

**Directorate of Education, GNCT of Delhi**  
**Practice Paper**  
**(Mid Term)**

(2026-27)

Class – XII

**Mathematics (Code: 041)**

**Time: 3 Hours**

**Maximum Marks: 80**

**General Instructions :**

- This Question paper contains - **Five sections A,B,C,D,E**. Each section is compulsory. However, there are internal choices in some questions.
- **Section A** has 18 **MCQ's and 02 Assertion-Reason** based questions of 1 mark each(20 Marks )
- **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each(10 Marks )
- **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each(18 Marks )
- **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each(20 Marks )
- Section E has 3 Source based/Case based/passage based/integrated units of assessment (4 marks each) with sub parts(12 Marks )

**Section A**

(Question number 1 – 18 are of MCQ type questions of one mark each.)

1. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = x^2 + 5x - 4$ . Then  $f$  is:

|                                   |                                      |
|-----------------------------------|--------------------------------------|
| (a) Injective but not surjective  | (b) Surjective but not injective     |
| (c) Both injective and surjective | (d) neither injective nor surjective |

2. If  $y = 2 \sin^{-1}(3x)$ , then the range of  $y$  is:

|                     |                 |
|---------------------|-----------------|
| (a) $[-\pi, \pi]$   | (b) $[0, \pi]$  |
| (c) $[-2\pi, 2\pi]$ | (d) $[0, 2\pi]$ |

3: The principal value of  $\cos^{-1} \frac{1}{2} + \sin^{-1} \frac{1}{2}$  is:

|             |             |
|-------------|-------------|
| (a) $\pi/6$ | (b) $\pi/3$ |
| (c) $\pi/2$ | (d) $\pi$   |

**4: The value of  $\sin\left(\sin^{-1}\left(\frac{2\pi}{3}\right)\right)$  is:**

|                      |                          |
|----------------------|--------------------------|
| (a) $\frac{2\pi}{3}$ | (b) $\frac{\sqrt{3}}{2}$ |
| (c) $\frac{3\pi}{2}$ | (d) $\frac{\pi}{3}$      |

**5: The number of  $2 \times 3$  matrices with entries chosen from  $\{0,1,2\}$  is:**

|         |        |
|---------|--------|
| (a) 9   | (b) 18 |
| (c) 729 | (d) 36 |

**6: If  $A = \begin{bmatrix} 4 & a & b \\ -2 & 5 & c \\ 3 & 7 & 6 \end{bmatrix}$  is symmetric, then  $a+b+c$  equals:**

|        |        |
|--------|--------|
| (a) 8  | (b) 10 |
| (c) 12 | (d) 14 |

**7. If P is a  $3 \times 1$  matrix and Q is a  $1 \times 3$  matrix, then PQ is:**

|                                |                           |
|--------------------------------|---------------------------|
| (a) a scalar                   | (b) a $3 \times 3$ matrix |
| (c) a $1 \times 1$ matrix only | (d) undefined             |

**8. If  $a_{ij} = i + j$  for a  $2 \times 2$  matrix A, then  $A^T$  is:**

|                                                    |                                                    |
|----------------------------------------------------|----------------------------------------------------|
| (a) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ | (b) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ |
| (c) $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ | (d) $\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$ |

**9. If  $A^2 = A$ , then  $(I - A)^2$  is equal to:**

|           |            |
|-----------|------------|
| (a) A     | (b) I + A  |
| (c) I - A | (d) 2A - I |

10. If  $x^2 + y^2 = 2xy$ , then  $\frac{dy}{dx}$  is:

|                           |                           |
|---------------------------|---------------------------|
| (a) $\frac{(y-x)}{(y-x)}$ | (b) $\frac{(y-x)}{(x-y)}$ |
| (c) $\frac{(x-y)}{(x+y)}$ | (d) $-1$                  |

11. The area between  $y = x^2$  and x-axis from  $x=0$  to  $x=2$  is:

|                         |                       |
|-------------------------|-----------------------|
| (a) $\int_0^2 x dx$     | (b) $\int_0^2 x^2 dx$ |
| (c) $2 \int_0^2 x^2 dx$ | (d) $\int_0^2 2x dx$  |

12.  $\frac{d}{dx} [\tan^{-1}(x^2)]$  is:

|                          |                          |
|--------------------------|--------------------------|
| (a) $\frac{2x}{(1+x^2)}$ | (b) $\frac{2x}{(1+x^4)}$ |
| (c) $\frac{x}{(1+x^4)}$  | (d) $\frac{1}{(1+x^2)}$  |

13. The derivative of  $\sin^3 x$  with respect to  $\cos^3 x$  is:

|                 |                |
|-----------------|----------------|
| (a) $-\tan^2 x$ | (b) $-\tan x$  |
| (c) $-\cot^2 x$ | (d) $\tan^2 x$ |

