Competency Focused Practice Questions

Mathematics (Volume 1) | Grade 12

Co-created by

CBSE Centre for Excellence in Assessment

and

Educational Initiatives

Preface

Assessments are an important tool that help gauge learning. They provide valuable feedback about the effectiveness of instructional methods; about what students have actually understood and also provide actionable insights. The National Education Policy, 2020 has outlined the importance of competency-based assessments in classrooms as a means to reform curriculum and pedagogical methodologies. The policy emphasizes on the development of higher order skills such as analysis, critical thinking and problem solving through classroom instructions and aligned assessments.

Central Board of Secondary Education (CBSE) has been collaborating with Educational Initiatives (Ei) in the area of assessment. Through resources like the Essential Concepts document and A- Question-A-Day (AQAD), high quality questions and concepts critical to learning have been shared with schools and teachers.

Continuing with the vision to ensure that every student is learning with understanding, Question Booklets have been created for subjects for Grade 10th and 12th. These booklets contain competency-based items, designed specifically to test conceptual understanding and application of concepts.

Process of creating competency-based items

All items in these booklets are aligned to the NCERT curriculum and have been created keeping in mind the learning outcomes that are important for students to understand and master. Items are a mix of Free Response Questions (FRQs) and Multiple-Choice Questions (MCQs). In case of MCQs, the options (correct answer and distractors) are specifically created to test for understanding and capturing specific errors/misconceptions that students may harbour. Each incorrect option can thereby inform teachers on specific gaps that may exist in student learning. In case of subjective questions, each question also has a detailed scoring rubric to guide evaluation of students' responses.

Each item has been reviewed by experts, to check for appropriateness of the item, validity of the item, conceptual correctness, language accuracy and other nuances.

How can these item booklets be used?

There are 267 questions in this booklet.

The purpose of these item booklets is to provide samples of high-quality competency-based items to teachers. The items can be used to–

- get an understanding of what good competency-based questions could look like
- give exposure to students to competency-based items
- assist in classroom teaching and learning
- get inspiration to create more such competency-based items

Students can also use this document to understand different kinds of questions and practice specific concepts and competencies. There will be further additions in the future to provide competency focused questions on all chapters.

The item booklets are aligned with the 2022-23 curriculum. However, a few questions from topic which got rationalized in 2023-24 syllabus are also there in the booklet which may be used as a reference for teachers and students.

Please write back to us to give your feedback.

Team CBSE

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Chapter - 1
Relations and Functions

Q: 1 **Given below is a relation R from the set X = { x,y,z } to itself.**

R = {(x,x), (x,y), (y,x), (y,z), (x,z)}

Which of the following is true about the relation R?

1. R is reflexive and transitive but not symmetric.
2. R is symmetric and transitive but not reflexive.
3. R is transitive but neither reflexive nor symmetric.
4. R is not reflexive, not symmetric and not transitive.

Q: 2 **A and B are two sets with m elements and n elements respectively ($m < n$).**

How many onto functions can be defined from set A to B?

1. 0
2. m !
3. n !
4. n^m

Q: 3 **Three students Aabha, Bhakti and Chirag were asked to define a function, f, from set X = {1, 3, 5, 7, 9} to set Y = {2, 4, 6, 8}. Their responses are shown below:**

Aabha: f = {(1, 2), (1, 4), (1, 6), (1, 8)}

Bhakti: f = {(1, 2), (3, 4), (5, 6), (7, 8)}

Chirag: f = {(1, 4), (3, 4), (5, 4), (7, 4), (9, 4)}
Who defined a function correctly?

1. only Chirag
2. only Aabha and Bhakti
3. only Bhakti and Chirag
4. only Chirag and Aabha

Q: 4 **Which of the following is an equivalence relation on the set P = {1, 4, 9}?**

1. R_1 = {(1, 1), (4, 4), (9, 9)}
2. R_2 = {(1, 1), (4, 4), (1, 4), (4, 1)}
3. R_3 = {(4, 4), (9, 9), (1, 1), (9, 1), (1, 9), (1, 4), (4, 1)}
4. R_4 = {(1, 4), (4, 4), (9, 4), (4, 1), (1, 1), (9, 9), (9, 1)}

Q: 5 **The power set of a set A = { a,b } is the set of all subsets of A. These subsets are given by:**

P(A) = {Φ, { a }, { b }, { a,b }}

A relation R is defined on P(A) as R = {(r,s) : $r \subseteq s$ }

Which of the following is the correct representation of R in its roster form?

1. {Φ, { a }, { b }, { a,b }}
2. {(Φ, { a }), (Φ, { b }), (Φ, { a,b })}
3. {(Φ, { a }), (Φ, { b }), (Φ, { a,b }), ({ a }, { a,b }), ({ b }, { a,b })}
4. {(Φ, { a }), (Φ, { b }), (Φ, { a,b }), ({ a }, { a,b }), ({ b }, { a,b }), ({ a,b }, { a,b })}

Q: 6 ***f*** **: X -> X is a function on the finite set X.**

Given below are two statements based on the above context - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion (A) **: If *f* is onto, then *f* is one-one and if *f* is one-one, then *f* is onto.**

Reason (R) **: Every one-one function is always onto and every onto function is always one-one.**

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true but (R) is not the correct explanation for (A).
3. (A) is true but (R) is false.
4. Both (A) and (R) are false.

Q: 7 **The set of non-negative integers, denoted by Z^*, is the set containing positive integers along with zero.**

On the set Z^*, a function *f* : Z^* -> Z^* is defined by:

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is an odd integer} \\ 0 \text{ , if } n \text{ is an even integer} \end{cases}$$

Which of the following is true about *f* ?

1. *f* is one-one but not onto
2. *f* is onto but not one-one
3. *f* is both one-one and onto
4. *f* is neither one-one nor onto

Q: 8 **Consider an operation * defined on the set { *a* , *b* , *c* } given by the following operation table.**

*	***a***	***b***	***c***
a	*a*	*a*	*a*
b	*a*	*a*	*a*
c	*a*	*a*	*a*

Which of the following is true about the operation *?

