

dt=sec2udn$dt=\sec^{2}udn$

∫1Iettdt $\int_{\frac{1}{I}} e^{t}tdt$

⇒∫ett−∫etdt ⇒ $\int e^{t}t-\ \int e^{t}dt$

tarxettanx−et−etanx+c$\text{tar}\ xe^{t}\tan\ x-e^{t}-e^{\tan\ x}+c$

yetanx=tanx⋅etanx−etanx+C$ye^{\tan\ x}=\tan\ x\cdot e^{\tan\ x}-e^{\tan\ x}+C$

(III) xdydx+ylogx=x+y$x\frac{dy}{dx}+y\log\ x=x+y$

Ans:

dydx=yx[logyx+1]$\frac{dy}{dx} = \frac{y}{x}\left[\log \frac{y}{x} + 1\right]$

y=Vx$y = \mathrm{V}x$

∫dVVlogV=∫dxx $\int \frac{d\mathrm{V}}{\mathrm{V}\log \mathrm{V}} = \int \frac{dx}{x}$

logV=xc$\log \mathrm{V} = xc$

yx=ecx⇒y=xecx$\frac{y}{x} = e^{cx} \Rightarrow y = xe^{cx}$

(IV) xdydx−3y=x2$x\frac{dy}{dx} - 3y = x^2$

Ans:

xdydx−3y=x2$x\frac{dy}{dx} - 3y = x^2$

xdydx+3y=x2$x\frac{dy}{dx} + 3y = x^2$

dydx+3x·y=x$\frac{dy}{dx} + \frac{3}{x} \cdot y = x$

y.I.F=∫I.F·q(x)·dx$y.I.F = \int I.F \cdot q(x) \cdot dx$

p(x)=3xq(x)=x $p(x) = \frac{3}{x} q(x) = x$

I.F.=e∫3x ·dx $I.F = e^{\int \frac{3}{x} \cdot dx}$

=e3lnx $= e^{3\ln x}$

=(e)lnx3 $= (e)^{\ln x^3}$

=x3 $= x^3$

y·x3=∫x3·x·dx$y \cdot x^3 = \int x^3 \cdot x \cdot dx$

y·x3=x55+c$y \cdot x^3 = \frac{x^5}{5} + c$

y=x25+cx3$y = \frac{x^2}{5} + \frac{c}{x^3}$

(V) dydx+ytanx=secx$\frac{dy}{dx} + y\tan x = \sec x$

Ans:

We have, dydx+ytanx=secx$\frac{dy}{dx} + y\tan x = \sec x$

which is a linear differential equation Here, P=tanx,Q=secx$P = \tan x, Q = \sec x$,

∴ ∴ I.F. =e∫tanxdx=elogsecx=secx$= e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

∴ ∴ The general solution is

ysecx=∫secx·secx+C$y\sec x = \int \sec x \cdot \sec x + C$

⇒ysecx=∫sec2xdx+C ⇒ $y\sec x = \int \sec^2 x dx + C$

⇒ysecx=tanx+C ⇒ $y\sec x = \tan x + C$

4. Write order of the differential equation of the family of following curves

(I)y = Aex + Bex+c$y = Ae^{x} + Be^{x + c}$

Ans:

y = Aex + Bex+c$y = Ae^{x} + Be^{x + c}$

y=ex(A+Bec)=Rex $y = e^{x}\left( A + Be^{c} \right) = \mathrm{Re}^{x}$

dydx = Aex + Bex+c dydx=y$\frac{dy}{dx} = Ae^{x} + Be^{x + c}\frac{dy}{dx} = y$

Therefore, order is 1.

(II)Ay = Bx2$Ay = Bx^{2}$

Ans:

Ay = Bx2$Ay = Bx^{2}$

Differentiating wrt x

Adydx=B$A\frac{dy}{dx} = B$

(2x)dydx=2BxA $\left( 2x \right)\frac{dy}{dx} = \frac{2Bx}{A}$

Therefore, order is 1.

(III)(x − a)2 + (y − b)2 = 9 $\left( x - a \right)^{2} + \left( y - b \right)^{2} = 9$

Ans:

(x − a)2 + (y − b)2 = 9 $\left( x - a \right)^{2} + \left( y - b \right)^{2} = 9$

Differentiate w.r.t x

ddx(x − a)2 + ddx(y − b)2 = 9$\frac{d}{dx}\left( x - a \right)^{2} + \frac{d}{dx}\left( y - b \right)^{2} = 9$

2(x−a)ddx(x−a)+2(y−b)ddx(y−b)=0$2\left( x - a \right)\frac{d}{dx}\left( x - a \right) + 2\left( y - b \right)\frac{d}{dx}\left( y - b \right) = 0$

2(x−a)(dydx−0)+2(y−b)(dydx−0)=0$2\left( x - a \right)\left( \frac{dy}{dx} - 0 \right) + 2\left( y - b \right)\left( \frac{dy}{dx} - 0 \right) = 0$

(x−a)+(y−b)dydx=0 $\left( x - a \right) + \left( y - b \right)\frac{dy}{dx} = 0$

(x−a)= −(y−b)dydx $\left( x - a \right) = - \left( y - b \right)\frac{dy}{dx}$

dydx=x−a−(y−b)$\frac{dy}{dx} = \frac{x - a}{- \left( y - b \right)}$

ddx(x−a)=ddx[(b−y)dydx]$\frac{d}{dx}\left( x - a \right) = \frac{d}{dx}\left[ \left( b - y \right)\frac{dy}{dx} \right]$

dxdx−ddx(a)=(b−y)d2ydx2+dydx$\frac{dx}{dx} - \frac{d}{dx}\left( a \right) = \left( b - y \right)\frac{d^{2}y}{dx^{2}} + \frac{dy}{dx}$

[ddx(b−y)]1−0=(b−y)d2ydx2+dydx(−dydx) $\left[ \frac{d}{dx}\left( b - y \right) \right]1 - 0 = \left( b - y \right)\frac{d^{2}y}{dx^{2}} + \frac{dy}{dx}\left( - \frac{dy}{dx} \right)$

1=(b−y)d2ydx2−(dydx)21 $= (b-y)\frac{d^2y}{dx^2} - (\frac{dy}{dx})^2$

Hence, the order is 2.

(IV)Ax + By2 = Bx2 − Ay$Ax + By^2 = Bx^2 - Ay$

Ans:

Ax + By2 = Bx2 − Ay$Ax + By^2 = Bx^2 - Ay$

Ax+By2=Bx2−Ay$Ax + By^2 = Bx^2 - Ay$

Ax+Ay=Bx2−By2$Ax + Ay = Bx^2 - By^2$

A(x+y)=B(x2−y2)$A(x+y) = B(x^2 - y^2)$

A(x+y)=B(x+y)(x−y)$A(x+y) = B(x+y)(x-y)$

A=B(x−y) $A = B(x-y)$

AB=x−y$\frac{A}{B} = x - y$

AB+y=x$\frac{A}{B} + y = x$

Differentiate w.r.t x

dydx(AB)+dydxy=dydxx$\frac{dy}{dx}(\frac{A}{B}) + \frac{dy}{dx}y = \frac{dy}{dx}x$

Therefore, the order is 1.

(V)x2a2−y2b2=0$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Ans:

x2a2−y2b2=0$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

x2a2=y2b2$\frac{x^2}{a^2} = \frac{y^2}{b^2}$

x2=a2b2y2$x^2 = \frac{a^2}{b^2}y^2$

x=±aby$x = \pm\frac{a}{b}y$

x=aby$x = \frac{a}{b}y$

1=abdydx$1 = \frac{a}{b}\frac{dy}{dx}$

dydx=ba$\frac{dy}{dx} = \frac{b}{a}$

−aby=x $-\frac{a}{b}y = x$

−abdydx=1 $-\frac{a}{b}\frac{dy}{dx} = 1$

dydx=−ba$\frac{dy}{dx} = \frac{-b}{a}$

Therefore, the order is 1.

(VI)y = a cos (x + b)$y = acos(x + b)$

Ans:

Given y=acos(x+b)$y = acos(x + b)$

Differentiating it w.r.t x

dxdy=−asin(x+b)$\frac{dx}{dy} = -asin(x + b)$

Therefore, the order is 1.

(VII)y = a + bex+c$y = a + be^{x+c}$

Ans:

y = a + bex+c$y = a + be^{x+c}$

y=a+bex+c$y = a + be^{x+c}$

bex+c=y−a$be^{x+c} = y - a$

dydx=bex+c$\frac{dy}{dx} = be^{x+c}$

dydx=bex+c=y−a$\frac{dy}{dx} = be^{x+c} = y - a$

diff - wrt x $\text{diff - wrt } x$

d2ydx2=dydx $\frac{d^2y}{dx^2} = \frac{dy}{dx}$

Hence, the order is 2.

5. (i) Show that y=emsin−1x$y = e^{m\sin^{-1}x}$is a solution of

(1−x2)d2ydx2−xdydx−m2y=0 $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$

Ans:

Using the Chain Rule, we get,

⇒y1=dydx=(emsin−1x)⋅ddx(msin−1x) $\Rightarrow y_1 = \frac{dy}{dx} = (e^{m\sin^{-1}x}) \cdot \frac{d}{dx}(m\sin^{-1}x)$

=y⋅{m1−x2−−−−−√}$= y \cdot \{\frac{m}{\sqrt{1 - x^2}}\}$

⇒y1⋅1−x2−−−−−√=my. $\Rightarrow y_1 \cdot \sqrt{1 - x^2} = my$.

y21(1−x2)=m2y2$y_1^2(1 - x^2) = m^2y^2$

W.r.t x, using the product and chain rule

y21⋅ddx(1−x2)+(1−x2)⋅ddx(y21)=m2ddx(y2),$y_1^2 \cdot \frac{d}{dx}(1 - x^2) + (1 - x^2) \cdot \frac{d}{dx}(y_1^2) = m^2\frac{d}{dx}(y^2)$ ,

⇒y21(−2x)+(1−x2)(2y1)·ddx(y1)=m2(2y)·ddx(y) ⇒ $y_1^2(-2x)+(1-x^2)(2y_1)\cdot\frac{d}{dx}(y_1)=m^2(2y)\cdot\frac{d}{dx}(y)$

⇒y21(−2x)+(1−x2)(2y1)(y2)=m2(2y)(y1) ⇒ $y_1^2(-2x)+(1-x^2)(2y_1)(y_2)=m^2(2y)(y_1)$

Dividing by 2y1≠0$2y_1\neq 0$, we get,

(1−x2)y2−xy1=m2y $(1-x^2)y_2-xy_1=m^2y$

Hence, the Proof.

(ii) Show that y=sin(sinx)$y=\sin(\sin x)$ is a solution of differential equation

d2ydx2+(tanx)dydx+ycos2x=0$\frac{d^2y}{dx^2}+(\tan x)\frac{dy}{dx}+y\cos^2x=0$

Ans:

y = sin(sin x) $y=\sin(\sin x)$

dy/dx = cos (sinx). cosx$dy/dx=\cos(\sin x).\cos x$

and d^2y/dx2 = -sin (sinx). cos2x - sinx cos (sinx) and $d^2y/dx^2$ = -sin (sinx) . cos2x - sinx cos (sinx)

LHS = - sin(sinx) cos2x - sinx cos(sinx) + sinx/ cosx cos (sinx) cosx + sin (sinx) cos2 x LHS = - sin ( sinx ) cos2x - sinx cos ( sinx ) + sinx/ cosx cos ( sinx ) cos

= 0 = RHS= 0 = RHS

y = sin(sinx)y = sin(sinx)

⇒dy/dx=cosxcos(sinx) ⇒ dy/dx=cosxcos(sinx)

⇒d2y/dx2= -cosx·cosx·sin(sinx)-sinx·cos(sinx) ⇒ $d^2y/dx^2$= -cosx · cosx · sin(sinx)-sinx · cos(sinx)

⇒d2y/dx2= -ycos2x-(sinx/cosx)dy/dx ⇒ $d^2y/dx^2$= -y$\cos^2$x-(sinx/cosx)dy/dx

= d2y/dx2+tanx(dy/dx)+ycos2x= $d^2y/dx^2$+tanx(dy/dx)+y$\cos^2$x

= 0= 0

(III) Show that y=Ax+Bx $y=Ax+\frac{B}{x}$is a solution x2d2ydx2+xdydx−y=0$\frac{x^2d^2y}{dx^2}+x\frac{dy}{dx}-y=0$

Ans:

Here ‘a’ and ‘b’ are arbitrary

Given solution

y = ax + (b/x), x ≠ 0$y=ax+(b/x), x\neq 0$

y = ax + (b/x) ... (1)$y=ax+(b/x)\ ...\ (1)$

xy = ax2 + b$xy=ax^2+b$

Differentiate with respect to ‘x’

xy + y . 1 = a (2x) = 2ax ... (2) $xy+y.1=a(2x)=2ax\ ...\ (2)$

Differentiate with respect to ‘x’

Differentiate again w.r.t ‘x’

xy + y . 1 + y = 2a ⇒ xy + 2y = 2a … (3)$xy+y.1+y=2a \Rightarrow xy+2y=2a\ ...\ (3)$

