Class 8 Maths Ganita Prakash Chapter 6 We Distribute, Yet Things Multiply NCERT Solutions

Figure it Out

1. Observe the multiplication grid below. Each number inside the grid is formed by multiplying two numbers. If the middle number of a 3 × 3 frame is given by the expression *pq*, as shown in the figure, write the expressions for the other numbers in the grid.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

3 × 5	3 × 6	3 × 7
4 × 5	4 × 6	4 × 7
5 × 5	5 × 6	5 × 7

	pq	

Solution:

3 × 5	3 × 6	3 × 7
4 × 5	4 × 6	4 × 7
5 × 5	5 × 6	5 × 7

(p – 1) (q – 1)	(p -1) q	(p -1) (q + 1)
p (q – 1)	pq	p (q + 1)
(p + 1) (q – 1)	(p + 1) q	(p + 1) (q +1)

2. Expand the following products.
(i) (3 + u) (v – 3)
(ii) 2/3 (15 + 6a)
(iii) (10a+ b) (10c + d)
(iv) (3 – x) (x– 6)
(v) (–5a + b) (c+ d)
(vi) (5 + z) (y+ 9)
Solution:

(i) (3 + u) (v – 3)
= (3 + u)v – (3 + u)3
= 3 + uv – (9 + 3u)
= 3 + uv – 9 + 3u
= uv + 3u + 3 – 9
= uv + 3u – 6.

$$\text{(ii) } \frac{2}{3}(15 + 6a)$$
$$= \frac{2}{3} \times 15 + \frac{2}{3} \times 6a$$
$$= 2 \times 5 + 2 \times 2a$$
$$= 10 + 4a.$$

(iii) (10a+ b) (10c + d)
= (10a + b)10c + (10 a + b)d
= 100ac + 10bc + 10ad + bd.

(iv) (3 – x) (x – 6)
= (3 – x)x – (3 – x)6
= 3x – x^2 – (18 – 6x)
= 3x – x^2 – 18 + 6x
= – x^2 + 6x + 3x – 18
= – x^2 + 9x – 18.

(v) (–5a + b) (c+ d)
= (–5a + b)c + (–5a + b)d
= – 5ac + bc – 5ad + bd
= – 5ac – 5ad + bc + bd.

(vi) (5 + z) (y+ 9)
= (5 + z)y + (5 + z)9
= 5y + zy + 45 + 9z
= 5y + 9z + zy + 45.

3. Find 3 examples where the product of two numbers remains unchanged when one of them is increased by 2 and the other is decreased by 4.
Solution:
If the two numbers are x and y, then:
x × y = (x + 2) × (y − 4)
xy = (x + 2)y – (x + 2)4
xy = xy + 2y – (4x + 8)
xy = xy + 2y – 4x – 8

$xy - xy = 2y - 4x - 8$
$0 = 2y - 4x - 8$
$4x + 8 = 2y$
$2(2x + 4) = 2y$
$y = 2x + 4$.
Examples:
(i) $x = 1$, $y = 6 \rightarrow$ Product $= 1 \times 6 = 6$
Check: $(1 + 2) \times (6 - 4) = 3 \times 2 = 6$.

(ii) $x = 2$, $y = 8 \rightarrow$ Product $= 16$
Check: $(2 + 2) \times (8 - 4) = 4 \times 4 = 16$.

(iii) $x = 5$, $y = 14 \rightarrow$ Product $= 5 \times 14 = 70$
Check: $(5 + 2) \times (14 - 4) = 7 \times 10 = 70$.

Therefore, (1, 6), (2, 8), and (5, 14) are three valid examples.

4. Expand (i) (*a* + ab – 3b2) (4 + b), and (ii) (4y + 7) (y + 11z – 3).
Solution:
(i) $(a + ab - 3b^2)(4 + b)$
$= (a + ab - 3b^2)4 + (a + ab - 3b^2)b$
$= 4a + 4ab - 12b^2 + ab + ab^2 - 3b^3$
$= -3b^3 - 12b^2 + ab^2 + 4ab + ab + 4a$
$= -3b^3 - 12b^2 + ab^2 + 5ab + 4a$.