1. * is not a binary operation
2. * is a binary operation that is commutative but not associative
3. * is a binary operation that is associative but not commutative
4. * is a binary operation that is both commutative and associative

Q: 9 **If $f(x) = x^3 + 1$ and $f(g(x)) = x$, then which of the following is $g(1)$?**

1. 0
2. 1
3. 2
4. (cannot be determined without knowing what is $g(x)$)

Q: 10 **A relation R on set G = {All the students in a certain mathematics class} is defined as, R = {(x, y): x and y have the same mathematics teacher}.**

Which of the following is true about R?

1. R is reflexive and transitive but not symmetric.
2. R is transitive and symmetric but not reflexive.
3. R is reflexive and symmetric but not transitive.
4. R is an equivalence relation.

Q: 11 **State whether the following statement is true or false. Justify your answer.** **[1]**

"The sine function is bijective in nature when the domain is set from 0 to 4π."

Q: 12 **[1]**

$$f(x) = \frac{2\tan x}{1-\tan^2 x}$$

Find the range of $f(x)$ for $x \in R$. Show your steps.

Q: 13 **State whether the following statement is true or false. Justify your answer.** **[1]**

"A function $f(x) = \ln x$ is invertible for all values of x."

Q: 14 **[1]**

A = {1, 3, 5, 7, ...}
B = {2, 4, 6, 8, ...}

Define a function from A to B that is neither one-one nor onto.

Q: 15 **X and Y are two sets with their number of elements being k and l respectively ($k < l$).** **[1]**

Find the number of onto functions that can be defined from set X to Y. Explain your answer.

Q: 16 **Is * defined on Q by $m * n = m^n$ a binary operation? Justify your answer.** **[1]**

(Note: Q is the set of rational numbers.)

Q: 17 **Find the domain of (*f* o *g*) if** **[2]**

$$f(x) = \frac{3}{x+1}$$

and

$$g(x) = \frac{x}{3x-2}$$

Show your steps and give reasons.

Q: 18 **Shreyas and Simran are playing a game in which they are trying to guess relations on set A = {-2, -3}. Simran tells Shreyas that she is thinking of an equivalence relation.** **[2]**

Shreyas guesses the relation as R = {(-2, -3), (-3, -2), (-3, -3)} .

Could Shreyas be correct? Justify your answer.

Q: 19 **Let *f* : A → B and *g* : B → C be two functions.** **[2]**

i) If *f* is not one-one, can *g* o *f* be one-one?
ii) If *f* is not onto, can *g* o *f* be onto?

Justify your answer.

Q: 20 **f: R -> R defined by f(*x*) = $\frac{3x}{8-5x}$ is not a function.** **[2]**

i) Why is f not a function?
ii) Based on your explanation in part i), what changes can be made so that f(*x*) = $\frac{3x}{8-5x}$ becomes a function?

Q: 21 The operations table for a binary operation * is shown below. **[2]**

*	**3**	**5**	**7**
3	3	3	3
5	3	5	5
7	3	5	7

i) Define the above binary operation *.
ii) Is the operation commutative?
iii) Is the operation associative?

Justify your answer.

Q: 22 State whether the following statement is true or false. If true, give a reason. If false, give an example. **[2]**

If $f(x)$ and $g(x)$ are two functions, then their composition is commutative. In other words, $f(g(x)) = g(f(x))$.

Q: 23 A teacher wrote $f(x) = x^9$ on the board and asked her student to examine whether the following equation is true or false. **[2]**

$[f(x)]^{\frac{1}{3}} = [f^{-1}(x)]^3$

Raghu said, "It is true".

Is he correct? Justify your answer.

Q: 24 f and g are real functions such that f is bijective and $g(x) = 3x + 4$, for all $x \in$ R. **[3]**

Is (gof) invertible? Justify your answer.

Q: 25 X = {2, 6, 12, 20, ...} **[3]**

Y = {2, 3, 4, 5, ...}

i) Define a one-one function from set X to set Y.
ii) Show that the function you defined is one-one.

Q: 26 **A relation R in set G = {All the countries in the world} is defined as R = {(x, y): x and y share a common boundary}.** **[3]**

Determine whether R is reflexive, symmetric and transitive. Hence, conclude if R is an equivalence relation. Show your work.

Q: 27 **Graph of a certain function f: R -> R is a straight line parallel to x -axis, where R is the set of real numbers.** **[3]**

i) Is the function one-one?
ii) Is the function onto?

Justify your answer.

Read the information given below and answer the questions that follow.

A confectionery shop is a place where sweets and chocolates are sold. The table below gives information on four varieties of chocolates sold there.

Chocolate name	Cost per piece (in Rs)
Daily Milk (D)	5
25-Star (S)	10
Crunch (C)	20
Ket-Kat (K)	50

Let A = {D, S, C, K} be the set containing the chocolates and B = {5, 10, 20, 50} be the set containing their costs.

A relation R is defined on set A as R = {(x, y) : cost of $x \leq$ cost of y }.