Substitute (3) in (2)

xy + y = (xy + 2y)x $xy + y = (xy + 2y)x$

xy + y = x2y + 2xy$xy + y = x^2y + 2xy$

=x2y + xy  y $= x^2y + xyy$

= 0 $= 0$

Hence, Proved

(IV) Show that y=acos(logx)+bsin(logx)$y = a\cos(\log x) + b\sin(\log x)$ is a solution of

x2d2ydx2+xdydx+y=0$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Ans:

y=acos(logx)+bsin(logx)$y = a\cos(\log x) + b\sin(\log x)$

Differentiating w.r.t. x,

dydx=−asin(logx)×1x+bcos(logx)×1xd2ydx2$\frac{dy}{dx} = -a\sin(\log x) \times \frac{1}{x} + b\cos(\log x) \times \frac{1}{x}\frac{d^2y}{dx^2}$

=−a[⎣⎢⎢cos(logx)×1x×x−1⋅sin(logx)x2⎤⎦⎥⎥+b[⎣⎢⎢−sin(logx)×1x×x−1⋅cos(logx)x2⎤⎦⎥⎥

$$= -a\left[\frac{\cos(\log x) \times \frac{1}{x} \times x - 1 \cdot \sin(\log x)}{x^2}\right] + b\left[\frac{-\sin(\log x) \times \frac{1}{x} \times x - 1 \cdot \cos(\log x)}{x^2}\right]$$

⇒x2d2ydx2=−acos(logx)+asin(logx)−bsin(logx)−bcos(logx) $\Rightarrow x^2\frac{d^2y}{dx^2} = -a\cos(\log x) + a\sin(\log x) - b\sin(\log x) - b\cos(\log x)$

⇒x2d2ydx2=−[acos(logx)+bsin(logx)]−[−asin(logx)+bcos(logx)]

$$\Rightarrow x^2\frac{d^2y}{dx^2} = -[a\cos(\log x) + b\sin(\log x)] - [-a\sin(\log x) + b\cos(\log x)]$$

=−y−xdydx $= -y - x\frac{dy}{dx}$

[ (1) and (2)] $[(1)\text{ and }(2)]$

⇒x2d2ydx2+xdydx+y=0 $\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(V) Verify that y=log(x+x2+a2−−−−−−√)$y = \log\left(x + \sqrt{x^2 + a^2}\right)$ satisfies the differential equation:

(a2+x2)d2ydx2+xdydx=0 $(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

Ans: Given

y = l o g ( x + x2+a2−−−−−−√)$y = log\left(x + \sqrt{x^2 + a^2}\right)$

On differentiating with x, we get

dydx=1x+x2+a2−−−−−−√(1+xx2+a2−−−−−−√)$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \frac{x}{\sqrt{x^2 + a^2}}\right)$

=1x2+a2−−−−−−√ $= \frac{1}{\sqrt{x^2 + a^2}}$

On differentiating again with xx, we get d2ydx2=−x(x2+a2)32$\frac{d^2y}{dx^2} = -\frac{x}{(x^2+a^2)^{\frac{3}{2}}}$

Now let's see what is the value of d2ydx2+xdydx$\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

=−x(x2+a2)32+xx2+a2−−−−−−√$= -\frac{x}{(x^2+a^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{x^2+a^2}}$

Conclusion: Therefore,

y=log(x+x2+a2−−−−−−√) $y = \log\ (x + \sqrt{x^2+a^2})$

Is not the solution of d2ydx2+xdydx=0$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

(VI) Find the differential equation of the family of curvesy = ex (A cos x + B sin x)$y = e^x\ (Acosx + Bsinx)$ , where A and B are arbitrary constants.

Ans:

y = ex(Acosx + Bsinx) .....(1)$y = e^x\ (Acosx + Bsinx)\ .\ .\ .\ .\ .\ (1)$

∴ (dy/dx) = ex[ A sin x + B cosx] + [Acosx + Bsinx]ex $\therefore\ (dy\ /\ dx)\ = e^x\ [Asinx + Bcosx]\ +\ [Acosx + Bsinx]\ ex$

∴ (dy/dx) = ex[(B A) sin x + (B + A) cos x] $\therefore\ (dy\ /\ dx)\ = e^x\ [\ (BA)\ sinx +\ (B + A)\ cosx\ ]$

∴ (dy/dx) = ex[(A + B) cos x (A B) sin x] .....(2) $\therefore\ (dy\ /\ dx)\ = e^x\ [\ (A + B)\ cosx\ (AB)\ sinx\ ]\ .\ .\ .\ .\ .\ (2)$

∴ (d2y/dx2) = ex[(A + B) cos x (A B) sin x] + ex[ (A + B) sin x (A B) cos x]
$\therefore\ (d^2y\ /\ dx^2)\ = e^x\ [\ (A + B)\ cosx\ (AB)\ sinx\ ]\ + e^x\ [\ (A + B)\ sinx\ (AB)\ cosx\ ]$

∴ (d2y/dx2) = ex[2B cos x 2A sin x] $\therefore\ (d^2y\ /\ dx^2)\ = e^x\ [\ 2Bcosx2Asinx\ ]$

∴ (d2y/dx2) = 2ex[B cos x A sin x] $\therefore\ (d^2y\ /\ dx^2)\ = 2e^x\ [\ BcosxAsinx\ ]$

∴ (d2y/dx2) × (1/2) = ex[B cos x A sin x] .....(3) $\therefore\ (d^2y\ /\ dx^2)\ \times\ (1\ /\ 2)\ = e^x\ [\ BcosxAsinx\ ]\ .\ .\ .\ .\ .\ (3)$

(1) + (3) y + (1/2)(d2y/dx2) = ex[(A + B) cosx (A B) sinx] $(1)\ +\ (3)\ y +\ (1\ /\ 2)\ (d^2y\ /\ dx^2)\ = e^x\ [\ (A + B)\ cosx\ (AB)\ sinx\ ]$

∴ y + (1/2)(d2y/dx2) = (dy/dx) −−−−−−− (2) $\therefore y +\ (1\ /\ 2)\ (d^2y\ /\ dx^2)\ =\ (dy\ /\ dx)\ -\ -\ -\ -\ -\ -\ -\ (2)$

∴ (d2y/dx2) 2(dy/dx) + 2y = 0 $\therefore\ (d^2y\ /\ dx^2)\ 2\ (dy\ /\ dx)\ + 2y = 0$

(vii) Find the differential equation of an ellipse with major and minor axes 2a and 2b respectively.

Ans:

Equation of ellipse

x2a2+y2b2=1$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Differentiate w.r.t x

2a2[yd2ydx2+(dydx)2]+2b2=0$2a^2\ [y\frac{d^2y}{dx^2} +\ (\frac{dy}{dx})^2\ ]\ + 2b^2 = 0$

yd2ydx2+(dydx)2=−pb2ka2=−b2a2=yxdydx$y\frac{d^2y}{dx^2} +\ (\frac{dy}{dx})^2 = \frac{-\ pb^2}{ka^2} = \frac{-\ b^2}{a^2} = \frac{y}{x}\frac{dy}{dx}$

yd2ydx2+(dydx)2=yxdydx$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$

xydydx2+x(dydx)2=ydydx$xy\frac{dy}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx}$

(viii) Form the differential equation representing the family of curves (y − b)2= 4(x − a). $(y-b)^2 = 4(x-a)$ .

Ans:

The general equation of the given family of curves

(y−b)2=4(x−a)…(i) $(y-b)^2 = 4(x-a) \;\ldots\; (i)$

Differentiating (i) w.r.t.x, we

get

2(y−b)dydx=4⇒(y−b)y′=2 ...(ii)$2(y-b)\frac{dy}{dx} = 4 \Rightarrow (y-b)y' = 2 \;...\; (ii)$

Where dydx=y′.$\frac{dy}{dx} = y'$ .

Differentiating (ii) w.r.t.y, we

get

$(y-b){y}"+{{\left( {{y}'} \right)}^{2}}=0\quad\text{ }\!\!~\!\!\text{ }\ldots

(iii)$,

where d2ydx2=y″$\frac{d^2y}{dx^2} = y''$

Putting (y−b)=2y′ $(y-b) = \frac{2}{y'}$ from (ii) in (iii),

we get 2y″y′+(y′)2=0⇒2y″+(y′)3=0$\frac{2y''}{y'} + (y')^2 = 0 \Rightarrow 2y'' + (y')^3 = 0$

Hence, 2d2ydx2+(dydx)3=0$2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$ is the required differential equation.

6. Solve the following differential equations.

(I)dydx+ycotx=sin2x$\frac{dy}{dx} + y\cot x = \sin 2x$

Ans:

dydx+ycotx=sin2x$\frac{dy}{dx} + y\cot x = \sin 2x$

Comparing equation (1) by dydx+Py=Q$\frac{dy}{dx} + Py = Q$

P=cotx,Q=sinx$P = \cot x$ , $Q = \sin x$

∴ ∴ I.F. =e∫cotxdx $= e^{\int \cot x dx}$

⇒ ⇒ I.F =elogsinx=sinx $= e^{\log \sin x} = \sin x$

Multiplying equation (1) by sinx$\sin x$ sinxdydx+sinx⋅cotxy=sin2x$\sin x\frac{dy}{dx} + \sin x \cdot \cot xy = \sin^2 x$

⇒sinxdydx+cosxy=1−cos2x ⇒ $\sin x\frac{dy}{dx} + \cos xy = 1 - \cos^2 x$

⇒ddx(sinx·y)=1−cos2x ⇒ $\frac{d}{dx}(\sin x \cdot y) = 1 - \cos^2 x$

∫[ddx(sinx·y)]dx=∫1dx−∫cos2xdx $\int \left[\frac{d}{dx}(\sin x \cdot y)\right] dx = \int 1 dx - \int \cos^2 x dx$

⇒ysinx=x−∫1+cos2x2dx ⇒ $y\sin x = x - \int \frac{1 + \cos 2x}{2} dx$

⇒ysinx=x−12∫dx−12∫cos2xdx ⇒ $y\sin x = x - \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x dx$

⇒ysinx=x−12x−12sin2x2+C ⇒ $y\sin x = x - \frac{1}{2}x - \frac{1}{2}\frac{\sin 2x}{2} + C$

⇒ysinx=12x−14sin2x+C ⇒ $y\sin x = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

(II)xdydx+2y=x2logx$x\frac{dy}{dx} + 2y = x^2 \log x$

Ans:

xdydx+2y=x2logx$x\frac{dy}{dx} + 2y = x^2 \log x$

⇒dydx+2xy=xlogx ......(1) ⇒ $\frac{dy}{dx} + \frac{2}{x}y = x\log x \ldots\ldots (1)$

Comparing equation (1) by dydx+Py=Q$\frac{dy}{dx} + Py = Q$

P=2x,Q=xlogx$P = \frac{2}{x}, Q = x\log x$

. I.F. =e∫2xdx=e2logx=elogx2=x2 $= e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

Multiplying equation (1) by x2$x^2$

x2dydx+x2×2xy=x3logx$x^2\frac{dy}{dx} + x^2 \times \frac{2}{x}y = x^3 \log x$

⇒x2dydx+2xy=x3logx ⇒ $x^2\frac{dy}{dx} + 2xy = x^3 \log x$

⇒ddx(x2y)=x3logx ⇒ $\frac{d}{dx}(x^2 y) = x^3 \log x$

Integrating both sides w.r.t. x$x$

x2y=∫x3logxdx+C$x^2 y = \int x^3 \log x dx + C$

=logx∫x3dx−∫{ddx(logx)∫x3dx}dx+C $= \log x \int x^3 dx - \int \left\{\frac{d}{dx}(\log x) \int x^3 dx\right\} dx + C$

(Taking logx$\log x$ as first function) =x44logx−∫1x×x44dx+C $= \frac{x^4}{4}\log x - \int \frac{1}{x} \times \frac{x^4}{4} dx + C$

=x44logx−14∫x3dx+C $= \frac{x^4}{4}\log x - \frac{1}{4}\int x^3 dx + C$

=x44logx−14×x44+C $= \frac{x^4}{4}\log x - \frac{1}{4} \times \frac{x^4}{4} + C$

⇒x2y=116x4[4logx−1]+C ⇒ $x^2 y = \frac{1}{16}x^4 [4\log x - 1] + C$

16x2y=4x4logx−x4+C$16x^2 y = 4x^4 \log x - x^4 + C$

(III) dxdy+1x.y=cosx+sinxx,x>0$\frac{dx}{dy}+\frac{1}{x}.y=\cos\ x+\frac{\sin\ x}{x}, x>0$

Ans: dxdy+1x.y=cosx+sinxx,x>0$\frac{dx}{dy}+\frac{1}{x}.y=\cos\ x+\frac{\sin\ x}{x}, x>0$