14. The diagonal square of side  $2\sqrt{2} \text{ cm}$  is increasing at the rate of  $2 \text{ cm/sec}$  which of the following is rate at which this area is increasing ?

|                                       |                                      |
|---------------------------------------|--------------------------------------|
| (a) $8 \text{ cm}^2/\text{s}$         | (b) $12 \text{ cm}^2/\text{s}$       |
| (c) $6\sqrt{2} \text{ cm}^2/\text{s}$ | (d) $\sqrt{2} \text{ cm}^2/\text{s}$ |

15. If  $\int \frac{4x}{1+x^2} dx = A \log(1+x^2) + C$ , then A is:

|       |                   |
|-------|-------------------|
| (a) 1 | (b) 2             |
| (c) 4 | (d) $\frac{1}{2}$ |

16. If  $\int_0^a \frac{dx}{(1+x^2)} = \frac{\pi}{4}$ , then a is:

|                |                     |
|----------------|---------------------|
| (a) 0          | (b) 1               |
| (c) $\sqrt{3}$ | (d) $\frac{\pi}{4}$ |

17. The product of order and degree of  $\frac{d}{dx} \left( \frac{d^2 y}{dx^2} \right) = 7$  is :

|       |       |
|-------|-------|
| (a) 4 | (b) 1 |
| (c) 2 | (d) 3 |

18. Which of the following is not homogeneous?

|                                      |                             |
|--------------------------------------|-----------------------------|
| (a) $\frac{(x+y)}{x}$                | (b) $\frac{(x^2+y^2)}{x^2}$ |
| (c) $\frac{y}{x} + \cos \frac{y}{x}$ | (d) $\sin x + \sin y$       |

#### Assertion-Reason Based Questions

*Note: For questions 19 and 20, choose the correct answer out of the following choices:*

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

19. Assertion (A): If f is onto, then f is one-to-one and if f is one-to-one, then f is onto.

Reason (R): Every one-to-one function is always onto and every onto function is always one-to-one.

|                                                                  |                                                                      |
|------------------------------------------------------------------|----------------------------------------------------------------------|
| (a) Both A and R are true and R is the correct explanation of A. | (b) Both A and R are true but R is not the correct explanation of A. |
| (c) A is true but R is false.                                    | (d) A is false but R is true.                                        |

20. Assertion (A):  $f(x) = e^{-x}$  is decreasing on R.

Reason (R): If  $f'(x) < 0$  on an interval, then f is decreasing on that interval.

|                                                                  |                                                                      |
|------------------------------------------------------------------|----------------------------------------------------------------------|
| (a) Both A and R are true and R is the correct explanation of A. | (b) Both A and R are true but R is not the correct explanation of A. |
| (c) A is true but R is false.                                    | (d) A is false but R is true.                                        |

### Section B

(This section contains 5 very short answer (VSA) type questions of 2 marks each.)

21. (a) Evaluate  $\tan \left[ \sin^{-1} \left( \frac{1}{2} \right) + \cos^{-1} \left( \frac{1}{\sqrt{2}} \right) \right]$ .

OR

(b) Evaluate  $\sin^{-1} \left( \sin \frac{5\pi}{6} \right) + \cos^{-1} \left( \cos \frac{7\pi}{6} \right)$

22. Construct a  $2 \times 2$  matrix A such that  $A^2 = -9I$ . Hence find  $A^4$ .

23. (a) Determine the intervals in which  $f(x) = \frac{(x+1)}{(x-2)}$  is increasing

or decreasing.

OR

(b) If  $f(a-x) = f(x)$ , show that  $\int_0^a f(x) dx = \frac{a}{2} \int_0^a f(x) dx$

24. Evaluate  $\int \frac{dx}{\sqrt{x^2 - 6x + 13}}$ .

25. Find the particular solution of  $\frac{dy}{dx} = y \tan 3x$ , given  $y(0) = 2$ .

## Section C

(This section contains 6 short answer (SA) type questions of 3 marks each.)

26. Let  $R$  on  $\mathbb{R}$  be defined by  $(x, y) \in R$  iff  $x - y$  is rational. Test whether  $R$  is reflexive, symmetric and transitive.

OR

On  $A = -3, -2, -1, 0, 1, 2, 3$ , define  $xRy$  iff  $x - y$  is divisible by 3. Show that  $R$  is an equivalence relation.

27. For  $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$  verify that  $(AB)^{-1} = B^{-1}A^{-1}$ .

28. (a) If  $x = e^t(\cos t - \sin t)$ ,  $y = e^t(\sin t + \cos t)$ , prove  $\frac{dy}{dx} = \tan t$ .

OR

(b) If  $y = (x^2 + 1)^x$ , find  $\frac{dy}{dx}$  using logarithmic differentiation.