(ii) $(4y + 7)(y + 11z - 3)$
$= (4y + 7)y + (4y + 7)11z - (4y + 7)3$
$= 4y^2 + 7y + 44yz + 77z - (12y + 21)$
$= 4y^2 + 7y + 44yz + 77z - 12y - 21$
$= 4y^2 + 7y - 12y + 44yz + 77z - 21$
$= 4y^2 - 5y + 44yz + 77z - 21$.

5. Expand (i) $(a - b)(a + b)$, (ii) $(a - b)(a^2 + ab + b^2)$, and (iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$, Do you see a pattern? What would be the next identity in the pattern that you see? Can you check it by expanding?
Solution:
(i) $(a - b)(a + b)$
$= (a - b)a + (a - b)b$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$. (ii) $(a - b)(a^2 + ab + b^2)$
$= (a - b)a^2 + (a - b)ab + (a - b)b^2$
$= a^3 - a^2b + a^2b - ab^2 + ab^2 - b^3$
$= a^3 - b^3$.

(iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$
$= (a - b)a^3 + (a - b)a^2b + (a - b)ab^2 + (a - b)b^3$
$= a^4 - a^3b + a^3b - a^2b^2 + a^2b^2 - ab^3 + ab^3 - b^4$

$= a^4 – b^4$.

The next identity would be: $(a − b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 − b^5$.

Textbook Page 149

Figure it Out

1. Which is greater: $(a – b)^2$ or $(b – a)^2$? Justify your answer.
Solution:
$(a – b)^2 = a^2 + b^2 – 2ab$
$(b – a)^2 = b^2 + a^2 – 2ba$
or $(b – a)^2 = a^2 + b^2 – 2ab$
Therefore, $(a – b)^2 = (b – a)^2$.

2. Express 100 as the difference of two squares.
Solution:
Let, $a^2 − b^2 = 100$
We know that $a^2 − b^2 = (a + b)(a − b)$
∴ $(a + b)(a − b) = 100$
$(a + b)(a – b) = 2 × 2 × 5 × 5$
$(a + b)(a – b) = 50 × 2$
So,
$(a + b) = 50$ ………. (i)
$(a – b) = 2$ …………(ii)
Adding (i) and (ii): $a + b + a – b = 50 + 2$
$2a = 52$
$a = 52/2 = 26$ ………. (iii)
Substituting a in (i): $26 + b = 50$
$b = 50 – 26 = 24$ ………(iv)
Therefore,
$26^2 − 24^2 = 100$.

3. Find 406^2, 72^2, 145^2, 1097^2, and 124^2 using the identities you have learnt so far.
Solution:
(i) 406^2
$= (400 + 6)^2$
$= 400^2 + 6^2 + 2 × 400 × 6$
$= 160000 + 36 + 4800$
$= 164836$.

(ii) 72^2
= (70 + 2)2
$= 70^2 + 2^2 + 2 × 70 × 2$
$= 4900 + 4 + 280$
$= 5184$.

(iii) 145^2
$= (150 - 5)^2$
$= 150^2 + 5^2 - 2 \times 150 \times 5$
$= 22500 + 25 - 1500$
$= 21025$.

(iv) 1097^2
$= (1100 - 3)^2$
$= 1100^2 + 3^2 - 2 \times 1100 \times 3$
$= 1210000 + 9 - 6600$
$= 1203409$.

(v) 124^2
$= (130 - 6)^2$
$= 130^2 + 6^2 - 2 \times 130 \times 6$
$= 16900 + 36 - 1560$
$= 15376$.

4. Do Patterns 1 and 2 hold only for counting numbers? Do they hold for negative integers as well? What about fractions? Justify your answer.
Solution:
Pattern 1: $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$.
(i) Let a = 6, b = 4
L.H.S $= 2(a^2 + b^2)$
$= 2(6^2 + 4^2)$
$= 2(36 + 16) = 2(52) = 104$.
R.H.S $= (a + b)^2 + (a - b)^2$
$= (6 + 4)^2 + (6 - 4)^2$
$= 10^2 + 2^2 = 100 + 4 = 104$.
Therefore, Pattern 1 holds for counting numbers.