P=1x,Q=cosx+sinxx$P=\frac{1}{x}, Q=\cos\ x+\frac{\sin\ x}{x}$

I.F=e∫Pdx=e∫1xdx=elogx=x$\text{I.F}=e^{\int Pdx}=e^{\int\frac{1}{x}dx}=e^{\log\ x}=x$

y(I.F)=∫Q⋅(I.F)dx$y\ (\text{ I.F })\ =\ \int Q\cdot(\text{I.F})dx$

=∫(cosx+sinxx)xdx $=\ \int\ (\cos\ x+\frac{\sin\ x}{x})\ xdx$

=∫cosxxdx+∫sinxdx $=\ \int \cos xxdx+\ \int \sin xdx$

=xsinx−∫sinxdx+∫sinxdx+c $=x\sin\ x-\ \int \sin\ xdx+\ \int \sin\ xdx+c$

=xsinx+c $=x\sin\ x+c$

(Iv)cos3xdydx+cosx=sinx$\cos^3 x\frac{dy}{dx}+\cos\ x=\sin\ x$

Ans:

cos3xdydx+cosx=sinx$\cos^3 x\frac{dy}{dx}+\cos\ x=\sin\ x$

Differentiate w.r.t y

dydx+ycos2n=sinxcos3x$\frac{dy}{dx}+\frac{y}{\cos^2 n}=\frac{\sin\ x}{\cos^3 x}$

dydx+y⋅sec2x=sinxcos3x$\frac{dy}{dx}+y\cdot\sec^2 x=\frac{\sin\ x}{\cos^3 x}$

I⋅F⋅=e∫sec2xdx=etanx$I\cdot F\cdot\ =e^{\int\sec^2 xdx}=e^{\tan\ x}$

y⋅etanx=∫ettanxtanx⋅sec2xdx$y\cdot e^{\tan\ x}=\ \int e^{t\tan\ x}\tan\ x\cdot\sec^2 xdx$

=∫et⋅tdt $=\ \int e^t\cdot tdt$

=et(t−1)+c $=e^t\ (\ t-1\ )\ +c$

y⋅tanx=etanx(tanx−1)+c$y\cdot\tan\ x=e^{\tan\ x}\ (\ \tan\ x-1\ )\ +c$

(V)ydx+(x−y3)dy=0$ydx+\ (\ x-y^3\ )\ dy=0$

Ans:

The given differential equation is

ydx+(x−y3)dy=0$ydx+\ (\ x-y^3\ )\ dy=0$

⇒ydxdy+(x−y3)=0 $\Rightarrow y\frac{dx}{dy}+\ (\ x-y^3\ )\ =0$

⇒ydxdy+x=y3 $\Rightarrow y\frac{dx}{dy}+x=y^3$

⇒dxdy+xy=y2 $\Rightarrow\frac{dx}{dy}+\frac{x}{y}=y^2$

which is a linear differential equation

Thus, IF=e∫dyy=elogy=yIF $= e^{\int \frac{dy}{y}} = e^{\log y} = y$

Multiplying both sides of E (i) by IF and integrating,

we get

x·(IF)=∫Q·(IF)dy+c$x \cdot (\text{IF}) = \int Q \cdot (\text{IF})\, dy + c$

⇒x·y=∫y3dy+c ⇒ $x \cdot y = \int y^3 dy + c$

=y44+c $= \frac{y^4}{4} + c$

which is the required solution.

(VI)yeydx=(y3+2xey)dy$ye^y dx = (y^3 + 2xe^y)\, dy$

Ans:

yeydx=(y3+2xey)dy$ye^y dx = (y^3 + 2xe^y)\, dy$

yey(y3+2xey)=dydx$\frac{ye^y}{(y^3 + 2xe^y)} = \frac{dy}{dx}$ or, dxdy+(−2y)x=y2ey$\frac{dx}{dy} + \left(-\frac{2}{y}\right) x = \frac{y^2}{e^y}$,

which is linear in x$x$.

I.F. =e∫−2ydy=e−2logy=1y2 $= e^{\int \frac{-2}{y} dy} = e^{-2\log y} = \frac{1}{y^2}$

Multiplying both sides by the I.F. and integrating, we get,

x1y2=∫e−ydy$x\frac{1}{y^2} = \int e^{-y} dy$

x1y2=−e−y+C$x\frac{1}{y^2} = -e^{-y} + C$ or x=−y2e−y+cy2$x = -y^2 e^{-y} + cy^2$

When x=0,y=1.0=−e−1+corc=1e$x = 0, y = 1.0 = -e^{-1} + \text{cor}\ \ c = \frac{1}{e}$.

Hence, the required particular solution is

x=−y2e−y+y2e$x = -y^2 e^{-y} + \frac{y^2}{e}$

7. Solve each of the following differential equations:

(I)y−xdydx=2(y2+dydx)$y - x\frac{dy}{dx} = 2\left(y^2 + \frac{dy}{dx}\right)$

Ans:

y−xdydx=2(y2+dydx)$y - x\frac{dy}{dx} = 2\left(y^2 + \frac{dy}{dx}\right)$

Given differential equation can be written as

(x+2)dydx=y−2y2 or dyy(1−2y)=dxx+2 $(x + 2)\frac{dy}{dx} = y - 2y^2$ or $\frac{dy}{y(1-2y)} = \frac{dx}{x+2}$

∫dyy(1−2y)=∫dxx+2 $\int \frac{dy}{y(1-2y)} = \int \frac{dx}{x+2}$

∫[1y+21−2y]dy=∫dxx+2 $\int \left[\frac{1}{y} + \frac{2}{1-2y}\right] dy = \int \frac{dx}{x+2}$

∴log|y|−log|1−ay| $\therefore \log |y| - \log |1 - ay|$

=log|x+2|+logC $= \log |x+2| + \log C$

y1−2y=C(x+2) $\frac{y}{1-2y} = C(x+2)$

y=C(x+2)(1−2y) $y = C(x+2)(1-2y)$

(II)cos y dx + (1 + 2e−x ) sin y dy = 0. $cos y dx + (1 + 2e^{-x}) sin y dy = 0.$

Ans:

cos y dx + (1 + 2e−x ) sin y dy = 0. $cos y dx + (1 + 2e^{-x}) sin y dy = 0.$

⇒∫dx1+2e−x=∫−sinycosydy $\Rightarrow \int \frac{dx}{1+2e^{-x}} = \int \frac{-\sin y}{\cos y} dy$

⇒∫ex2+exdx=∫−sinycosydy $\Rightarrow \int \frac{e^x}{2+e^x} dx = \int \frac{-\sin y}{\cos y} dy$

⇒ln(ex+2)=ln|cosy|+lnC $\Rightarrow \ln (e^x + 2) = \ln |\cos y| + \ln C$

⇒ln(ex+2)=ln|cosy|C $\Rightarrow \ln (e^x + 2) = \ln |\cos y| C$

⇒ex+2=Ccosy⋯(1) $\Rightarrow \mathrm{e}^{\mathrm{x}} + 2 = \mathrm{C}\cos \mathrm{y} \cdots (1)$

⇒ex+2=±Ccosy⇒ex+2=kcosy $\Rightarrow \mathrm{e}^{\mathrm{x}} + 2 = \pm \mathrm{C}\cos \mathrm{y} \Rightarrow \mathrm{e}^{\mathrm{x}} + 2 = \mathrm{k}\cos \mathrm{y}$

x=0,y=π4 in (1) $\mathrm{x} = 0, \mathrm{y} = \frac{\pi}{4}$ in (1)

We get

1+2=kcosπ4 $1 + 2 = \mathrm{k}\cos\frac{\pi}{4}$

⇒k=32–√ $\Rightarrow \mathrm{k} = 3\sqrt{2}$

∴ex+2=32–√cosy $\therefore \mathrm{e}^{\mathrm{x}} + 2 = 3\sqrt{2}\cos \mathrm{y}$

Is the particular solution.

(III) x1+y2−−−−−√dx+y1+x2−−−−−√dy=0 $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$

Ans:

x1+y2−−−−−√dx+y1+x2−−−−−√dy=0 $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$

It is given that

x1+y2−−−−−√dx+y1+x2−−−−−√dy=0 $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$

We can write it as

x1+x2−−−−−√dx+y1+y2−−−−−√dy=0 $\frac{x}{\sqrt{1+x^2}}dx + \frac{y}{\sqrt{1+y^2}}dy = 0$

By integrating the given terms, we get

∫x1+x2−−−−−√dx+∫y1+y2−−−−−√dy=c $\int \frac{x}{\sqrt{1+x^2}}dx + \int \frac{y}{\sqrt{1+y^2}}dy = c$

Using the formula

ddx(1+x2−−−−−√)=2x2·1+x2−−−−−√=x1+x2−−−−−√ $\frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{2x}{2\cdot\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$

We get

1+x2−−−−−√+1+y2−−−−−√=C $\sqrt{1+x^2} + \sqrt{1+y^2} = C$

(IV)(1−x2)(1−y2)−−−−−−−−−−−−−√dy+xydx=0 $\sqrt{(1-x^2)(1-y^2)}\,dy + xydx = 0$

Ans:

(1−x2)(1−y2)−−−−−−−−−−−−−√dy+xydx=0 $\sqrt{(1-x^2)(1-y^2)}\,dy + xydx = 0$

By simplifying the equation, we get

xydydx=−1−x2−y2+x2y2−−−−−−−−−−−−−−−√ $xy\frac{dy}{dx} = -\sqrt{1-x^2-y^2+x^2y^2}$

⇒xydydx=−(1−x2)(1−y2)−−−−−−−−−−−−−√ $\Rightarrow xy\frac{dy}{dx} = -\sqrt{(1-x^2)(1-y^2)}$

=−(1−x2)−−−−−−−√(1−y2)−−−−−−−√ $= -\sqrt{(1-x^2)}\sqrt{(1-y^2)}$

⇒y1−y2−−−−−√dy=−1−x2−−−−−√xdx $\Rightarrow \frac{y}{\sqrt{1-y^2}}dy = -\frac{\sqrt{1-x^2}}{x}dx$

Integrating both sides, we get

∫y1−y2−−−−−√dy=−∫1−x2−−−−−√xdx $\int \frac{y}{\sqrt{1-y^2}}dy = -\int \frac{\sqrt{1-x^2}}{x}dx$

1−y2=t⇒2ydy=dt $1-y^2 = t \Rightarrow 2ydy = dt$

1−x2=m2⇒2xdx=2mdm⇒xdx=mdm $1-x^2 = m^2 \Rightarrow 2xdx = 2mdm \Rightarrow xdx = mdm$

∴ (i) ⇒12∫1t√dt=−∫mm2−1·mdm $\therefore$ (i) $\Rightarrow \frac{1}{2}\int \frac{1}{\sqrt{t}}dt = -\int \frac{m}{m^2-1}\cdot mdm$

⇒12t1/21/2+∫m2m2−1dm=0⇒t√+∫m2+1−1m2−1dm=0 $\Rightarrow \frac{1}{2}\frac{t^{1/2}}{1/2} + \int \frac{m^2}{m^2-1}dm = 0 \Rightarrow \sqrt{t} + \int \frac{m^2+1-1}{m^2-1}dm = 0$

⇒t√+∫(1+1m2−1)dm=0⇒t√+m+12log|||m−1m+1|||=0 $\Rightarrow \sqrt{t} + \int\left(1+\frac{1}{m^2-1}\right)dm = 0 \Rightarrow \sqrt{t} + m + \frac{1}{2}\log\left|\frac{m-1}{m+1}\right| = 0$

Now substituting this value of tt and mm, we get

1−y2−−−−−√−1−x2−−−−−√+12log|||1−y2−−−−−√−11−y2−−−−−√+1|||+C=0 $\sqrt{1-y^2} - \sqrt{1-x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1-y^2}-1}{\sqrt{1-y^2}+1}\right| + C = 0$

(V)(xy2 + x) dx + (yx2 + y) dy = 0; y(0) = 1. $(xy^2+x)\,dx + (yx^2+y)\,dy = 0\,;\, y(0) = 1.$

Ans:

(xy2 + x) dx + (yx2 + y) dy = 0 $(xy^2+x)\,dx + (yx^2+y)\,dy = 0$

(xy2+x)dx=−(x2y+y)dy $(xy^2+x)\,dx = -(x^2y+y)\,dy$

x(y2+1)dx=−y(x2+1)dy $x\left(y^2+1\right)dx=-y\left(x^2+1\right)dy$

x(x2+1)dx=−y1+y2dy=2x(x2+1)dx=−2y1+y2dy $\frac{x}{\left(x^2+1\right)}dx=\frac{-y}{1+y^2}dy=\frac{2x}{\left(x^2+1\right)}dx=\frac{-2y}{1+y^2}dy$

Integrating both sides

∫2x1+x2dx=−∫2y1+y2dy $\int\frac{2x}{1+x^2}dx=-\int\frac{2y}{1+y^2}dy$

This is the integrals of the type

∫f′(x)f(x)dx=log|f(x)|+clog||x2+1|| $\int\frac{f'\left(x\right)}{f\left(x\right)}dx=\log\ \left|f\left(x\right)\right|+c\log\ \left|x^2+1\right|$

=−log||y2+1||+logclog||x2+1||+log||y2+1|| $=-\log\ \left|y^2+1\right|+\log\ c\log\ \left|x^2+1\right|+\log\ \left|y^2+1\right|$