29. A spherical balloon is inflated at  $2 \text{ cm}^3/\text{sec}$ . Find the rate of increase of its radius when  $r = 3 \text{ cm}$ .

30. (a) Evaluate  $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin |x| + \cos |x|) dx$

OR

(b) Evaluate  $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1 + \sqrt{\cot x}}$ .

31. Solve the following differential equation

$$x \frac{dy}{dx} = y - x \sin^2\left(\frac{y}{x}\right), \text{ given that } y(1) = \frac{\pi}{6}$$

## Section D

(This section contains 4 Long answer (LA) type questions of 5 marks each.)

32. Using integration, find the area of the ellipse  $2x^2 + 9y^2 = 36$  bounded between the lines  $y=0$  and  $y = \frac{\sqrt{2}}{3}x$  in the first quadrant.

33. (a) Let  $A = \mathbb{R} - \{3\}$ ,  $B = \mathbb{R} - \{1\}$  and define  $f: A \rightarrow B$  by  $f(x) = \frac{x-2}{x-3}$

(i) Explain why  $f$  is defined, i.e., why  $f(x) \in B$  for every  $x \in A$  1

(ii) Prove that  $f$  is one-one 2

(iii) Prove that  $f$  is onto 2

OR

(b) Find the domain and range of  $p(x) = \sin^{-1}(2x - 1)$ . Also find  $x$  when  $p(x) = \frac{\pi}{6}$ . 5

34. Given  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$  and  $C = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ , find  $D$  such that  $AB - CD$  is the null matrix.

35. (a) Evaluate  $\int_0^{\frac{\pi}{2}} \log(\cos x) dx$

OR

(b) Integrate  $\frac{(2x^2 + 3x + 4)}{(x+1)(x^2+1)} dx$ .

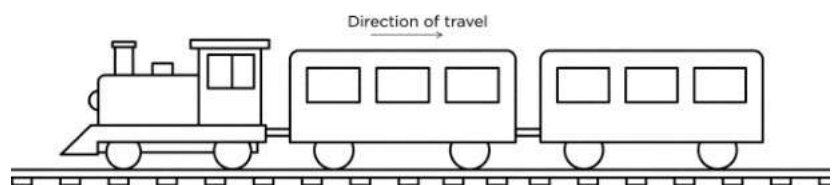
### Section E

**(Case Study Based Questions carry 4 marks each.)**

36. A train travels along a straight track. Its speed  $v(t)$  (in km/h) at time  $t$  (in minutes) after it starts moving is given by :

$$v(t) = 5t^2 - t^3 \text{ for } 0 \leq t < 5$$

and  $v(t) = 4t^2 - 15t - 25 \text{ for } t \geq 5$



Based on the above information answer the following:

(a) Find  $v'(t)$  for  $0 < t < 5$  [1]

(b) Find  $v'(t)$  for  $t > 5$  [1]

(c) Check whether  $v(t)$  is continuous at  $t=5$  [2]

OR

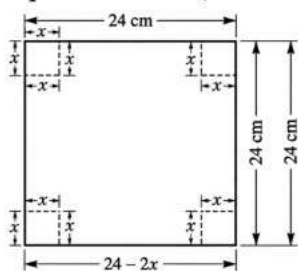
Check whether  $v(t)$  is differentiable at  $t=5$  [2]

37. A manufacturer wants to make an open rectangular box from a square sheet of cardboard of side 24 cm, by cutting equal squares of side  $x$  cm from each corner and folding up the sides.

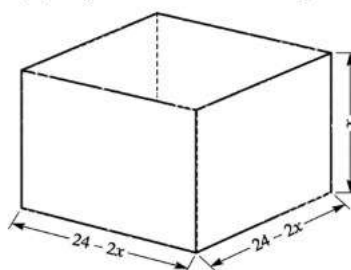
The volume of the box formed is given by:

$$V(x) = x(24 - 2x)^2, 0 < x < 12$$

(a) Square Cardboard (Plan View)



(b) Open Box After Folding



Based on the above information, answer the following:

(a) Find  $V'(x)$ . [1]

(b) Find the critical point(s) of  $V(x)$  [1]

(c) Use the second derivative test to find the value of  $x$  for which the volume is maximum, and hence find the maximum volume. [2]

OR

Verify using the second derivative test whether  $x=12$  gives a maximum or minimum volume. [2]

38. A school conducted a Mathematics and Science Olympiad for students of Classes XI and XII. The number of students who qualified at the School Level and District Level is represented by the matrix  $A = \begin{bmatrix} 20 & 15 \\ 12 & 18 \end{bmatrix}$

where the rows represent Class XI and Class XII, while the columns represent School Level and District Level qualifiers respectively.

For further analysis, the school uses the matrix  $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Based on the above information, answer the following questions:

(a) Find the inverse of matrix  $B$  [2]

(b) If  $C = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$  find  $BC$  [2]