(ii) Let a = -10, b = -5
L.H.S $= 2(a^2 + b^2)$
$= 2[(-10)^2 + (-5)^2]$
$= 2[100 + 25]$
$= 2(125) = 250$.
R.H.S $= (a + b)^2 + (a - b)^2$
$= [(-10) + (-5)]^2 + [(-10) - (-5)]^2$
$=(-15)^2 + (-5)^2$
$= 225 + 25 = 250$.
Therefore, Pattern 1 holds for negative integers.

(iii)

Let $a = \frac{1}{2}$, $b = \frac{1}{3}$

L.H.S = $2(a^2 + b^2)$

$= 2\left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2\right]$

$= 2\left(\frac{1}{4} + \frac{1}{9}\right) = 2\left(\frac{9+4}{36}\right) = 2 \times \frac{13}{36} = \frac{13}{18}.$

R.H.S = $(a + b)^2 + (a - b)^2$

$= \left(\frac{1}{2} + \frac{1}{3}\right)^2 + \left(\frac{1}{2} - \frac{1}{3}\right)^2$

$= \left(\frac{3+2}{6}\right)^2 + \left(\frac{3-2}{6}\right)^2 = \left(\frac{5}{6}\right)^2 + \left(\frac{1}{6}\right)^2$

$= \frac{25}{36} + \frac{1}{36} = \frac{25+1}{36} = \frac{26}{36} = \frac{13}{18}.$

Therefore, Pattern 1 holds for fractions.

Pattern 2: $(a + b) \times (a - b) = a^2 - b^2$.

(i) Let a = 6, b = 4

L.H.S = $(a + b) \times (a - b)$

$= (6 + 4)(6 - 4)$

$= 10 \times 2 = 20.$

R.H.S = $a^2 - b^2$

$= 6^2 - 4^2$

$= 36 - 16 = 20.$

Therefore, Pattern 2 holds for counting numbers.

(ii) Let a = -10, b = -5

L.H.S = $(a + b) \times (a - b)$

$= [(-10) + (-5)] \times [(-10) - (-5)]$

$= [-15] \times [-5] = 75.$

R.H.S = $a^2 - b^2$

$= [(-10)^2 - (-5)^2]$

$= 100 - 25 = 75.$

Therefore, Pattern 2 holds for negative numbers.

(iii)

Let a = $\frac{1}{2}$, b = $\frac{1}{3}$

L.H.S = (a + b) × (a – b)

$= \left(\frac{1}{2} + \frac{1}{3}\right) \times \left(\frac{1}{2} - \frac{1}{3}\right)$

$= \left(\frac{3+2}{6}\right) \times \left(\frac{3-2}{6}\right) = \left(\frac{5}{6}\right) \times \left(\frac{1}{6}\right) = \frac{5}{36}.$

R.H.S = $a^2 - b^2$

$= \left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2$

$= \frac{1}{4} - \frac{1}{9} = \frac{9-4}{36} = \frac{5}{36}$

Therefore, Pattern 2 holds for fractions.

Textbook Page 154

Figure it Out

1. Compute these products using the suggested identity.
(i) 46^2 using Identity 1A for $(a + b)^2$
Solution:
Identity $(a + b)^2 = a^2 + 2ab + b^2$
46^2
$= (40 + 6)^2$
$= (40)^2 + (6)^2 + 2 \times 40 \times 6$
= 1600 + 36 + 480 = 2116. **(ii) 397 × 403 using Identity 1C for (a + b) (a – b)**
Solution:
Identity $(a + b)(a - b) = a^2 - b^2$.
397 × 403
= (400 + 3) (400 – 3)
$= 400^2 - 3^2$
= 160000 – 9 = 159991.

(iii) 91^2 using Identity 1B for $(a - b)^2$
Solution:
Identity $(a - b)^2 = a^2 - 2ab + b^2$
91^2
$= (100 - 9)^2$
$= (100)^2 + (9)^2 - 2 \times 100 \times 9$
= 10000 + 81 – 1800 = 8281.

(iv) 43 × 45 using Identity 1C for (a + b) (a – b)

Solution:

Identity $(a + b)(a - b) = a^2 - b^2$.

43×45

$= (44 - 1)(44 + 1)$

$= 44^2 - 1^2$

$= 1936 - 1 = 1935$.