Q: 28 **Express the relation R in roster form.** **[1]**

Q: 29 **Is R a reflexive relation? Justify your answer.** **[1]**

Q: 30 **Is R a symmetric relation? Justify your answer.** **[1]**

Q: 31 **Is R a transitive relation? Justify your answer.** **[1]**

Q: 32 **Define a function from set A to set B.** **[1]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	1
3	1
4	1
5	4
6	3
7	2
8	4
9	1
10	4

Q.No	Teacher should award marks if students have done the following:	Marks
11	**Writes False(F).**	**0.5**
	Writes that the sine function is onto but not one-one (gives an example such as sin($\frac{\pi}{2}$) = sin($\frac{5\pi}{2}$) = 1), therefore it is not bijective in nature.	**0.5**
12	**Rewrites *f* (*x*) as tan(2 *x*).**	**0.5**
	Writes that the range of tan(2 *x*) is R(all real numbers).	**0.5**
13	**Writes True(T).**	**0.5**
	Writes that the logarithmic function is both onto and one-one. Hence, its inverse exists. **(Award full marks for any other logical explanation.)**	**0.5**
14	**Defines a function from A to B that is neither one-one nor onto. For example,** **f: A -> B defined by f(*x*) = 4 for all *x* ∈ A.**	**1**
15	**Writes that the number of onto functions from set X to Y is zero.**	**0.5**
	Reasons that, since set Y contains more elements than set X, at least one element of Y will always remain unmapped.	**0.5**
16	**Writes no.**	**0.5**
	writes a pair of rational numbers *m* and *n* such that m^n is not rational. **For example, 2 * $\frac{1}{2}$ = $2^{\frac{1}{2}}$ = √2 ∉ Q.**	**0.5**
17	**Finds the composite function (*f* o *g*)(*x*) as:** $\frac{3(3x-2)}{2(2x-1)}$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that $\frac{1}{2} \notin$ domain of (f o g) as (f o g)(x) is not defined at $x = \frac{1}{2}$.	0.5
	Writes that the domain of (f o g) will not contain $\frac{2}{3}$ as the domain of (f o g) is a subset of the domain of g, and $\frac{2}{3} \notin$ domain of g.	0.5
	Concludes that the domain of (f o g) is the set of all real numbers except $\frac{1}{2}$ and $\frac{2}{3}$.	0.5
18	Writes that Shreyas is not correct.	0.5
	Writes that Shreyas' relation is not reflexive as (-2, -2) is not a part of it.	1
	Writes that, since the relation is not reflexive, it cannot be an equivalence relation.	0.5
19	i) Writes that g o f cannot be one-one, since g o f is one-one implies f is one-one.	1
	ii) Writes that g o f can be onto, since g o f is onto implies g is onto and there is no restriction on f to be one-one or onto.	1
20	i) Writes that f($\frac{8}{5}$) = $\frac{24}{0} \notin$ R. Thus, f is not a function as it is not well defined.	1
	ii) Writes that f can be made a function by removing the element $\frac{8}{5}$ from the domain R.	1
21	i) Defines the given binary operation as $a * b$ = min{ a, b }.	0.5
	ii) Writes that the function is commutative as min{ a, b} = min{ b, a }. or Writes that the function is commutative since first row is identical to first column in the operations table.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	iii) Writes that, $a * (b * c) = a * \min\{b, c\} = \min\{a, b, c\}$. Similarly, $(a * b) * c = \min\{a, b\} * c = \min\{a, b, c\}$. Therefore, $a * (b * c) = (a * b) * c$ and the binary operation is associative.	1
22	Writes false.	0.5
	Gives an example: Let $f(x) = x^2$ and, $g(x) = x + 2$. Then, $f(g(x)) = (x + 2)^2$ and, $g(f(x)) = x^2 + 2$. $\Rightarrow f(g(x)) \neq g(f(x))$	1.5
23	Writes that Raghu is wrong.	0.5
	Finds $f^{-1}(x)$ as $x^{\frac{1}{9}}$.	0.5
	Finds RHS as $[f^{-1}(x)]^3 = x^{\frac{1}{3}}$.	0.5
	Finds LHS as $[f(x)]^{\frac{1}{3}} = x^3$ and compares it with the RHS. Concludes that $[f(x)]^{\frac{1}{3}} \neq [f^{-1}(x)]^3$.	0.5
24	Writes that $g(x)$ is one-one as: $g(x_1) = g(x_2)$ $\Rightarrow 3x_1 + 4 = 3x_2 + 4$ $\Rightarrow x_1 = x_2$.	0.5
	Writes that for any real value of y in R, there exists $\frac{y-4}{3}$ in R such that: $g\left(\frac{y-4}{3}\right)$ $= 3 \times \frac{y-4}{3} + 4$ $= y - 4 + 4$ $= y$ Thus, $g(x)$ is onto. Concludes that $g(x)$ is bijective therefore invertible.	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that $f(x)$ is bijective and therefore invertible.	**0.5**
	Uses the theorem that if f and g are invertible, then (gof) is also invertible to conclude that (gof) is invertible.	**1**
25	**i) Rewrites set X as $\{1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5, ..., n(n+1), ...\}$.**	**1**
	Defines a function f: X -> Y as: $f(n(n+1)) = n + 1$	**1**
	ii) Shows that the above function is one-one as: $f(n(n+1)) = f(m(m+1))$ $\Rightarrow n + 1 = m + 1$ $\Rightarrow n = m$	**1**
26	**Writes that every country shares its boundary with itself.** **That is, $(x, x) \in R$, for each element $x \in G$. Hence, R is reflexive.**	**0.5**
	Writes that, whenever x shares a boundary with y, y also shares a boundary with x. **That is, $(x, y) \in R \Rightarrow (y, x) \in R$. Hence, R is symmetric.**	**1**
	Writes that, if x shares a boundary with y and y shares a boundary with z, then x need not share a boundary with z. **That is, $(x, y) \in R$, $(y, z) \in R$ need not imply $(x, z) \in R$. Hence, R is not transitive.**	**1**
	From the above steps, concludes that R is not an equivalence relation.	**0.5**
27	**i) Writes that the function must be of the form $f(x) = k$, where 'k' is a real number.**	**1**
	Writes that f is not one-one. **Justifies by giving an example as follows:** **$f(1) = k = f(2)$, but $1 \neq 2$.**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Writes that f is not onto. **Justifies by giving an example as follows:** **Consider β ($\neq k$) $\in$ R (codomain),** **there is no element $x \in$ R (domain) such that f(x) = β.**	**1**
28	**Expresses the relation R in roster form as R = {(D, S), (D, C), (D, K), (S, C), (S, K), (C, K)} .**	**1**
29	**Writes yes.**	**0.5**
	Justifies the answer. For example, the cost of every chocolate is equal to its own cost i.e. (x, x) $\in$ R, for every $x \in$ A.	**0.5**
30	**Writes no.**	**0.5**
	Justifies the answer. For example, (D, S) $\in$ R but (S, D) $\notin$ R. Hence, R is not symmetric.	**0.5**
31	**Writes yes.**	**0.5**
	Justifies the answer. For example: **Let (x, y) and (y, z) $\in$ R** **=> cost of $x \leq$ cost of y** **and cost of $y \leq$ cost of z** **Uses the above set of inequalities to show that cost of $x \leq$ cost of z. Hence, concludes that (x, z) $\in$ R.**	**0.5**
32	**Defines a function from set A to set B. For example:** **f : A -> B, defined by, f (x) = cost of x.** **(Award full marks if any function is written correctly in set-builder form or in roster form.)**	**1**