=logclog||x2+1||||y2+1||=logc $=\log\ c\log\ \left|x^2+1\right|\left|y^2+1\right|=\log\ c$

⇒c=||x2+1||||y2+1|| $\Rightarrow c=\left|x^2+1\right|\left|y^2+1\right|$

(VI) dydx=ysin3xcox2x+xex $\frac{dy}{dx}=y\sin^3 x cox^2 x+xe^x$

Ans:

dydx=ysin3xcox2x+xex $\frac{dy}{dx}=y\sin^3 x cox^2 x+xe^x$

dydx=sin3x⋅cos2x+x⋅ex⋅(1) $\frac{dy}{dx}=\sin^3 x\cdot\cos^2 x+x\cdot e^x\cdot\ \left(1\right)$

dy=[(sin3x⋅cos2x)+x⋅ex]⋅dx→2 $dy=\left[\ \left(\sin^3 x\cdot\cos^2 x\right)+x\cdot e^x\right]\cdot dx\rightarrow 2$

Taking the integral sign in both side of equation (2)

∫dy=∫sin3x⋅cos2xdx+∫x⋅ex⋅dx+c $\int dy=\int\sin^3 x\cdot\cos^2 xdx+\int x\cdot e^x\cdot dx+c$

∫sin3x⋅cos2xdx wt cosx=t $\int\sin^3 x\cdot\cos^2 xdx$ wt $\cos\ x=t$

∫sinx⋅sin2x⋅cos2xdx $\int\sin x\cdot\sin^2 x\cdot\cos^2 xdx$

∫sinx(1−cos2x)⋅cos2xdx $\int\sin x\left(1-\cos^2 x\right)\cdot\cos^2 xdx$

−∫(1−t2)⋅t2dt⇒−∫t2−t4dt $-\int\ \left(1-t^2\right)\cdot t^2dt\Rightarrow-\int t^2-t^4dt$

=−[t33−t55] $=-\left[\frac{t^3}{3}-\frac{t^5}{5}\right]$

⇒−[cos3x3−cos5ts] $\Rightarrow-\left[\frac{\cos^3 x}{3}-\frac{\cos^5 t}{s}\right]$

∫x⋅exdx=x∫endx−∫ddxx⋅∫exdx]dx $\int x\cdot e^xdx=x\int e^ndx-\int\frac{d}{dx}x\cdot\int e^xdx\ ]\ dx$

=xex−∫exdx $=xe^x-\int e^xdx$

=xex−ex $=xe^x-e^x$

=ex(x−1) $=e^x\left(x-1\right)$

From Equation (3)

y=−[cos3x3−cos5x5]+en(x−1)+C$y = -\left[\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5}\right] + e^n(x-1) + C$

(VII)sec2xtanydx+sec2ytanxdy=0$\sec^2 x\tan y dx + \sec^2 y\tan x dy = 0$

Ans:

Given equation:

sec2xtanydx+sec2ytanxdy=0$\sec^2 x\tan y dx + \sec^2 y\tan x dy = 0$

Dividing both sides by tanytanx$\tan y\tan x$

sec2xtanydx+sec2ytanxdytanytanx=0tanxtany$\frac{\sec^2 x\tan y dx + \sec^2 y\tan x dy}{\tan y\tan x} = \frac{0}{\tan x\tan y}$

sec2xtanydxtanytanx+sec2ytanxdytanytanx=0$\frac{\sec^2 x\tan y dx}{\tan y\tan x} + \frac{\sec^2 y\tan x dy}{\tan y\tan x} = 0$

sec2xtanxdx+sec2ytanydy=0$\frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy = 0$

Integrating both sides

∫(sec2xtanxdx+sec2ytanydy)=0$\int\left(\frac{\sec^2 x}{\tan x}dx + \frac{\sec^2 y}{\tan y}dy\right) = 0$

∫sec2xtanxdx+∫sec2ytanydy=0$\int\frac{\sec^2 x}{\tan x}dx + \int\frac{\sec^2 y}{\tan y}dy = 0$

u=tanx , v=tany$u = \tan x$ , $v = \tan y$

Diff u w.r.t. x and v w.r.t y

dudx=sec2x$\frac{du}{dx} = \sec^2 x$

dusec2x=dx$\frac{du}{\sec^2 x} = \mathrm{dx}$

dvdy=sec2y$\frac{dv}{dy} = \sec^2 y$

dvsec2y=dy$\frac{dv}{\sec^2 y} = \mathrm{dy}$

Therefore, our equation becomes

∫sec2xtanxdx+∫sec2ytanydy=0$\int\frac{\sec^2 x}{\tan x}dx + \int\frac{\sec^2 y}{\tan y}dy = 0$

∫sec2xtanxdusec2x+∫sec2yvdvsec2y=0$\int\frac{\sec^2 x}{\tan x}\frac{du}{\sec^2 x} + \int\frac{\sec^2 y}{v}\frac{dv}{\sec^2 y} = 0$

∫duu+∫dvv=0$\int\frac{du}{u} + \int\frac{dv}{v} = 0$

log|u|+log|v|=logc$\log|u| + \log|v| = \log c$

u=tanx and v=tany$u = \tan x$ and $v = \tan y$

log|tanx|+log|tany|=logc$\log|\tan x| + \log|\tan y| = \log c$

log|tanx+tany|=logc$\log|\tan x + \tan y| = \log c$

tanxtany=C$\tan x \tan y = C$

8. Solve the following differential equations:

(I)x2ydx−(x3+y3)dy=0$x^2 ydx - (x^3 + y^3) dy = 0$

Ans:

(x3+y3)dy−x2ydx=0 $(x^3 + y^3) dy - x^2 ydx = 0$

Is rearranged as

dydx=x2yx3+y3$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$

yx=v⇒y=vx$\frac{y}{x} = v \Rightarrow y = vx$

dydx=v+xdvdx$\frac{dy}{dx} = v + x\frac{dv}{dx}$

∴v+xdvdx=v1+v3 $\therefore v + x\frac{dv}{dx} = \frac{v}{1 + v^3}$

⇒xdvdx=v1+v3−v=−v41+v3 $\Rightarrow x\frac{dv}{dx} = \frac{v}{1 + v^3} - v = \frac{-v^4}{1 + v^3}$

⇒1+v3v4dv=−dxx $\Rightarrow \frac{1 + v^3}{v^4}dv = -\frac{dx}{x}$

Integrating both sides, we get ∫(1v4+1v)dv=−∫dxx $\int \left(\frac{1}{v^4} + \frac{1}{v}\right) dv = -\int \frac{dx}{x}$

⇒−13v3+log|v|=−log|x|+C $\Rightarrow -\frac{1}{3v^3} + \log |v| = -\log |x| + C$

⇒−x33y3+log||(yx)||=−log|x|+C $\Rightarrow -\frac{x^3}{3y^3} + \log \left|\left(\frac{y}{x}\right)\right| = -\log |x| + C$

⇒−x33y3+log|y|=C $\Rightarrow \frac{-x^3}{3y^3} + \log |y| = C$

Is the solution of the given differential equation.

(II)x2dydx=x2+xy+y2$x^2\frac{dy}{dx} = x^2 + xy + y^2$

Ans:

Solution of the given differential equation Re-writing the given equation as

x2dydx=x2+xy+y2$x^2\frac{dy}{dx} = x^2 + xy + y^2$

dydx= 1+y2x2+yx$\frac{dy}{dx} = 1 + \frac{y^2}{x^2} + \frac{y}{x}$

It is clearly a homogenous differential equation

Assuming y=vx$y = vx$

Differentiating both sides

dydx=v+xdvdx $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substituting dydx $\frac{\mathrm{dy}}{\mathrm{dx}}$ from the given equation

1+y2x2+yx=v+xdvdx $1 + \frac{y^2}{x^2} + \frac{y}{x} = v + x\frac{dv}{dx}$

1+v2+v=v+xdvdx $1 + \mathrm{v}^2 + \mathrm{v} = \mathrm{v} + \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}$

1+v2=xdvdx $1 + \mathrm{v}^2 = \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}$

dxx=dv1+v2 $\frac{dx}{x} = \frac{dv}{1 + v^2}$

Now integrating both sides

∫dxx=∫dv1+v2 $\int \frac{dx}{x} = \int \frac{dv}{1 + v^2}$

lnx=arctanv+c $\ln\ \mathrm{x} = \arctan\ \mathrm{v} + \mathrm{c}$

Formula

∫dxx=lnx∫dv1+v2=tan−1v $\int \frac{\mathrm{dx}}{\mathrm{x}} = \ln\ \mathrm{x} \int \frac{\mathrm{dv}}{1 + \mathrm{v}^2} = \tan^{-1}\mathrm{v}$

v=yx $\mathrm{v} = \frac{\mathrm{y}}{\mathrm{x}}$

lnx=tan−1yx+c $\ln\ x = \tan^{-1}\frac{y}{x} + c$

Is the solution of given differential equation.

(III) (x2−y2)dx+2xydy=0,y(1)=1 $(x^2 - y^2)\ dx + 2xydy = 0\ ,\ y(1) = 1$

Ans:

(x2−y2)dx+2xydy=0 $(x^2 - y^2)\ dx + 2xydy = 0$

dydx=−(x2−y2)2xy $\frac{dy}{dx} = -\frac{(x^2 - y^2)}{2xy}$

It is homogeneous differential equation.

Putting y=ux⇒u+xdudx=dydx $y = ux \Rightarrow u + \frac{xdu}{dx} = \frac{dy}{dx}$

From (I)

u+xdudx=−x2(1−u2)2x2u=−(1−u22u) $u + x\frac{du}{dx} = -x^2\frac{(1 - u^2)}{2x^2u} = -\left(\frac{1 - u^2}{2u}\right)$

⇒xdudx=−[1−u22u+u] $\Rightarrow \frac{xdu}{dx} = -\left[\frac{1 - u^2}{2u} + u\right]$

⇒xdudx=−[1+u22u] ⇒ $\frac{xdu}{dx} = -\left[\frac{1+u^2}{2u}\right]$

⇒2u1+u2du=−dxx ⇒ $\frac{2u}{1+u^2}du = -\frac{dx}{x}$

Integrating both sides, we get

⇒∫2udu1+u2=−∫dxx ⇒ $\int \frac{2udu}{1+u^2} = -\int \frac{dx}{x}$

⇒log||1+u2||=−log|x|+logC ⇒ $\log\ |1+u^2| = -\log\ |x| + \log\ C$

⇒log|||x2+y2x2||||x|=logC ⇒ $\log\ \left|\frac{x^2+y^2}{x^2}\right|\ |x| = \log\ C$

⇒x2+y2x=C ⇒ $\frac{x^2+y^2}{x} = C$

⇒x2+y2=Cx ⇒ $x^2 + y^2 = Cx$

Given that y=1$y = 1$ when x=1$x = 1$ ⇒1+1=C⇒C=2 ⇒ $1 + 1 = C \Rightarrow C = 2$

∴ ∴ Solution is x2+y2=2x$x^2 + y^2 = 2x$.

(IV)dydx=yx+tan(yx)$\frac{dy}{dx} = \frac{y}{x} + \tan\ \left(\frac{y}{x}\right)$

Ans:

xdydx=y+xtan(yx)$\frac{xdy}{dx} = y + x\tan\ \left(\frac{y}{x}\right)$

dydx=yx+tan(yx)$\frac{dy}{dx} = \frac{y}{x} + \tan\ \left(\frac{y}{x}\right)$

y=vx$y = vx$

dydx=v+xdvdx$\frac{dy}{dx} = v + x\frac{dv}{dx}$

v+xdvdx=v+tanv$v + x\frac{dv}{dx} = v + \tan\ v$

xdvdx=tanv$x\frac{dv}{dx} = \tan\ v$

∫dvtanv=∫dxx⇒ $\int \frac{dv}{\tan\ v} = \int \frac{dx}{x} \Rightarrow$

∫cosvsinvdv=∫dxx $\int \frac{\cos\ v}{\sin\ v}dv = \int \frac{dx}{x}$

logsinv=logx+logc$\log\ \sin\ v = \log\ x + \log\ c$

logsin(yx)=logcx$\log\ \sin\ \left(\frac{y}{x}\right) = \log\ \frac{c}{x}$

⇒sin(yx)=cx ⇒ $\sin\ \left(\frac{y}{x}\right) = \frac{c}{x}$

sin(yx)=cx$\sin\ \left(\frac{y}{x}\right) = cx$

(V) dydx=2xyx2+y2$\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$

Ans:

(x2+y2)dydx=2xy $(x^2+y^2)\frac{dy}{dx}=2xy$

dydx=2xyx2+y2$\frac{dy}{dx}=\frac{2xy}{x^2+y^2}$

dydx=(x2+y2)/2xy….(i)$\frac{dy}{dx}=(x^2+y^2)/2xy\ldots.(i)$

Let x=vy$x=vy$

Here, differentiating w.r.t. yy,

dxdy=v.(dydx)+y(dvdy)$\frac{dx}{dy}=\mathrm{v}.\left(\frac{dy}{dx}\right)+\mathrm{y}\left(\frac{dv}{dy}\right)$

dxdy=(v+y)·(dydx)$\frac{dx}{dy}=(\mathrm{v}+\mathrm{y})\cdot\left(\frac{dy}{dx}\right)$