2. Use either a suitable identity or the distributive property to find each of the following products.

(i) (p – 1) (p + 11)

Solution:

Using distributive property

$(p - 1)(p + 11)$

$= p(p + 11) - 1(p + 11)$

$= p^2 + 11p - p - 11$

$= p^2 + 10p - 11$.

(ii) (3a – 9b) (3a + 9b)

Solution:

Using Identity $(a + b)(a - b) = a^2 - b^2$.

$(3a - 9b)(3a + 9b)$

$= (3a)^2 - (9b)^2$

$= 9a^2 - 81b^2$.

(iii) –(2*y* + 5) (3*y* + 4)

Solution:

Using distributive property

$-(2y + 5)(3y + 4)$

$= -[\ 2y(3y + 4) + 5(3y + 4)\ ]$

$= -[\ 6y^2 + 8y + 15y + 20\ ]$

$= -[\ 6y^2 + 23y + 20\ ]$

$= -6y^2 - 23y - 20$.

(iv) (6*x* + 5y)^2

Solution:

Identity $(a + b)^2 = a^2 + 2ab + b^2$

$(6x + 5y)^2$

$= (6x)^2 + (5y)^2 + 2 \times 6x \times 5y$

$= 36x^2 + 25y^2 + 60xy$.

(v) (2x – 1/2)^2

Solution:

Using Identity $(a - b)^2 = a^2 - 2ab + b^2$

$$\left(2x - \frac{1}{2}\right)^2$$

$$= (2x)^2 + \left(\frac{1}{2}\right)^2 - 2.\ 2x.\ \frac{1}{2}$$

$$= 4x^2 + \frac{1}{4} - 2x.$$

(vi) (7p) × (3r) × (p + 2)

Solution:

(7p) × (3r) × (p + 2)

= 21pr (p + 2)

= $21p^2r + 42pr$.

3. For each statement identify the appropriate algebraic expression(s).

(i) Two more than a square number.

2 + s ; $(s + 2)^2$; $s^2 + 2$; $s^2 + 4$; $2s^2$; 2^2s

Solution:

Let the number be s.

Sqaure number = s^2

Therefore, two more than a square number = $s^2 + 2$.

(ii) The sum of the squares of two consecutive numbers

$m^2 + n^2$; $(m + n)^2$; $m^2 + 1$; $m^2 + (m + 1)^2$; $m^2 + (m - 1)^2$; $(m + (m + 1))^2$; $(2m)^2 + (2m + 1)^2$

Solution:

Let the two consecutive numbers be m and (m + 1)

Therefore, the sum of two consecutive numbers = $m^2 + (m + 1)^2$.

4. Consider any 2 by 2 square of numbers in a calendar, as shown in the figure.

February						
Su	M	Tu	W	Th	F	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

Find products of numbers lying along each diagonal — 4 × 12 = 48, 5 × 11 = 55. Do this for the other 2 by 2 squares. What do you observe about the diagonal products? Explain why this happens.

Hint: Label the numbers in each 2 by 2 square as

a	$(a + 1)$
$a + 7$	$(a + 8)$

Solution:

(i)

16	17
23	24

First diagonal: 16 × 24 = 384.
Second diagonal: 23 × 17 = 391.
Difference: 391 – 384 = 7.

(ii)

7	8
14	15

First diagonal: 7 × 15 = 105.
Second diagonal: 14 × 8 = 112.
Difference: 112 – 105 = 7.
We observe that the difference of the digonal products of any 2 by 2 square of numbers from the calender is always 7.

5. Verify which of the following statements are true.
(i) (k + 1) (k + 2) – (k + 3) is always 2.
Solution:
The statement is false.
Explantion:
(k + 1) (k + 2) – (k + 3)
= k(k + 2) + 1(k + 2) – k – 3
= k^2 + 2k + k + 2 – k – 3
= k^2 + 2k – 1.
If k = 2, k^2 + 2k – 1 = $(2)^2$ + 2(2) – 1 = 4 + 4 – 1 = 7.
if k = 3, k^2 + 2k – 1 = $(3)^2$ + 2(3) – 1 = 9 + 6 – 1 = 14.