Chapter - 2
Inverse Trigonometric Functions

Q: 1 **What is the domain of the function $y = \sec^{-1} x + \sin^{-1} x$?**

1 -1 and 1 **2** [-1, 1] **3** (-∞, -1] ∪ [1, ∞) **4** Φ

Q: 2

Given: $\tan^{-1}\left(\sqrt{1 + \tan^2\theta} \times \sin(\pi - \theta)\right) = \dfrac{1 - a^2}{(1 + a)^2}$

where $a \in R$ and $a \neq -1$

Which of the following gives the value of θ in terms of a?

$\tan\left(\dfrac{a-1}{a+1}\right)$ Expression 1

$\left(\dfrac{1+a}{1-a}\right)$ Expression 2

$\dfrac{1-a^2}{(1+a)^2}$ Expression 3

$\left(\dfrac{a-1}{1+a}\right)$ Expression 4

1 Expression 1 **2** Expression 2 **3** Expression 3 **4** Expression 4

Q: 3 **Which of the following is the domain of the function given below?**

$y = \cos^{-1}\left(\frac{1}{x-3}\right)$

1 [-1, 1] **2** [2, 4] **3** (-∞, 2] ∪ [4, ∞) **4** (-∞, -1] ∪ [1, ∞)

Q: 4 **The domain of $f(x) = \sin^{-1} x - \cos^{-1} x$ is [-1, 1] while its range is $[\frac{-3\pi}{2}, \frac{\pi}{2}]$.**

If $g(x)$ is the inverse of $f(x)$, which of the following is true about the domain of the function $g(x)$?

1 It is [-1, 1].
2 It is $[\frac{-3\pi}{2}, \frac{\pi}{2}]$.
3 It is independent of the domain and range of $f(x)$.
4 (cannot be said without knowing $g(x)$)

Q: 5 **If $\sec^{-1}(-x) = \frac{\pi}{8}$, which of the following could be the value of $\sec^{-1}(x)$?**

1 $(-\frac{\pi}{8})$
2 $\frac{7\pi}{8}$
3 $\frac{9\pi}{8}$
4 (cannot be determined without knowing the value of x)

Q: 6 **Akash says that, since the domain of the sine function is (-∞, ∞), $\sin^{-1} x$ is well defined in the domain (-∞, ∞).** [1]

Is Akash right or wrong? Justify your answer.

Q: 7 **Simplify:** **[1]**

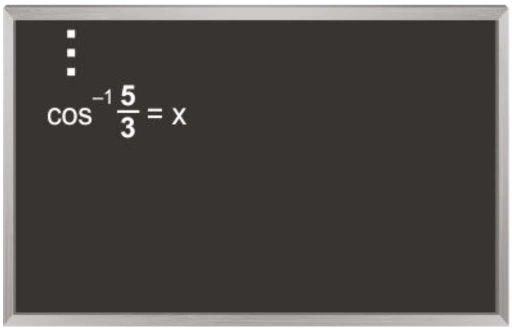
$\cos\left(\frac{\pi}{2} + \sin^{-1}\frac{1}{\sqrt{3}}\right)$

Show your work.

Q: 8 $\text{cosec}\,\frac{\pi}{6} = \text{cosec}\,\frac{5\pi}{6} = 2$, **but** $\frac{\pi}{6} \neq \frac{5\pi}{6}$. **[1]**

Since the cosecant function is not one-one, how can it be made invertible? Give a reason for your answer.

Q: 9 **While solving an inverse trigonometry problem on the blackboard, Satish wrote the following as part of his solution:** **[1]**



His teacher stopped him and said that he must have made a mistake in the solution.

How did the teacher recognise that Satish had made a mistake? Justify your answer.

Q: 10 **What would be the value of** $\cos^{-1}\left(\frac{24}{25}\right) + \tan^{-1}\left(\frac{24}{7}\right)$**?** **[2]**

Q: 11 **Prove that:** **[2]**

$$\sin^{-1}\left[\frac{2^{x+1}}{1+4^{x}}\right] = 2\tan^{-1}(2^{x}), \text{ where } x \leq 0$$

Q: 12 **Chirag asked his students to find the value of:** **[2]**

$\tan^{-1}(-x) - \tan^{-1}\left(\frac{1}{x}\right)$

Rahul said that the value of the above expression can be found ONLY if x **is known.**

Is Rahul correct? Justify your answer.