Here, from e (i),

v+y(dvdy)=(v2y2+y2)/2vy2$v+y\left(\frac{dv}{dy}\right)=(v^2y^2+y^2)/2vy^2$

v+y(dvdy)=y2(v2+1)/y22v$v+y\left(\frac{dv}{dy}\right)=y^2(v^2+1)/y^22v$

v+y(dvdy)=(v2+1)/2v$v+y\left(\frac{dv}{dy}\right)=(v^2+1)/2v$

y(dvdy)=((v2+1)/2v)−(v/1)$y\left(\frac{dv}{dy}\right)=((v^2+1)/2v)-(v/1)$

y(dvdy)=(v2+1−2v2)/2v$y\left(\frac{dv}{dy}\right)=(v^2+1-2v^2)/2v$

y(dvdy)=(−v2+1)/2v$y\left(\frac{dv}{dy}\right)=(-v^2+1)/2v$

y(dvdy)=(1−v2)/2v$y\left(\frac{dv}{dy}\right)=(1-v^2)/2v$

2v/(1−v2)dv=1ydy$2v/(1-v^2)dv=\frac{1}{y}dy$

Integrating both sides

∫2v(v2−1)dv=−∫dyy $\int 2v(v^2-1)dv=-\int\frac{dy}{y}$

log||v2−1||=−logy+logc$\log|v^2-1|=-\log y+\log c$

log||v2−1||y=logc$\log|v^2-1|y=\log c$

v2y−y=c$v^2y-y=c$

(x2y2)x(y−y)=c$\left(\frac{x^2}{y^2}\right)x(y-y)=c$

(x2y)−y=c$\left(\frac{x^2}{y}\right)-y=c$

x2−y2y=c$\frac{x^2-y^2}{y}=c$

x2−y2=cy$x^2 - y^2 = cy$

(VI) dydx=ex+y+x2ey$\frac{dy}{dx} = e^{x+y} + x^2 e^y$

Ans:

From the integral

dydx=ex+y+x2⋅ey$\frac{dy}{dx} = e^{x+y} + x^2 \cdot e^y$

dydx=ex⋅ey˙+x2⋅ey$\frac{dy}{dx} = e^x \cdot \dot{e^y} + x^2 \cdot e^y$

dydx=ey(ex+x2)$\frac{dy}{dx} = e^y \left( e^x + x^2 \right)$

∫dyey=∫(ex+x2)dx $\int \frac{dy}{e^y} = \int \left( e^x + x^2 \right) dx$

⇒logey=ex+x33+C $\Rightarrow \log\ e^y = e^x + \frac{x^3}{3} + C$

∴y=ex+x33+C $\therefore y = e^x + \frac{x^3}{3} + C$

(VII) dydx=1−y21−x2−−−−−−√$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Ans:

dydx=1−y21−x2−−−−−−√$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

It is given that

dydx−1−y21−x2−−−−−−√=0$\frac{dy}{dx} - \sqrt{\frac{1-y^2}{1-x^2}} = 0$

We can write it as

dydx=1−y21−x2−−−−−−√$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

By cross multiplication

11−y2−−−−−√dy=11−x2−−−−−√dx$\frac{1}{\sqrt{1-y^2}} dy = \frac{1}{\sqrt{1-x^2}} dx$

By integrating both sides w.r.t x$x$

∫11−y2−−−−−√dy=∫11−x2−−−−−√dx $\int \frac{1}{\sqrt{1-y^2}} dy = \int \frac{1}{\sqrt{1-x^2}} dx$

We get

sin−1y=sin−1x+c$\sin^{-1} y = \sin^{-1} x + c$

(VIII) (3xy+y2)dx+(x2+xy)dy $\left( 3xy + y^2 \right) dx + \left( x^2 + xy \right) dy$

Ans:

dydx=−(3xy+y2x2+xy) $\frac{dy}{dx} = -\left(\frac{3xy + y^2}{x^2 + xy}\right)$

y=vx⇒dydx=v+xdvdx $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

v+xdvdx=−(3x·vx+v2x2x2+x·vx) $v + x\frac{dv}{dx} = -\left(\frac{3x \cdot vx + v^2x^2}{x^2 + x \cdot vx}\right)$

xdvdx=−3v−v21+v−v $x\frac{dv}{dx} = \frac{-3v - v^2}{1 + v} - v$

xdvdx=−3v−v2−v−v21+v $x\frac{dv}{dx} = \frac{-3v - v^2 - v - v^2}{1 + v}$

xdvdx=−2v2−4v1+v $x\frac{dv}{dx} = \frac{-2v^2 - 4v}{1 + v}$

1+v2v2+4vdv=−1xdx $\frac{1 + v}{2v^2 + 4v}dv = -\frac{1}{x}dx$

∫1+v2v2+4vdv=∫−1xdx $\int \frac{1 + v}{2v^2 + 4v}dv = \int -\frac{1}{x}dx$

14∫2+2v2v+v2dv=−∫1xdx $\frac{1}{4}\int \frac{2 + 2v}{2v + v^2}dv = -\int \frac{1}{x}dx$

14log||v2+2v||=−log|x|+c $\frac{1}{4}\log\left|v^2 + 2v\right| = -\log|x| + c$

14log(y2x2+2yx)·x=c $\frac{1}{4}\log\left(\frac{y^2}{x^2} + 2\frac{y}{x}\right) \cdot x = c$

log(y2x+2y)=4c $\log\left(\frac{y^2}{x} + 2y\right) = 4c$

9. (I) Form the differential equation of the family of circles touching y-axis at (0, 0).

Ans:

The centre of the circle touching the y-axis at origin lies on the x-axis.

Let (a, 0) be the centre of the circle.

Since it touches the y-axis at origin, its radius is a.

Now, the equation of the circle with centre (a, 0) and radius (a) is

(x−a)2+y2=a2. $(x - a)^2 + y^2 = a^2$.

⇒x2+y2=2ax $\Rightarrow x^2 + y^2 = 2ax$

(Image will be uploaded soon)

Differentiating equation (1) with respect to x$x$, we get:

2x+2yy′=2a $2x + 2yy' = 2a$

⇒x+yy′=a $\Rightarrow x + yy' = a$

Now, on substituting the value of a in equation (1), we get:

x2+y2=2(x+yy′)x$x^2+y^2=2(x+yy')x$

⇒x2+y2=2x2+2xyy′ ⇒ $x^2+y^2=2x^2+2xyy'$

⇒2xyy′+x2=y2 ⇒ $2xyy'+x^2=y^2$

This is the required differential equation.

(ii) Form the differential equation of family of parabolas having vertex at (0, 0) $(0,0)$ and axis along the

(i) positive y-axis

Ans:

The equation of the parabola having the vertex at origin and the axis along the positive

y$y$-axis is:

x2=4ay$x^2=4ay$…….1

(Image will be uploaded soon)

Differentiating equation (1) with respect to x$x$, we get:

2x=4ay′$2x=4ay'$……(2)

Dividing equation (2) by equation (1), we get:

2xx2=4ay′4ay$\frac{2x}{x^2}=\frac{4ay'}{4ay}$

⇒2x=y′y ⇒ $\frac{2}{x}=\frac{y'}{y}$

⇒xy′=2y ⇒ $xy'=2y$

⇒xy′−2y=0 ⇒ $xy'-2y=0$

This is the required differential equation.

(ii) Positive x-axis

Ans:

Since parabola has axis along positive x$x$-axis,

its Equation: y2=4ax$y^2=4ax$...(1)

(Image will be uploaded soon)

Diff. w.r.t. x$x$

ddx(y2)=ddx(4ax)$\frac{d}{dx}(y^2)=\frac{d}{dx}(4ax)$

2ydydx=4a$2y\frac{dy}{dx}=4a$

Putting Value of 4 a in (1) $(1)$

y2=2ydydx×x$y^2=2y\frac{dy}{dx}\times x$

y2−2xydydx=0$y^2-2xy\frac{dy}{dx}=0$

(iii) Form differential equation of family of circles passing through origin and whose Centre lie on x-axis.

Ans:

Equation of circle is (x − a) + y = a ... (1) $(x-a)+y=a\ldots(1)$ ( ' a' arbitrary constant)

Differentiate with respect to 'x'

2(x−a)+2ydydx=0 $2(x-a)+2y\frac{dy}{dx}=0$

(x−a)+ydydx=0⇒a=x+ydydx $(x-a)+y\frac{dy}{dx}=0 \Rightarrow a=x+y\frac{dy}{dx}$

Substituting in (1) y2(dydx)2+y2=(x+ydydx)2 $y^2\left(\frac{dy}{dx}\right)^2+y^2=\left(x+y\frac{dy}{dx}\right)^2$

y2(dydx)2+y2=x2+2xydydx+y2(dydx)2 $y^2\left(\frac{dy}{dx}\right)^2+y^2=x^2+2xy\frac{dy}{dx}+y^2\left(\frac{dy}{dx}\right)^2$

x2+2xydydx−y2=0 $x^2+2xy\frac{dy}{dx}-y^2=0$

(iv) Form the differential equation of the family of circles in the first quadrant and touching the coordinate axes.

Ans:

Let the equation of the circle be

(x−a)2+(y−a)2=a2 $(x-a)^2+(y-a)^2=a^2$

⇒x2+y2−2a(x+y)+a2=0 ⇒ $x^2+y^2-2a(x+y)+a^2=0$

⇒2x+2y−2a(1+y1)=0 ⇒ $2x+2y-2a(1+y_1)=0$

⇒x+y−a(1+y1)=0 ⇒ $x+y-a(1+y_1)=0$

⇒a=x+y1+y1 ⇒ $a=\frac{x+y}{1+y_1}$

Hence, the required differential equation is

x2+y2−2(x+y1+y1)(x+y)+(x+y1+y1)2=0 $x^2+y^2-2\left(\frac{x+y}{1+y_1}\right)(x+y)+\left(\frac{x+y}{1+y_1}\right)^2=0$

⇒(1+y1)2(x2+y2)−2(x+y)2(1+y1) ⇒ $(1+y_1)^2(x^2+y^2)-2(x+y)^2(1+y_1)$ +(x+y)2=0 $+(x+y)^2=0$

10. Show that the differential equation dydx=x+2yx−2y $\frac{dy}{dx}=\frac{x+2y}{x-2y}$ is homogeneous and solve it.

Ans:

Step 1: Find dydx $\frac{dy}{dx}$

(x−2y)dydx=x+2y $(x-2y)\frac{dy}{dx}=x+2y$

dydx=(x+2yx−2y) $\frac{dy}{dx}=\left(\frac{x+2y}{x-2y}\right)$

Step 2: Put F(x,y)=dydx $F(x,y)=\frac{dy}{dx}$ Find F(λx,λy) $F(\lambda x,\lambda y)$

dydx=(x+2yx−2y) $\frac{dy}{dx}=\left(\frac{x+2y}{x-2y}\right)$

Put F(x,y)=(x+2yx−2y) $F(x,y)=\left(\frac{x+2y}{x-2y}\right)$

Finding

F(λx,λy) $F(\lambda x, \lambda y)$

F(λx,λy)$F(\lambda x, \lambda y)$ =λx+2(λy)λx−2λy $= \frac{\lambda x + 2(\lambda y)}{\lambda x - 2\lambda y}$

=λ(x+2y)λ(x−2y) $= \frac{\lambda(x + 2y)}{\lambda(x - 2y)}$

=(x+2y)x−2y $= \frac{(x + 2y)}{x - 2y}$

=F(x,y) $= F(x, y)$

Thus, F(λx,λy) =F(x,y)=λ∘F(x,y) $F(\lambda x, \lambda y) = F(x, y) = \lambda \circ F(x, y)$

Thus, F(x,y)$F(x, y)$ is Homogeneous function of degree zero

Therefore, the given Differential Equation is Homogeneous

differential Equation

Step 3: Solving dydx$\frac{dy}{dx}$ by Putting y=vx$y = vx$

dydx=(x+2yx−2y)$\frac{dy}{dx} = \left(\frac{x + 2y}{x - 2y}\right)$

Let y=vx$y = vx$

So, dydx=d(vx)dx$\frac{dy}{dx} = \frac{d(vx)}{dx}$

dydx=dvdx⋅x+vdxdx$\frac{dy}{dx} = \frac{dv}{dx} \cdot x + v\frac{dx}{dx}$

dydx=dvdxx+v$\frac{dy}{dx} = \frac{dv}{dx}x + v$

Putting in eqn, (1)

dydx , yx $\frac{dy}{dx}$, $\frac{y}{x}$

dydx=x+2yx−2y$\frac{dy}{dx} = \frac{x + 2y}{x - 2y}$

dvdxx+v=x+2vxx−2vx$\frac{dv}{dx}x + v = \frac{x + 2vx}{x - 2vx}$

dvdxx+v=x(1+2v)x(1−2v)$\frac{dv}{dx}x + v = \frac{x(1 + 2v)}{x(1 - 2v)}$

dvdxx+v=1+2v1−2v$\frac{dv}{dx}x + v = \frac{1 + 2v}{1 - 2v}$

dvdxx=1+2v1−2v−v$\frac{dv}{dx}x = \frac{1 + 2v}{1 - 2v} - v$

dvdx⋅x=1+2v−v+2v21−2v$\frac{dv}{dx} \cdot x = \frac{1 + 2v - v + 2v^2}{1 - 2v}$

dvdx⋅x=2v2+v+11−2v$\frac{dv}{dx} \cdot x = \frac{2v^2 + v + 1}{1 - 2v}$

dvdxx=−(2v2+v+12v−1)$\frac{dv}{dx}x = -\left(\frac{2v^2 + v + 1}{2v - 1}\right)$

dv(2v−12v2+v+1)=−dxx$dv\left(\frac{2v-1}{2v^2+v+1}\right) = \frac{-dx}{x}$

Integrating Both Sides

∫2v−12v2+v+1dv=∫−dxx $\int \frac{2v-1}{2v^2+v+1}dv = \int \frac{-dx}{x}$

∫2v−12v2+v+1dv=−∫dxx $\int \frac{2v-1}{2v^2+v+1}dv = -\int \frac{dx}{x}$

∫(2v−1)dv2v2+v+1dv=−log|x|+c $\int \frac{(2v-1)\,dv}{2v^2+v+1}dv = -\log |x| + c$