(ii) (2q + 1) (2q – 3) is a multiple of 4.
Solution:
The statement is false.
Explanation:
(2q + 1) (2q – 3)
= 2q (2q – 3) + 1 (2q – 3)
= $4q^2$ – 6q + 2q – 3
= $4q^2$ – 4q – 3.
If k = 1, $4q^2$ – 4q – 3 = $4(1)^2$ – 4(1) – 3 = 4 – 4 – 3 = -3.
If k = 2, $4q^2$ – 4q – 3 = $4(2)^2$ – 4(2) – 3 = 4(4) – 8 – 3 = 16 – 8 – 3 = 5.

(iii) Squares of even numbers are multiples of 4, and squares of odd numbers are 1 more than multiples of 8.

Solution:

The statement is true.

Explanation:

1.For even numbers:

Let the even number be 2n.

Then, $(2n)^2 = 4n^2$.

Clearly, $4n^2$ is a multiple of 4.

So, squares of even numbers are multiples of 4.

2. For odd numbers:

Let the odd number be 2n + 1.

Then, $(2n + 1)^2 = 4n^2 + 4n + 1 = 4n\ (n + 1) + 1$.

Now, n(n + 1) is always even, so let n(n + 1) = 2k.

Then, 4n(n + 1) + 1 = 4(2k) + 1 = 8k + 1.

This means the square of an odd number is 1 more than a multiple of 8.

(iv) $(6n + 2)^2 – (4n + 3)^2$ is 5 less than a square number.

Solution:

The statement is false.

Expalnation:

$(6n + 2)^2 – (4n + 3)^2$

$= [(6n)^2 + 2 \times 6n \times 2 + (2)^2\ ] – [(4n)^2 + 2 \text{ x } 4n \times 3 + (3)^2\ ]$

$= (36n^2 + 24n + 4) – (16n^2 + 24n + 9)$

$= 36n^2 + 24n + 4 – 16n^2 – 24n – 9$

$= 20n^2 – 5$.

Here, 20 is not a sqaure number. So, $20n^2$ is not a square number.

6. A number leaves a remainder of 3 when divided by 7, and another number leaves a remainder of 5 when divided by 7. What is the remainder when their sum, difference, and product are divided by 7?

Solution:

Let the first number be 7a + 3 and the second number be 7b + 5.

(i) Sum:

(7a + 3) + (7b + 5) = 7a + 3 + 7b + 5 = 7a + 7b + 3 + 5

= 7(a + b) + 8

= 7(a + b) + 7 + 1

= 7(a + b + 1) + 1.

So, the remainder is 1.

(ii) Difference:

(7a + 3) − (7b + 5) = 7a + 3 – 7b – 5 = 7a – 7b + 3 – 5

= 7(a − b) − 2

$= 7(a - b) – 7 + 5$
$= 7(a – b – 1) + 5.$
So, the remainder is 5.

(iii) Product:
$(7a + 3)(7b + 5) = 7a(7b + 5) + 3(7b + 5)$
$= 49ab + 35a + 21b + 15$
$= 49ab + 35a + 21b + 14 + 1$
$= 7(7ab + 5a + 3b + 2) + 1.$
So, the remainder is 1.

7. Choose three consecutive numbers, square the middle one, and subtract the product of the other two. Repeat the same with other sets of numbers. What pattern do you notice? How do we write this as an algebraic equation? Expand both sides of the equation to check that it is a true identity.
Solution:
Let three consecutive numbers: $(n - 1), n, (n + 1)$.
Squaring the middle one and subtracting the product of the other two:
$n^2 - (n - 1)(n + 1)$
$= n^2 - \{ n (n + 1) – 1 (n + 1) \}$
$= n^2 - \{ n^2 + n – n – 1 \}$
$= n^2 – \{ n^2 – 1\}$
$= n^2 – n^2 + 1 = 1.$

Let another set of three consecutive numbers: $n, n + 1, n + 2$.
Squaring the middle one and subtracting the product of the other two:
$(n + 1)^2 – n (n + 2)$
$= n^2 + 1 + 2n – (n^2 + 2n)$
$= n^2 + 1 + 2n – n^2 – 2n = 1.$

Therefore, for any three consecutive numbers, the square of the middle number minus the product of the other two is always 1.
Algebraic identity: $n^2 - (n - 1)(n + 1) = 1$

Check with a numerical example: take 2, 3, 4 :
$3^2 - (2 \cdot 4) = 9 - 8 = 1.$
Thus, the identity is true for all integers.