Q: 13 **Considering the principal value branch, prove that the property below is true ONLY for $xy > (-1)$.** **[3]**

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	1
2	3
3	3
4	2
5	2

Q.No	Teacher should award marks if students have done the following:	Marks
6	Writes that Akash is wrong.	0.5
	Writes that $\sin^{-1} x$ cannot be defined in the domain $(-\infty, \infty)$ as $\sin x$ is not one-one in that domain. (Award full marks for any other valid reason.)	0.5
7	Simplifies the above expression as: $\cos(\frac{\pi}{2} + \sin^{-1}\frac{1}{\sqrt{3}}) = -\sin(\sin^{-1}\frac{1}{\sqrt{3}})$	0.5
	Simplifies the above expression as $(-\frac{1}{\sqrt{3}})$.	0.5
8	Writes that the cosecant function can be made invertible when its domain is restricted to $[n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}] - \{0\}$, where n is an integer.	1
9	Writes that the $\cos^{-1}$ function is not defined for $\frac{5}{3}$ as it is outside the domain of the inverse cosine function. Therefore, the teacher stopped Satish here.	1
10	Assumes $\cos^{-1}(\frac{24}{25})$ as x and writes that $\cos x = \frac{24}{25}$.	0.5
	Uses the above step and writes: $\cot x = \frac{24}{7}$ $\Rightarrow x = \cot^{-1}(\frac{24}{7})$	1
	Uses the property, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, for $x \in R$, and evaluates the given expression as: $\cot^{-1}(\frac{24}{7}) + \tan^{-1}(\frac{24}{7}) = \frac{\pi}{2}$ (Award full marks if the problem is solved correctly using any other method.)	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
11	**Rewrites the LHS of the given equation as:** $\sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right] = \sin^{-1}\left[\frac{2.\,2^x}{1+(2^x)^2}\right]$	1
	Uses the property of inverse trigonometric functions and writes: $\sin^{-1}\left[\frac{2.\,2^x}{1+(2^x)^2}\right] = 2\tan^{-1}(2^x)$	1
12	**Writes that Rahul is incorrect.**	0.5
	Writes that $\tan^{-1}(-x) - \tan^{-1}\frac{1}{x}$ **can be written as** $-(\tan^{-1}x + \cot^{-1}x)$.	1
	Finds the value of the given expression as $(-\frac{\pi}{2})$ **as** $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$.	0.5
13	**Assumes** $xy < (-1)$, $x = \tan\theta$ **and** $y = \tan\Phi$.	0.5
	Rewrites the above inequality as $\tan\theta < \tan(\Phi - \frac{\pi}{2})$.	0.5
	Writes that, since tangent is an increasing function in the principal value branch, $\theta < (\Phi - \frac{\pi}{2})$.	0.5
	Uses steps 1 and 3 to write $\tan^{-1}x - \tan^{-1}y < -\frac{\pi}{2}$.	0.5
	Uses the above step to conclude that: **If** $xy < (-1)$, **the value of** $\tan^{-1}x - \tan^{-1}y \notin (-\frac{\pi}{2}, \frac{\pi}{2})$.	0.5
	Hence, proves that the given property is true only for $xy > (-1)$.	0.5

Chapter - 3
Matrices

Q: 1 **T is a matrix given by:**

$$T = \begin{bmatrix} 4 & 0 & 4 \\ 4 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

On performing which of the following individual operations, will the matrix T remain the same?

i) $C_1 \leftrightarrow C_3$

ii) $R_2 \rightarrow R_2 + 99R_3$

iii) $C_2 \rightarrow (-1)C_2$

1 only (i) and (ii)
2 only (ii) and (iii)
3 only (i) and (iii)
4 all - (i), (ii) and (iii)

Q: 2 **A teacher gave the following problem to his students.**

$$\begin{bmatrix} 3 & 7 & 2 \\ -1 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} 5 & 4 & -2 \\ 6 & 3 & 8 \end{bmatrix}$$

Three students' solutions are shown below.

$\begin{bmatrix} 15 & 28 & -4 \\ -6 & 12 & 0 \end{bmatrix}$ Ram's solution

$\begin{bmatrix} 39 & 55 \\ 11 & 6 \end{bmatrix}$ Aviraj's solution

$\begin{bmatrix} 57 & 33 & 50 \\ 19 & 8 & 34 \end{bmatrix}$ Shyama's solution

Who gave the correct answer?

1 only Ram
2 only Aviraj
3 only Shyama
4 none of them

Q: 3

A and B are two matrices such that the transpose of (A + B) is $\begin{bmatrix} 2 & -1 \\ 7 & 6 \end{bmatrix}$

If $B = \begin{bmatrix} 3 & -2 \\ 6 & 0 \end{bmatrix}$, which of the following is A?

1 $\begin{bmatrix} -1 & 1 \\ 1 & 6 \end{bmatrix}$
2 $\begin{bmatrix} -1 & 9 \\ -7 & 6 \end{bmatrix}$
3 $\begin{bmatrix} 1 & -9 \\ 7 & -6 \end{bmatrix}$
4 $\begin{bmatrix} -4 & 4 \\ 0 & 7 \end{bmatrix}$

Q: 4 **In an online advertisement (ad) campaign, there are two options for publicity: picture ads and video ads.**

The cost per ad (in Rs) is given by the matrix:

$$A = \begin{array}{c} \text{Cost per ad} \\ \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Picture} \\ \text{Video} \end{array}$$

The number of ads run by two companies X and Y is given by the matrix:

$$B = \begin{array}{cc} \text{Picture} & \text{Video} \end{array} \begin{bmatrix} 10000 & 15000 \\ 8000 & 20000 \end{bmatrix} \begin{array}{l} X \\ Y \end{array}$$

To find the total cost of ads for the two companies, Nahush performs the matrix operation A × B while Divyesh performs the matrix operation B × A.

Who is correct?

1. Only Nahush
2. Only Divyesh
3. Both Nahush and Divyesh
4. Neither Nahush nor Divyesh

Q: 5

$A = \begin{bmatrix} 3 & 4 \\ -5 & 2 \end{bmatrix}$ and B is the inverse of A.

Which of the following is AB?

1. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
4. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Q: 6 **A and B are two matrices such that both products, AB and BA, exist.** **[1]**

Write a condition on the order of the matrices A and B for the above statement to be true.