By equating

log||x2+xy+y2||=23–√tan−1(x+2y3x−−√)+c$\log |x^2+xy+y^2| = 2\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3x}}\right)+c$

12. Solve the following differential equations:

(i) dydx−2y=cos3x$\frac{dy}{dx} - 2y = \cos 3x$

Ans:

dydx+(−2)y=cos3x……(1)$\frac{dy}{dx} + (-2)y = \cos 3x \ldots \ldots (1)$

This is a linear differential equation of the form

dydx+Py=Q$\frac{dy}{dx} + Py = Q$, where P=−2$P = -2$ and Q=cos3x$Q = \cos 3x$

Multiplying both sides by (1), we get

e−2xdydx−2ye−2x=cos3xe−2x$e^{-2x}\frac{dy}{dx} - 2ye^{-2x} = \cos 3xe^{-2x}$

Integrating both sides w.r.t. x$x$, we get

ye−2x=∫e−2xcos3xdx+C$ye^{-2x} = \int e^{-2x}\cos 3x dx + C$

⇒ye−2x=I+C, $\Rightarrow ye^{-2x} = I + C$ ,

I=e−txcos3x….$I = e^{-tx}\cos 3x \ldots$ .

I=∫e−2xcos3xdx$I = \int e^{-2x}\cos 3xdx$

=13e−2xsin3x−∫(−2)3e−2xsin3xdx $= \frac{1}{3}e^{-2x}\sin 3x - \int \frac{(-2)}{3}e^{-2x}\sin 3xdx$

=13e−2xsin3x+23[−13e−2xcos3x−∫(−2)e−2t(−cos3x3)dx $= \frac{1}{3}e^{-2x}\sin 3x + \frac{2}{3}\left[\frac{-1}{3}e^{-2x}\cos 3x - \int (-2)e^{-2t}\left(\frac{-\cos 3x}{3}\right)dx\right.$

=13e−2πsin3x+23[−13e−2xcos3x−23∫e−2xcos3xax] $= \frac{1}{3}e^{-2\pi}\sin 3x + \frac{2}{3}\left[\frac{-1}{3}e^{-2x}\cos 3x - \frac{2}{3}\int e^{-2x}\cos 3xax\right]$

=13e−2xsin3x−29e−2xcos3x−49I $= \frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x - \frac{4}{9}I$

⇒(I+49I)=e−2x9(3sin3x−2cos3x) $\Rightarrow \left(I + \frac{4}{9}I\right) = \frac{e^{-2x}}{9}(3\sin 3x - 2\cos 3x)$

⇒I=e−2x13(3sin3x−2cos3x) ⇒ $I = \frac{e^{-2x}}{13}(3\sin 3x - 2\cos 3x)$

Substituting the value of 1 in (2)

ye−2x=e−2x13(3sin3x−2cos3x)+C$ye^{-2x} = \frac{e^{-2x}}{13}(3\sin 3x - 2\cos 3x) + C$

y=3sin3x13−2cos3x13+ce2x$y = \frac{3\sin 3x}{13} - \frac{2\cos 3x}{13} + ce^{2x}$

(II) sinxdydx+ycosx=2sin2xcosx if y(π2)=1$\sin x\frac{dy}{dx} + y\cos x = 2\sin^2 x\cos x$ if $y\left(\frac{\pi}{2}\right) = 1$

Ans:

Given

(sinx)dydx+ycosx=2sin2xcosx $(\sin x)\frac{dy}{dx} + y\cos x = 2\sin^2 x\cos x$

⇒1sinx[(sinx)dydx+ycosx]=2sin2xcosx×1sinx ⇒ $\frac{1}{\sin x}\left[(\sin x)\frac{dy}{dx} + y\cos x\right] = 2\sin^2 x\cos x \times \frac{1}{\sin x}$

⇒dydx+(cosxsinx)y=2sinxcosx ⇒ $\frac{dy}{dx} + \left(\frac{\cos x}{\sin x}\right)y = 2\sin x\cos x$

⇒dydx+(cotx)y=2sinxcosx ⇒ $\frac{dy}{dx} + (\cot x)y = 2\sin x\cos x$

This is a first order linear differential equation of the form

dydx+Py=Q$\frac{dy}{dx} + Py = Q$

Here, P=cotx$P = \cot x$ and Q=2sinxcosx$Q = 2\sin x\cos x$

The integrating factor (I.F) of this differential equation is,

I. F=e∫Pdx$F = e^{\int Pdx}$

⇒ ⇒ I. F=e∫cotxdx$F = e^{\int \cot x dx}$

We have

∫cotxdx=log(sinx)+c $\int \cot x dx = \log(\sin x) + c$

⇒ I. F=elog(sinx) ⇒ $\text{I.}F = e^{\log(\sin x)}$

∴ I. F=sinx[∵elogx=x] ∴ $\text{I. F} = \sin x\ [\because e^{\log x} = x]$

Hence, the solution of the differential equation is,

y($y$ ( I. F)=∫(Q×I.F)dx+c$F) = \int (Q \times I.F)\, dx + c$

⇒y(sinx)=∫(2sinxcosx×sinx)dx+c ⇒ $y(\sin x) = \int (2\sin x\cos x \times \sin x)\, dx + c$

⇒ysinx=2∫sin2xcosxdx+c ⇒ $y\sin x = 2\int \sin^2 x\cos x\, dx + c$

Let sinx=t$\sin x = t$

⇒cosxdx= ⇒ $\cos x dx =$ dt (Differentiating both sides)

By substituting this in the above integral, we get yt=2∫t2dt+c $yt = 2\int t^2 dt + c$

∫xndx=xn+1n+1+c $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

⇒yt=2(t2+12+1)+c $\Rightarrow yt = 2\left(\frac{t^{2+1}}{2+1}\right) + c$

⇒yt=2(t33)+c $\Rightarrow yt = 2\left(\frac{t^3}{3}\right) + c$

⇒yt=2t33+c $\Rightarrow yt = \frac{2t^3}{3} + c$

⇒y=2t23+ct $\Rightarrow y = \frac{2t^2}{3} + \frac{c}{t}$

⇒y=2(sinx)23+csinx[∵t=sinx] $\Rightarrow y = \frac{2(\sin x)^2}{3} + \frac{c}{\sin x}\ [\because t = \sin x]$

∴y=23sin2x+ccosecx $\therefore y = \frac{2}{3}\sin^2 x + c\operatorname{cosec} x$

Thus, the solution of the given differential equation is

y=23sin2x+c cosec x $y = \frac{2}{3}\sin^2 x + c \operatorname{cosec} x$

(III) 3extanydx+(1−ex)sec2ydy=0 $3e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0$

Ans:

The given differential equation is

3extanydx+(1−ex)sec2ydy=0 $3e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0$

3ex(1−ex)dx=sec2ytanydy $\frac{3e^x}{(1-e^x)}dx = \frac{\sec^2 y}{\tan y}dy$

On Integrating, we get

∫3ex(1−ex)dx=∫sec2ytanydy $\int \frac{3e^x}{(1-e^x)}dx = \int \frac{\sec^2 y}{\tan y}dy$

−3log|1−ex|=log|tany|+c $-3\log|1 - e^x| = \log|\tan y| + c$

tany=k(1−ex)3 $\tan y = k(1 - e^x)^3$ which is the required solution of the given differential equation.

13. Solve the following differential equations:

(i) (x3+y3)dy−x2ydx=0 $(x^3 + y^3)dy - x^2 y dx = 0$

Ans:

(x3+y3)dy−x2ydx=0 $(x^3 + y^3)dy - x^2 y dx = 0$

⇒dydx=x2yx3+y3 $\Rightarrow \frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$

⇒dydx=1xy+y2x2 ⇒ $\frac{dy}{dx} = \frac{1}{\frac{x}{y} + \frac{y^2}{x^2}}$

Let yx=v⇒y=vx⇒dydx=v+xdvdx Let $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

So,our differential equations becomes

v+xdvdx=11v+v2=v1+v3$v + x\frac{dv}{dx} = \frac{1}{\frac{1}{v} + v^2} = \frac{v}{1 + v^3}$

⇒xdvdx=v1+v3−v=−v41+v3 ⇒ $x\frac{dv}{dx} = \frac{v}{1 + v^3} - v = -\frac{v^4}{1 + v^3}$

⇒−(1+v3v4)dv=dxx ⇒ $-\left(\frac{1 + v^3}{v^4}\right) dv = \frac{dx}{x}$

Integrating both sides

⇒−(1+v3v4)dv=dxx ⇒ $-\left(\frac{1 + v^3}{v^4}\right) dv = \frac{dx}{x}$

⇒∫−(1+v3v4)dv=∫dxx ⇒ $\int -\left(\frac{1 + v^3}{v^4}\right) dv = \int \frac{dx}{x}$

⇒∫−(v−4+1v)dv=∫dxx ⇒ $\int -\left(v^{-4} + \frac{1}{v}\right) dv = \int \frac{dx}{x}$

⇒v−33−logv=logx+c ⇒ $\frac{v^{-3}}{3} - \log v = \log x + c$

⇒13v3−c=logx+logv ⇒ $\frac{1}{3v^3} - c = \log x + \log v$

⇒13v30−c=log(vx) ⇒ $\frac{1}{3v_0^3} - c = \log (vx)$

⇒x33y3=logy+c, ⇒ $\frac{x^3}{3y^3} = \log y + c,$

Which is required solution.

(ii) xdy−ydx=x2+y2−−−−−−√dx$xdy - ydx = \sqrt{x^2 + y^2}dx$

Ans:

Given Given

xdy−ydx=x−−√2+y2dx$xdy - ydx = \sqrt{x}^2 + y^2dx$

xdy=(y+x2+y2−−−−−−√)dx⇒dydx=y+x2+y2−−−−−−√x$xdy = \left(y + \sqrt{x^2 + y^2}\right) dx \Rightarrow \frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$

F(x,y)=y+x2+y2−−−−−−√x $F(x,y) = \frac{y + \sqrt{x^2 + y^2}}{x}$

F(λx,λy)=λy+λ2x2+λ2y2−−−−−−−−−−√λx $F(\lambda x, \lambda y) = \frac{\lambda y + \sqrt{\lambda^2 x^2 + \lambda^2 y^2}}{\lambda x}$

=λ{y+x2+y2−−−−−−√}λx $= \frac{\lambda\{y + \sqrt{x^2 + y^2}\}}{\lambda x}$

=λ∘.F(x,y) $= \lambda^\circ . F(x,y)$

F(x,y) $F(x,y)$ is a homogeneous function of degree zero.

Now, dydx=y+x2+y2−−−−−−√x $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$

Let y=vx $y = vx$

⇒dydx=v+x⋅dvdx $\Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

Putting above value, we have

v+x⋅dvdx=vx+x2+v2x2−−−−−−−−−√x $v + x \cdot \frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2 x^2}}{x}$

v+x⋅dvdx=v+1+v2−−−−−√ $v + x \cdot \frac{dv}{dx} = v + \sqrt{1 + v^2}$

⇒x⋅dvdx=1+v2−−−−−√ $\Rightarrow x \cdot \frac{dv}{dx} = \sqrt{1 + v^2}$

dxx=dv1+v2−−−−−√ $\frac{dx}{x} = \frac{dv}{\sqrt{1 + v^2}}$

Integrating both sides, we get

∫dxx=∫dv1+v2−−−−−√ $\int \frac{dx}{x} = \int \frac{dv}{\sqrt{1 + v^2}}$

logx+logc=log||v+1+v2−−−−−√||[∵∫dxx2+a2−−−−−−√=log||x+x2+a2−−−−−−√||+c]
$\log x + \log c = \log |v + \sqrt{1 + v^2}| \; [\because \int \frac{dx}{\sqrt{x^2 + a^2}} = \log |x + \sqrt{x^2 + a^2}| + c]$

cx=v+1+v2−−−−−√ $cx = v + \sqrt{1 + v^2}$

⇒cx=yx+1+y2x2−−−−−−√ $\Rightarrow cx = \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}$

cx=yx+x2+y2−−−−−−√x $cx = \frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x}$

⇒cx2=y+x2+y2−−−−−−√ $\Rightarrow cx^2 = y + \sqrt{x^2 + y^2}$

11. Show that the differential equation (x2+2xy−y2)dx+(y2+2xy−x2)dy=0 $(x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy = 0$

is homogeneous and solve it.