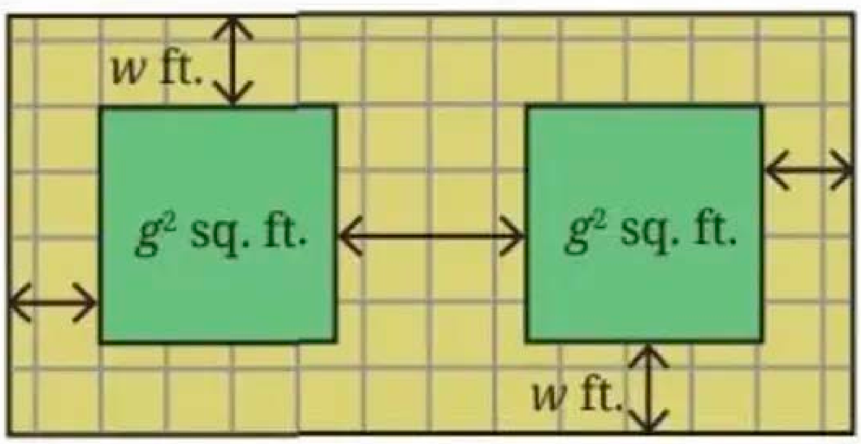
8. What is the algebraic expression describing the following steps—add any two numbers. Multiply this by half of the sum of the two numbers? Prove that this result will be half of the square of the sum of the two numbers.
Solution:
Let the two numbers be a and b.
Adding these two numbers = $(a + b)$.

Multiplying this by half of the sum of the numbers = (a + b) × 1/2 (a + b)
= 1/2 $(a + b)^2$.
Therefore, the result is half of the square of the sum of the two numbers.

9. Which is larger? Find out without fully computing the product.
(i) 14 × 26 or 16 × 24
(ii) 25 × 75 or 26 × 74
Solution:
(i) 14 × 26 or 16 × 24
Let a = 14 × 26
b = 16 × 24
b = (14 + 2) (26 – 2)
b = 14 × 26 + 2 × 26 – 14 × 2 – 2 × 2
b = 14 x 26 + 2(26 – 14 – 2)
b = 14 x 26 + 2 × 10
b = a + 2 × 10
Therefore, b > a
or 16 × 24 > 14 × 26.

(ii) 25 × 75 or 26 × 74
Let a = 25 × 75
b =26 x 74
b = (25 + 1) (75 – 1)
b = 25 × 75 + 75 × 1 – 25 × 1 – 1 × 1
b = a + (75 – 25 – 1)
b = a + 49
Therefore, b > a
or 26 × 74 > 25 × 75.

10. A tiny park is coming up in Dhauli. The plan is shown in the figure. The two square plots, each of area g^2 sq. ft., will have a green cover. All the remaining area is a walking path w ft. wide that needs to be tiled. Write an expression for the area that needs to be tiled.



Solution:

Area of square plots is g^2 sq. ft., then each side of the plot is 'g' ft.

Length of the park = w + g + w + w + g + w = (4w + 2g) ft.

Breadth of the park = w + g + w = (2w + g) ft.

Area of the park = (4w + 2g) × (2w + g)

= 4w(2w + g) + 2g(2w + g)

= $8w^2 + 4gw + 4gw + 2g^2$

= $8w^2 + 8gw + 2g^2$ sq. ft.

Area to be tiled = $8w^2 + 8gw + 2g^2 - 2 \times g^2$

= $8w^2 + 8gw + 2g^2 - 2g^2$.

= $8w^2 + 8gw = 8w(w + 1)$ sq. ft.