Q: 7 **Identify if the statement below is true or false. If true, give a reason. If false, give a counter-example.** **[1]**

"If A is a non-zero matrix such that A × K is a zero matrix, then K is definitely a zero matrix."

Q: 8 **The matrix obtained after applying the column operation [$C_1 \to C_1 + (-3)C_3$] on matrix A is shown below.** **[1]**

$$\begin{bmatrix} 5 & 5 & -2 \\ 1 & -6 & 1 \\ -6 & -4 & 3 \end{bmatrix}$$

Find matrix A. Show your work.

Q: 9 **A is a 4 × 3 matrix, which when multiplied by matrix B, results in a 4 × 2 matrix, C.** **[1]**

How many rows and columns does matrix B have? Justify your answer.

Q: 10 **[2]**

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix}$$

If $pA^2 + qA + rI = O$, where O is the zero matrix and p, q and r are integers, find the value of ($\frac{-q}{p}$). Show your steps.

Q: 11 **If B is a symmetric matrix, prove that BB' is a symmetric matrix.** **[2]**

Q: 12 **D is a matrix of order 3 which is both symmetric and skew symmetric.** **[2]**

Find D. Show your work.

Q: 13 **A, B and C are three matrices that are compatible for multiplication. Under what condition(s) will the following statement be true?** **[2]**

If AB = AC, then B = C.

Show your working.

Q: 14 **D is a diagonal matrix and BD = I, where B is a matrix.** **[2]**

Is B a diagonal matrix? Justify your answer.

Q: 15 **Two distributors of a chips company distribute two varieties of chips packets - Spicy Chips (SC) and Cheesy Chips (CC).** **[3]**

At the beginning of a certain month, the number of chips packets available with the distributors is shown in matrix M. The number of chips packets distributed during that month by them is shown in matrix N.

$$M = \begin{array}{c} \begin{array}{cc} SC & CC \end{array} \\ \begin{bmatrix} 965 & 498 \\ 872 & 689 \end{bmatrix} \end{array} \begin{array}{l} \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{array}$$

$$N = \begin{array}{c} \begin{array}{cc} SC & CC \end{array} \\ \begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix} \end{array} \begin{array}{l} \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{array}$$

i) Find the number of chips packets remaining with the distributors at the end of that month.

ii) A packet of Spicy Chips and Cheesy Chips costs Rs 10 and Rs 20 respectively. Find the total cost of the chips distributed by each distributor that month using matrix multiplication.

Show your work and give your answer in the matrix form.

Q: 16 **[3]**

$$A = \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix}; \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 9 \end{bmatrix}$$

For what value(s) of β, is $A^2 = B$? Show your work.

Q: 17 **[3]**

Given:

$$(AB) \times C = \begin{bmatrix} -13 & 12 \\ 32 & 11 \end{bmatrix} \text{ and } A = \begin{bmatrix} 2 & -3 \\ 7 & 5 \end{bmatrix}$$

Find BC. Show your work with valid reasons.

Q: 18 **[3]**

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 4 \\ 4 & 2 \end{bmatrix}$$

i) Find the relation between the elements of matrix B, such that AB = BA. Show your steps.
ii) Use the relations from part i) to justify if the matrix C satisfies AC = CA.

Q: 19 **A is a square matrix of order m.** **[3]**

If $(A^2 - A)$ is invertible, then is A invertible? Justify your answer.

Q: 20 **Achal wants to purchase 2 kg of sugar, 10 kg of wheat and 5 kg of rice. In a general store near his house, these groceries were priced at Rs 50, Rs 35 and Rs 40 per kg whereas in a supermarket, these groceries were priced at Rs 44, Rs 30 and Rs 38 per kg respectively. The cost of travelling to the supermarket is Rs 20.** **[3]**

Using matrix multiplication, find Achal's total savings if he buys the groceries from supermarket. Show your work.

Q: 21 **A company wanted to outsource the creation of its video and picture content for social media.** **[5]**

They were approached by two firms offering the following rates per post (in Rs).

$$\begin{array}{cc} \begin{array}{cc} \text{Video} & \text{Picture} \end{array} & \\ \begin{bmatrix} 30 & 10 \\ 25 & 15 \end{bmatrix} & \begin{array}{l} \text{Firm 1} \\ \text{Firm 2} \end{array} \end{array}$$

They gave the contract to both the firms for an equal number of posts. After 3 months, each firm created the following number of posts:

$$\begin{array}{cc} \begin{bmatrix} 70 \\ 100 \end{bmatrix} & \begin{array}{l} \text{Video} \\ \text{Picture} \end{array} \end{array}$$

Each firm's posts created the following number of sales for the company:

$$\begin{array}{cc} \begin{array}{cc} \text{Video} & \text{Picture} \end{array} & \\ \begin{bmatrix} 500 & 200 \\ 400 & 300 \end{bmatrix} & \begin{array}{l} \text{Firm 1} \\ \text{Firm 2} \end{array} \end{array}$$