Ans:

(x2+2xy−y2)·dx+(y2+2xy−x2)·dy=0 $(x^2 + 2xy - y^2) \cdot dx + (y^2 + 2xy - x^2) \cdot dy = 0$

or. dydx=(y2−2xy−x2)(y2+2xy−x2) or. $\frac{dy}{dx} = \frac{(y^2 - 2xy - x^2)}{(y^2 + 2xy - x^2)}$

y=V·X $y = V \cdot X$

dydx=v.1+x·dvdx$\frac{dy}{dx} = v.1 + x \cdot \frac{dv}{dx}$

v+x·dvdx=v2·x2−2vx2−x2v2·x2+2vx2−x2$v + x \cdot \frac{dv}{dx} = \frac{v^2 \cdot x^2 - 2vx^2 - x^2}{v^2 \cdot x^2 + 2vx^2 - x^2}$

=(v2−2v−1)/(v2+2v−1) $= (v^2 - 2v - 1) / (v^2 + 2v - 1)$

x·dvdx=v2−2v−1v2+2v−1$x \cdot \frac{dv}{dx} = \frac{v^2 - 2v - 1}{v^2 + 2v - 1}$

x·dvdx=v2−2v−1−v3−2v2+vv2+2v−1$x \cdot \frac{dv}{dx} = \frac{v^2 - 2v - 1 - v^3 - 2v^2 + v}{v^2 + 2v - 1}$

or. [v∧2+2v−1−v3−v2−v−1]·dv=1x·dx or. $\left[ \frac{v^\wedge 2 + 2v - 1}{-v^3 - v^2 - v - 1} \right] \cdot dv = \frac{1}{x} \cdot dx$

v2+2v−1v+1·(v2+1)=Av+1+Bv+C(v2+1)$\frac{v^2 + 2v - 1}{v + 1} \cdot (v^2 + 1) = \frac{A}{v + 1} + \frac{Bv + C}{(v^2 + 1)}$

Equating the coeff. of v2,v$v^2$ , $v$ and constant terms.

A+B=1……………..(1)$A + B = 1 \dots \dots \dots \dots \dots . . (1)$

B+C=2……………..(2)$B + C = 2 \dots \dots \dots \dots \dots . . (2)$

A+C=−1……………..(3)$A + C = -1 \dots \dots \dots \dots \dots . . (3)$

Subtracting eqn. (2) from (1)

A−C=−1…………. (4) $A - C = -1 \dots \dots \dots \dots . . (4)$

by eqn. (3) and (4) A=−1,C=0$A = -1, C = 0$

But

A+B=1. $A + B = 1.$

−1+B=1=>B=2$-1 + B = 1 => B = 2$

[−1v+1+2vv2+1]·dv=−1x·dx $\left[ \frac{-1}{v + 1} + \frac{2v}{v^2 + 1} \right] \cdot dv = -\frac{1}{x} \cdot dx$

or. Integ.of [1/(v+1)−2v/(v∧2+1)]·dv= $[1 / (v + 1) - 2v / (v^\wedge 2 + 1)] \cdot dv =$ integ. of 1/x.dx$1 / x . dx$

or. log(v+1)−log(v+1)=logx+logC$\log (v + 1) - \log (v + 1) = \log x + \log C$

or. log(v+1)/(v2+1)=logx.C$\log (v + 1) / (v^2 + 1) = \log x . C$

(IV) x2dy+y(x+y)dx=0 $x^2 dy + y(x + y) dx = 0$given that y=1 $y = 1$whenx=1. $x = 1.$

Ans:

Given, x2dy+(xy+y2)dx=0$x^2 dy + (xy + y^2) dx = 0$

dydx=−(xy+y2)x2 $\frac{dy}{dx} = \frac{-(xy + y^2)}{x^2}$

Put y=vx $y = vx$

or dydx=v+xdvdx $\frac{dy}{dx} = v + x\frac{dv}{dx}$

The differential equation becomes

v+xdvdx=−(v+v2) $v + x\frac{dv}{dx} = -(v + v^2)$

or dvv2+2v=−dxx $\frac{dv}{v^2 + 2v} = -\frac{dx}{x}$

or ∫dv(v+1)2−12=−∫dxx $\int \frac{dv}{(v+1)^2 - 1^2} = -\int \frac{dx}{x}$

or. 12logvv+2=−logx+logC $\frac{1}{2}\log \frac{v}{v+2} = -\log x + \log C$

or Cx=yy+2x−−−−−−√ $\frac{C}{x} = \sqrt{\frac{y}{y+2x}}$

If x=1,y=1, C=13–√ If $x = 1, y = 1, C = \frac{1}{\sqrt{3}}$

or or

13–√x=yy+2x−−−−−−√ $\frac{1}{\sqrt{3}x} = \sqrt{\frac{y}{y+2x}}$

(V) xeyx+y+xdydx=0 $xe^{\frac{y}{x}} + y + x\frac{dy}{dx} = 0$ ify(e)=0 $y(e) = 0$

Ans: Given differential equation is, (xeyx+y)dx=xdy $\left(xe^{\frac{y}{x}} + y\right)dx = xdy$

⇒dydx=x⋅eyx+yx…..(i) $\Rightarrow \frac{dy}{dx} = \frac{x \cdot e^{\frac{y}{x}} + y}{x} \ldots (i)$

Let F(x,y)=x⋅eyx+yx $F(x, y) = \frac{x \cdot e^{\frac{y}{x}} + y}{x}$

∴F(λx,λy)=λx⋅eλyλx+λyλx=λ0x⋅eyx+yx=λ0F(x,y) $\therefore F(\lambda x, \lambda y) = \frac{\lambda x \cdot e^{\frac{\lambda y}{\lambda x}} + \lambda y}{\lambda x} = \lambda^0 \frac{x \cdot e^{\frac{y}{x}} + y}{x} = \lambda^0 F(x, y)$

Hence, given differential equation (i) is homogenous.

Let y=vx⇒dydx=v+x⋅dvdx $y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

Now, given differential equation (i) would become

v+x⋅dvdx=x⋅evxx+vxx $v + x \cdot \frac{dv}{dx} = \frac{x \cdot e^{\frac{vx}{x}} + vx}{x}$

⇒v+x⋅dvdx=ev+v ⇒ $v + x \cdot \frac{dv}{dx} = e^{v} + v$

⇒x⋅dvdx=ev ⇒ $x \cdot \frac{dv}{dx} = e^{v}$

dvev=dxx$\frac{dv}{e^{v}} = \frac{dx}{x}$

⇒∫e−vdv=∫dvv ⇒ $\int e^{-v} dv = \int \frac{dv}{v}$

⇒e−v−1=logx+C ⇒ $\frac{e^{-v}}{-1} = \log\ x + C$

−e−yx=logx+C $-e^{-\frac{y}{x}} = \log\ x + C$

⇒−1eyx=logx+C ⇒ $-\frac{1}{e^{\frac{y}{x}}} = \log\ x + C$

⇒eyx⋅logx+Ceyx+1=0 ⇒ $e^{\frac{y}{x}} \cdot \log\ x + Ce^{\frac{y}{x}} + 1 = 0$

Putting y(e)=0$y(e) = 0$, we get

∴1yxlogx+C1yx+1=0 ∴ $1^{\frac{y}{x}}\log\ x + C1^{\frac{y}{x}} + 1 = 0$

⇒C=−1e ⇒ $C = -\frac{1}{e}$

The required particular solution is

eyx⋅logx−1eeyx+1=0$e^{\frac{y}{x}} \cdot \log\ x - \frac{1}{e}e^{\frac{y}{x}} + 1 = 0$

or eyxlogx−eyx−1+1=0$e^{\frac{y}{x}}\log\ x - e^{\frac{y}{x}-1} + 1 = 0$

(VI) (x3−3xy2)dx=(y3−3x2y)dy $(x^{3} - 3xy^{2})\ dx = (y^{3} - 3x^{2}y)\ dy$

Ans:

Given differential equation is

(x3−3xy2)dx=(y3−3x2y)dy $(x^{3} - 3xy^{2})\ dx = (y^{3} - 3x^{2}y)\ dy$

⇒dydx=x3−3xy2y3−3x2y ⇒ $\frac{dy}{dx} = \frac{x^{3} - 3xy^{2}}{y^{3} - 3x^{2}y}$

=1−3(yx)2(y3)3−3(yx) = $\frac{1 - 3\left(\frac{y}{x}\right)^{2}}{\left(\frac{y}{3}\right)^{3} - 3\left(\frac{y}{x}\right)}$

⇒v+xdvdx=1−3v2v3−3v, Let v=yx) ⇒ $v + x\frac{dv}{dx} = \frac{1 - 3v^{2}}{v^{3} - 3v}$, Let $v = \frac{y}{x}$)

⇒xdvdx=1−3v2v3−3v−v ⇒ $x\frac{dv}{dx} = \frac{1 - 3v^{2}}{v^{3} - 3v} - v$

=1−3v2−v4+3v2v3−3v $= \dfrac{1-3v^2-v^4+3v^2}{v^3-3v}$

=1−v4v3−3v $= \dfrac{1-v^4}{v^3-3v}$

⇒(v3−3v1−v4)dv=dxx $\Rightarrow \left(\dfrac{v^3-3v}{1-v^4}\right) dv = \dfrac{dx}{x}$

⇒(v31−v4)dv−(3v1−v4)dv=dxx $\Rightarrow \left(\dfrac{v^3}{1-v^4}\right) dv - \left(\dfrac{3v}{1-v^4}\right) dv = \dfrac{dx}{x}$

Integrating, we get

⇒(v31−v4)dv−(3v1−v4)dv=dxx $\Rightarrow \left(\dfrac{v^3}{1-v^4}\right) dv - \left(\dfrac{3v}{1-v^4}\right) dv = \dfrac{dx}{x}$

Integrating, we get

−14log||1−v4||+34log|||v2−1v2+1|||=log|x|+logc $-\dfrac{1}{4}\log\left|1-v^4\right| + \dfrac{3}{4}\log\left|\dfrac{v^2-1}{v^2+1}\right| = \log|x| + \log c$

−14log|||1−(yx)4|||+34log|||||(yx)2−1(yx)2+1|||||=log|xc| $-\dfrac{1}{4}\log\left|1-\left(\dfrac{y}{x}\right)^4\right| + \dfrac{3}{4}\log\left|\dfrac{\left(\dfrac{y}{x}\right)^2-1}{\left(\dfrac{y}{x}\right)^2+1}\right| = \log|xc|$

which is the required solution

(VII) dydx−yx+cosec(yx)=0 $\dfrac{dy}{dx} - \dfrac{y}{x} + \operatorname{cosec}\left(\dfrac{y}{x}\right) = 0$given that y=0$y=0$ when x=1$x=1$

Ans:

Differential equation is

dydx=yx−cosec(yx)$\dfrac{dy}{dx} = \dfrac{y}{x} - \operatorname{cosec}\left(\dfrac{y}{x}\right)$

Let F(x,y)=dydx=yx−cosec(yx)$F(x,y) = \dfrac{dy}{dx} = \dfrac{y}{x} - \operatorname{cosec}\left(\dfrac{y}{x}\right)$

Finding F(λx,λy)$F(\lambda x, \lambda y)$

F(λx,λy)=λyλx−cosec(λyλx)=yx−cosec(yx)=λ∘F(x,y)$F(\lambda x, \lambda y) = \dfrac{\lambda y}{\lambda x} - \operatorname{cosec}\left(\dfrac{\lambda y}{\lambda x}\right) = \dfrac{y}{x} - \operatorname{cosec}\left(\dfrac{y}{x}\right) = \lambda \circ F(x,y)$