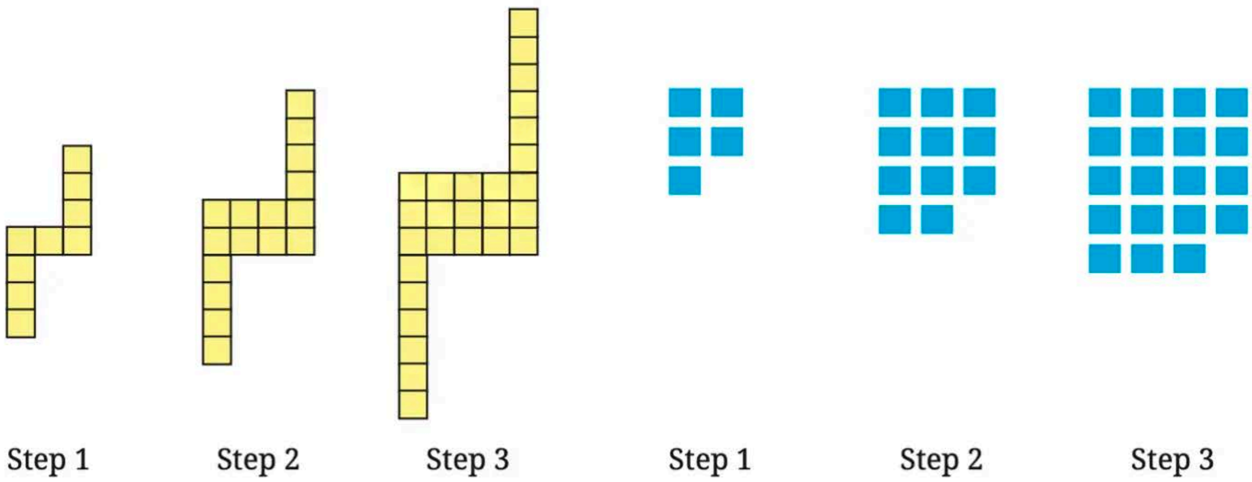
Therefore, 8w(w + 1) sq. ft. is the area that needs to be tiled.

11. For each pattern shown below,

(i) Draw the next figure in the sequence.

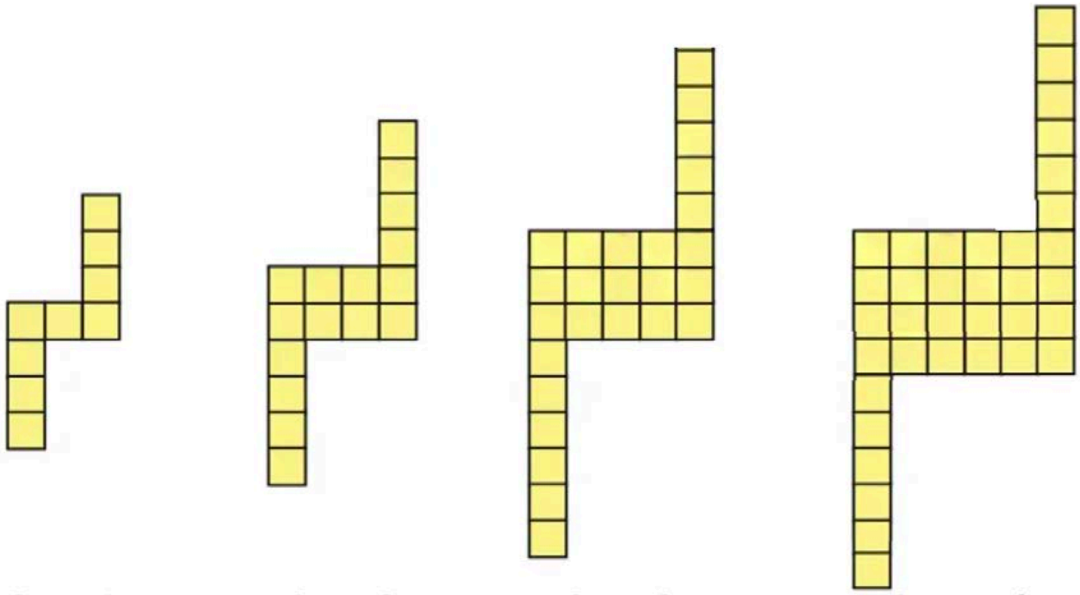
(ii) How many basic units are there in Step 10?

(iii) Write an expression to describe the number of basic units in Step y.



Solution:

(i)



Step 1 Step 2 Step 3 Step 4

Step 1 Step 2 Step 3 Step 4