∴F(x,y) $\therefore F(x,y)$ is a$a$ homogenous function of degree zero

F(λx,λy)=λ∘F(x,y)$F(\lambda x, \lambda y) = \lambda \circ F(x,y)$

Putting y=vx$y=vx$

Diff w.r.t. x$x$

dydx=xdvdx+v$\dfrac{dy}{dx} = x\dfrac{dv}{dx} + v$

Putting value of dydx$\dfrac{dy}{dx}$ and y=vx$y=vx$ in (1)

dydx=yx−cosec(yx)$\dfrac{dy}{dx} = \dfrac{y}{x} - \operatorname{cosec}\left(\dfrac{y}{x}\right)$

v+xdvdx=vxx−cosec(vxx)$v + x\frac{dv}{dx} = \frac{vx}{x} - \operatorname{cosec}\ \left(\frac{vx}{x}\right)$

v+xdvdx=v−cosecv$v + x\frac{dv}{dx} = v - \operatorname{cosec}\ v$

xdvdx=−cosecv$\frac{xdv}{dx} = -\operatorname{cosec}\ v$

−dvcosecv=dxx$\frac{-dv}{\operatorname{cosec}\ v} = \frac{dx}{x}$

Integrating both sides

∫−dvcosecv=∫dxx $\int \frac{-dv}{\operatorname{cosec}\ v} = \int \frac{dx}{x}$

∫−sinvdv=log|x|+c $\int -\sin\ v dv = \log\ |x| + c$

Put value of v=yx$v = \frac{y}{x}$

cosyx=log|x|+C$\cos\ \frac{y}{x} = \log\ |x| + C$

Putting x=1 y=0$x = 1 y = 0$

cos01=log1+C$\cos\ \frac{0}{1} = \log\ 1 + C$

1=0+C[log1=0]$1 = 0 + C\ [\log\ 1 = 0]$

Putting value in (2)

c=1$c = 1$

cosy2=log|x|+1$\cos\ \frac{y}{2} = \log\ |x| + 1$

cosy2=log|x|+loge$\cos\ \frac{y}{2} = \log\ |x| + \log\ e$

cosy2=log|x|$\cos\ \frac{y}{2} = \log\ |x|$

16. Solve the following differential equations:

(I)cos2xdydx=tanx−y$\cos^2 x\frac{dy}{dx} = \tan\ x - y$

Ans:

Given differential equation is

cos2x·dydx+y=tanx$\cos^2 x \cdot \frac{dy}{dx} + y = \tan\ x$

⇒dydx+ysec2x=tanx·sec2x $\Rightarrow \frac{dy}{dx} + y\sec^2 x = \tan\ x \cdot \sec^2 x$

Given differential equation is a linear differential equation of the type dydx+py=Q$\frac{dy}{dx} + py = Q$

I.F. =e∫Pdx=e∫sec2xdx=etanx$= e^{\int Pdx} = e^{\int \sec^2 x dx} = e^{\tan\ x}$

∴ ∴ Solution is given by etanxy=∫tanx·sec2x·etanxdx$e^{\tan\ x} y = \int \tan x \cdot \sec^2 x \cdot e^{\tan\ x} dx$

Let I=∫tanx·sec2x·etanxdx$I = \int \tan x \cdot \sec^2 x \cdot e^{\tan\ x} dx$

Let tanx=t,sec2xdx=dt$\tan x = t$ , $\sec^2 x dx = dt$

⇒I=∫tetdt ⇒ $I = \int te^t dt$

Integrating by parts ∴I=tet−∫etdt=tet−et+C ∴ $I = te^t - \int e^t dt = te^t - e^t + C$

⇒I=tanxetanx−etanx+C ⇒ $I = \tan x e^{\tan x} - e^{\tan x} + C$,

Hence etanxy=etanx(tanx−1)+C$e^{\tan x} y = e^{\tan x}(\tan x - 1) + C$

⇒y=tanx−1+Ce−tanx ⇒ $y = \tan x - 1 + Ce^{-\tan x}$

(II) xcosxdydx+y(xsinx+cosx)=1$x\cos x \frac{dy}{dx} + y(x\sin x + \cos x) = 1.$

Ans:

Given xcosx(dy/dx)+y(xsinx+cosx)=1$x\cos x (dy/dx) + y(x\sin x + \cos x) = 1.$

dydx+y(xsinx+cosx)xcosx=1xcosx$\frac{dy}{dx} + \frac{y(x\sin x + \cos x)}{x\cos x} = \frac{1}{x\cos x}$

dydx+(xsinxxcosx+cosxxcosx)y=1xcosx$\frac{dy}{dx} + \left(\frac{x\sin x}{x\cos x} + \frac{\cos x}{x\cos x}\right)^y = \frac{1}{x\cos x}$

dydx+(tanx+1x)y=1xcosx$\frac{dy}{dx} + \left(\tan x + \frac{1}{x}\right) y = \frac{1}{x\cos x}$

It is linear differential equation in the form dydx+Py=Q$\frac{dy}{dx} + Py = Q$

where P=tanx+(1x)$P = \tan x + \left(\frac{1}{x}\right)$ and secxx$\frac{\sec x}{x}$

∴(I.F.)=e∫(tanx+(1/x))dx ∴ $(I.F.) = e^{\int (\tan x + (1/x))dx}$

=e∫tanx+∫dx/x=elogsecx+logx=elog(xsecx) $= e^{\int \tan x + \int dx/x} = e^{\log \sec x + \log x} = e^{\log (x\sec x)}$

=xsecx $= x\sec x$

Now, multiplying (1) by I.F. and integration, we get

y×I.F=∫Q×I.F+C$y \times I.F = \int Q \times I.F + C$

yxsecx=∫(secx/x)x(xsecx)dx+c$yx\sec x = \int (\sec x / x) x (x\sec x) dx + c$

xysecx=∫sec2xdx+c=tanx+c$xy\sec x = \int \sec^2 x dx + c = \tan x + c$

xysecx=tanx+C$xy\sec x = \tan x + C$

Which is the required solution

(III).⎛⎝⎜ 1+exy⎞⎠⎟ ⎟dx+exy(1−xy)dy=0 $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$

Ans:

⎛⎝⎜ 1+exy⎞⎠⎟ ⎟dx=(xy−1)exydy $\left(1 + e^{\frac{x}{y}}\right) dx = \left(\frac{x}{y} - 1\right) e^{\frac{x}{y}} dy$

dxdy=(xy−1)exy(1+ey)=f(xy)$\frac{dx}{dy} = \frac{\left(\frac{x}{y} - 1\right) e^{\frac{x}{y}}}{(1 + e^y)} = f\left(\frac{x}{y}\right)$

Hence, homogeneous

dxdy=(xy−1)exy(1+ey)=f(xy)$\frac{dx}{dy} = \frac{\left(\frac{x}{y} - 1\right) e^{\frac{x}{y}}}{\left(1 + e^{y}\right)} = f\left(\frac{x}{y}\right)$

Equating Homogeneous,

x−vy⇒dxdy=v+ydvdy$x - vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

v+ydvdy=(v−1)eγ1+ep$v + y\frac{dv}{dy} = \frac{(v-1)e^{\gamma}}{1 + e^{p}}$

∫1+evev+vdv=−∫dyy $\int \frac{1 + e^{v}}{e^{v} + v} dv = -\int \frac{dy}{y}$

loge|ev+v|=−loge|y|+logeClog$\log_e |e^{v} + v| = -\log_e |y| + \log_e C$

loge|(ev+v)y|=logcClog$\log_e |(e^{v} + v) y| = \log_c C$

(eσ+v)y=C=A $(e^{\sigma} + v) y = C = A$

(xey+xy)y=A, $\left(\frac{x}{e^{y}} + \frac{x}{y}\right) y = A$ ,

The General solution

(IV) (y−sinx)dx+tanxdy=0,y(0)=0. $(y - \sin x) dx + \tan xdy = 0, y(0) = 0.$

Ans:

The given diff. equation can be written as

dxdy+(cotx)y=cosx$\frac{dx}{dy} + (\cot x) y = \cos x$

This is linear differential equation.

I.F. =e∫cotxdy=elogsinx=sinx $= e^{\int \cot x dy} = e^{\log \sin x} = \sin x$

The solution is:

ysinx=∫sinxcosxdy+C$y\sin x = \int \sin x \cos xdy + C$

=12∫sin2xdy+Cysinx $= \frac{1}{2}\int \sin 2xdy + Cy\sin x$

=−14cos2x+C $= \frac{-1}{4}\cos 2x + C$

It is given that , when

c−14=0 or c=14$c - \frac{1}{4} = 0 \text{ or } c = \frac{1}{4}$

ysinx=14(1−cos2x)=12sin2x$y\sin x = \frac{1}{4}(1 - \cos 2x) = \frac{1}{2}\sin^2 x$

2y=sinx$2y = \sin x$

which is the required solution.

LONG ANSWER TYPE QUESTIONS (6 MARKS EACH)

17. Solve the following differential equations:

(I) (xdy−ydx)ysin(yx)=(ydx+xdy)xcos(yx) $(xdy-ydx)y\sin\left(\frac{y}{x}\right)=(ydx+xdy)x\cos\left(\frac{y}{x}\right)$

Ans:

Given Differential equation can be written as

(xdy−ydx)ysin(yx)=(ydx+xdy)xcos(yx) $(xdy-ydx)y\sin\left(\frac{y}{x}\right)=(ydx+xdy)x\cos\left(\frac{y}{x}\right)$

xysin(yx)dy−y2sin(yx)dx$xy\sin\left(\frac{y}{x}\right)dy-y^2\sin\left(\frac{y}{x}\right)dx$

=yxcos(yx)dx+x2cos(yx)dy $=yx\cos\left(\frac{y}{x}\right)dx+x^2\cos\left(\frac{y}{x}\right)dy$

xysin(yx)dydx−y2sin(yx)$xy\sin\left(\frac{y}{x}\right)\frac{dy}{dx}-y^2\sin\left(\frac{y}{x}\right)$

=xycos(yx)+x2cos(yx)dydx $=xy\cos\left(\frac{y}{x}\right)+x^2\cos\left(\frac{y}{x}\right)\frac{dy}{dx}$

dydx=y2sin(y/x)+xycos(y/x)xysin(y/x)−x2cos(y/x)$\frac{dy}{dx}=\frac{y^2\sin(y/x)+xy\cos(y/x)}{xy\sin(y/x)-x^2\cos(y/x)}$

Put (yx)=v $\left(\frac{y}{x}\right)=v$ to get y=vx$y=vx$ and dydx=v+xdydx$\frac{dy}{dx}=v+x\frac{dy}{dx}$

v+xdvdx=v2sinv+vcosvvsinv−cosv$v+x\frac{dv}{dx}=\frac{v^2\sin v+v\cos v}{v\sin v-\cos v}$

or xdvdx=2vcosvvsinv−cosv$x\frac{dv}{dx}=\frac{2v\cos v}{v\sin v-\cos v}$

or ∫cosv−vsinvvcosvdv=−2∫dxx $\int\frac{\cos v-v\sin v}{v\cos v}dv=-2\int\frac{dx}{x}$

or log|vcosv|+logx2$\log|v\cos v|+\log x^2$

=logC $=\log C$

or x2vcosv=C$x^2v\cos v=C$ or xycos(yx)=C$xy\cos\left(\frac{y}{x}\right)=C$

(II)

3extanydx+(1−ex)sec2ydy=0 $3e^x\tan y dx+(1-e^x)\sec^2 y dy=0$

Given that y=π4$y=\frac{\pi}{4}$, when x=1$x=1$

Ans:

The given differential equation is

3extanydx+(1−ex)sec2ydy=0$3e^x\tan y dx+(1-e^x)\sec^2 y dy=0$

3ex(1−ex)dx=sec2ytanydy$\frac{3e^x}{(1-e^x)}dx=\frac{\sec^2 y}{\tan y}dy$

On Integrating, we get

∫3ex(1−ex)dx=∫sec2ytanydy $\int \frac{3e^{x}}{(1-e^{x})}dx = \int \frac{\sec^{2}y}{\tan y}dy$

−3log|1−ex|=log|tany|+c $-3\log |1-e^{x}| = \log |\tan y| + c$

By putting

y=π4 , and x=1 $y = \frac{\pi}{4}, and x = 1$

(1−e)3tany=(1−ex)3 $(1-e)^{3}\tan y = (1-e^{x})^{3}$

which is the required solution of the given differential equation.

(III) dydx+ycotx=2x+x2cotx $\frac{dy}{dx} + y\cot x = 2x + x^{2}\cot x$Given that y(0)=0 $y(0) = 0$

Ans:

dydx+y·cotx=2x+x2·cotx $\frac{dy}{dx} + y \cdot \cot x = 2x + x^{2} \cdot \cot x$

Let P=cotx,Q=2x+x2cotx $P = \cot x, Q = 2x + x^{2}\cot x$

I.F =e∫pdx=e∫cotxdx=e∫log(sinx)=sinx $= e^{\int pdx} = e^{\int cotxdx} = e^{\int \log(\sin x)} = \sin x$

Sol is

y(I.F)=∫(Qx|.F)dx+C $y(I.F) = \int (Qx| .F)dx + C$

ysinx=∫(2x+x2cotx)sindx+c $y\sin x = \int (2x + x^{2}\cot x)\sin dx + c$

=∫(2x+x2cotx)dx+c $= \int (2x + x^{2}\cot x)dx + c$

=2∫xsinxdx+∫x2cosxdx+c $= 2\int x\sin xdx + \int x^{2}\cos xdx + c$

=2[x(−cosx)−(1)(−sinx)]+[x2sinx−2x(−cosx)+2c(−sinx)] $= 2[x(-\cos x) - (1)(-\sin x)] + [x^{2}\sin x - 2x(-\cos x) + 2c(-\sin x)]$

=2xcosx+2sinx+x2sinxysinx $= 2x\cos x + 2\sin x + x^{2}\sin xy\sin x$

=x2sinx+c $= x^{2}\sin x + c$

Then Y (0) =0

Y=x2 $Y = x^{2}$

This is the Required General solution for the given differential Equation.