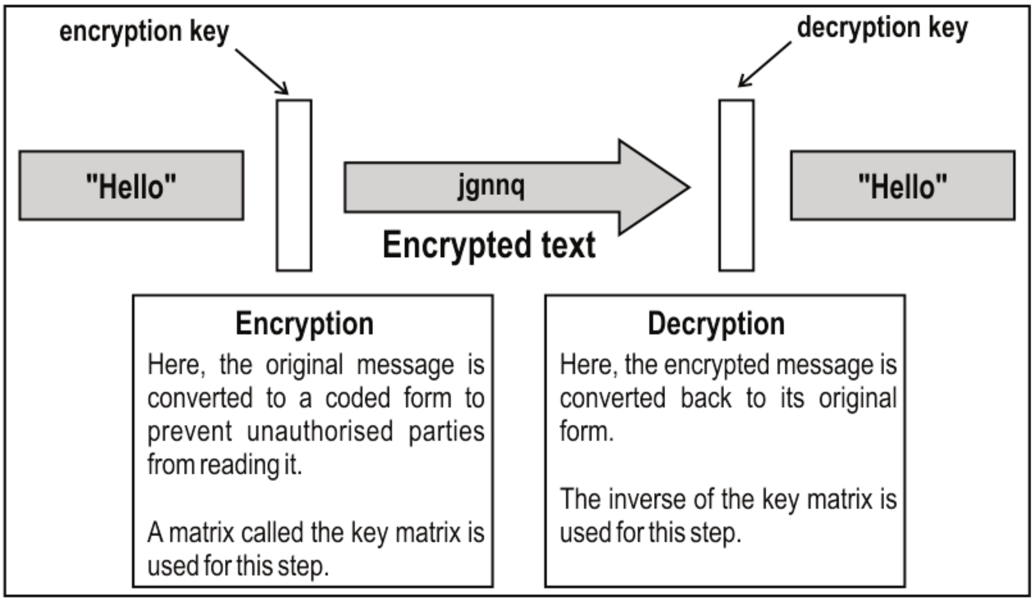
i) Which firm cost more to the company?

ii) If each sale was worth Rs 300, which firm was more profitable for the company?

Show your work.

Study the given information and answer the questions that follow.

One of the prominent applications of matrices is in cryptography. Cryptography is a type of secure communication where a message is transmitted from a sender to a receiver.



An encryption process with a key matrix, K = $\begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix}$ is shown below.

Assume the coordinates of a location, P = (4, 18), is to be encrypted. Matrix multiplication is performed between the key matrix and the coordinates to obtain the encrypted form of P as:

$$K \times P = \begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 18 \end{bmatrix} = \begin{bmatrix} 138 \\ 34 \end{bmatrix}$$

Decryption is carried out by using the inverse of the key matrix as:

K^{-1} × encrypted matrix = original matrix

Q: 22 **Using the same key matrix, K, find the encrypted form of Q = (19, -20).** **[1]**

Q: 23 **Find the matrix used for decryption corresponding to K. Show your work.** **[2]**

Q: 24 **If M = (160, 105) is the encrypted coordinates corresponding to the key matrix K, find the original coordinates of M.** **[1]**

Q: 25 **K_1 is the new key matrix obtained by performing the elementary row operation $R_2 \to R_2 + R_1$ on K, where R_1 and R_2 denote rows 1 and 2 respectively.** **[1]**

Find K_1.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	4
3	2
4	2
5	2

Q.No	Teacher should award marks if students have done the following:	Marks
6	**Writes that the above statement will be true if the orders of matrices A and B are of the form $m \times n$ and $n \times m$ respectively, where m and n are positive integers.** **(Award 0.5 marks if the condition 'both the matrices A and B are square matrices of the same order' is written.)** **OR** **(Award 0.5 marks if particular orders are written instead. For example, 2 × 3 and 3 × 2.)**	1
7	**Writes False(F).**	0.5
	Gives a counterexample. For example: $\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ where $A = \begin{bmatrix} 0 & 2 \\ 0 & 7 \end{bmatrix}$ & $K = \begin{bmatrix} 5 & -3 \\ 0 & 0 \end{bmatrix}$	0.5
8	**Applies the reverse operation $[C_1 \rightarrow C_1 + 3C_3]$ on the given matrix to find matrix A as:** $\begin{bmatrix} -1 & 5 & -2 \\ 4 & -6 & 1 \\ 3 & -4 & 3 \end{bmatrix}$	1
9	**Writes that B has 3 rows and 2 columns.**	0.5
	Gives reason that, since A is a 4 × 3 matrix, B must have 3 rows. Further, since the product C is a 4 × 2 matrix, B must have 2 columns.	0.5
10	**Finds A^2 as:** $A^2 = \begin{bmatrix} 1 & -6 \\ 2 & -3 \end{bmatrix}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes the given equation as: $$p\begin{bmatrix}1 & -6\\2 & -3\end{bmatrix}+q\begin{bmatrix}2 & -3\\1 & 0\end{bmatrix}+r\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$	**0.5**
	Writes the equation 2 $p + q = 0$ to find the value of ($\frac{-q}{p}$) as 2. **(Award full marks if any other valid equation is correctly used.)**	**0.5**
11	**Writes that, since B is a symmetric matrix, B = B'.**	**0.5**
	Proves that BB' is a symmetric matrix as: **(BB')' = (B')' × B' = BB'** **(Award 0.5 marks if only an example is written instead of a proof.)**	**1.5**
12	**Writes that D' = D as D is a symmetric matrix.**	**0.5**
	Writes that D' = -D as D is a skew symmetric matrix.	**0.5**
	Uses the above steps to conclude that D is a null matrix of order 3. The working may look as follows: **D = -D** **⇒ 2D = 0** **⇒ D = 0** **(Award 0.5 marks if the correct conclusion is written without any working.)**	**1**
13	**Writes that the given statement will be true if A is invertible or if A^{-1} exists.**	**1**
	Shows the working as follows: **Let AB = AC** **Pre multiplying both sides by A^{-1}, we get:** **⇒ A^{-1} (AB) = A^{-1} (AC)** **⇒ (A^{-1} A)B = (A^{-1} A)C** **⇒ (I)B = (I)C** **⇒ B = C**	**1**

<table>
<tr><th>Q.No</th><th>Teacher should award marks if students have done the following:</th><th>Marks</th></tr>
<tr><td rowspan="3">14</td><td>Writes yes.</td><td>0.5</td></tr>
<tr><td>Writes a justification. For example, assumes B to not be a diagonal matrix and writes an equation as:

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} b_{11}d_{11} & b_{12}d_{22} \\ b_{21}d_{11} & b_{22}d_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(Award full marks if a counter example is written.)</td><td>1</td></tr>
<tr><td>Uses the above equality to write that, as d_{11} and d_{22} are non-zero, b_{21} and b_{12} must be zero. Hence, concludes that B is a diagonal matrix.</td><td>0.5</td></tr>
<tr><td rowspan="2">15</td><td>i) Finds the number of chips packets remaining with the distributors at the end of the month in the matrix form as:

$$M - N = \begin{bmatrix} 965 & 498 \\ 872 & 689 \end{bmatrix} - \begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{cc} SC & CC \end{array} \\ \begin{bmatrix} 9 & 99 \\ 222 & 178 \end{bmatrix} \end{array} \begin{array}{l} \\ \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{array}$$</td><td>1</td></tr>
<tr><td>ii) Finds the total cost of the chips distributed by each distributor that month using matrix multiplication as:

$$\begin{array}{c} \begin{array}{cc} SC & CC \end{array} \\ \begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix} \end{array} \times \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 9560 + 7980 \\ 6500 + 10220 \end{bmatrix} = \begin{bmatrix} 17540 \\ 16720 \end{bmatrix} \begin{array}{l} \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{array}$$</td><td>2</td></tr>
</table>

Q.No	Teacher should award marks if students have done the following:	Marks
16	**Finds A^2 as:** $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} = \begin{bmatrix} 1 & 1+\beta \\ 0 & \beta^2 \end{bmatrix}$	1
	Equates A^2 to B and writes the matrix equation as: $\begin{bmatrix} 1 & 1+\beta \\ 0 & \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 9 \end{bmatrix}$ **Equates the corresponding elements and writes the equations $1 + \beta = 2$ and $\beta^2 = 9$.**	0.5
	Solves the first equation to get $\beta = 1$ and writes that $\beta = 1$ doesn't satisfy the equation $\beta^2 = 9$.	1
	Concludes that $A^2 = B$ is not possible for any value of β.	0.5
17	**Writes that by associative law of matrix multiplication $(AB) \times C = A \times (BC)$.**	0.5
	Finds A^{-1} as: $A^{-1} = \frac{1}{31} \begin{bmatrix} 5 & 3 \\ -7 & 2 \end{bmatrix}$	1
	Pre-multiplies A^{-1} on both sides of the equation $(AB) \times C = A \times (BC)$ to find BC as: **$A^{-1} \times [(AB) \times C] = A^{-1} \times [A \times (BC)] = (A^{-1} A) \times (BC) = I \times (BC) = BC$** $BC = \frac{1}{31} \begin{bmatrix} 5 & 3 \\ -7 & 2 \end{bmatrix} \times \begin{bmatrix} -13 & 12 \\ 32 & 11 \end{bmatrix}$ $\Rightarrow BC = \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
18	**i) Assumes a square matrix, B, of order 2 and writes the equation:** $$\begin{bmatrix} b_{11}+2b_{21} & b_{12}+2b_{22} \\ 2b_{11} & 2b_{12} \end{bmatrix} = \begin{bmatrix} b_{11}+2b_{12} & 2b_{11} \\ b_{21}+2b_{22} & 2b_{21} \end{bmatrix}$$	1
	Equates the corresponding terms of the matrices to conclude $b_{21} = b_{12}$.	0.5
	Equates the corresponding terms of the matrices and uses the above step to conclude $b_{22} = (2b_{11} - b_{12}) \div 2$.	0.5
	ii) Writes that C does not satisfy AC = CA as $c_{22} \neq (2c_{11} - c_{12}) \div 2$.	1
19	**Writes that since $(A^2 - A)$ is invertible, there exists a unique square matrix B of order *m* such that:** **$(A^2 - A)B = I$**	1
	Writes the above equation as: **A(A - I)B = I**	0.5
	Assumes (A - I)B as C where C is a square matrix of order *m*.	0.5
	Rewrites the equation in step 2 as: **AC = I**	0.5
	Writes that, similarly, CA = I and concludes that if, $(A^2 - A)$ is invertible, then A is invertible.	0.5
20	**Represents the quantity of sugar, wheat and rice to be purchased by Achal by the matrix:** $$\begin{bmatrix} 2 & 10 & 5 \end{bmatrix}$$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Represents the prices of sugar, wheat and rice at the general store and the supermarket by the matrix: $$\begin{bmatrix} 50 & 44 \\ 35 & 30 \\ 40 & 38 \end{bmatrix}$$	**1**
	Finds the total cost at the two places as: $$\begin{bmatrix} 2 & 10 & 5 \end{bmatrix} \begin{bmatrix} 50 & 44 \\ 35 & 30 \\ 40 & 38 \end{bmatrix} = \begin{bmatrix} 650 & 578 \end{bmatrix}$$	**1**
	Finds Achal's total savings if he buys groceries from the supermarket as 650 - 578 - 20 = Rs 52.	**0.5**
21	**i) Writes that, the cost for each firm can be found as:** $$\begin{bmatrix} 30 & 10 \\ 25 & 15 \end{bmatrix} \times \begin{bmatrix} 70 \\ 100 \end{bmatrix}$$	**0.5**
	Calculates the cost for firm 1 as Rs 3100 and the cost for firm 2 as Rs 3250.	**1**
	Writes that firm 2 cost more to the company.	**0.5**
	ii) Finds the total revenue generated by each firm from the outsourcing as: $$300 \times \begin{bmatrix} 500 & 200 \\ 400 & 300 \end{bmatrix}$$	**0.5**
	Writes that posts from firm 1 generated 1,50,000 + 60,000 = Rs 2,10,000 in revenue and the posts from firm 2 generated 1,20,000 + 90,000 = Rs 2,10,000.	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	Calculates the profit generated by firm 1 as 2,10,000 - 3,100 = Rs 206,900 and firm 2 as 2,10,000 - 3,250 = Rs 2,06,750.	1
	Writes that firm 1 was more profitable for the company than firm 2.	0.5
22	Finds the encrypted form of Q as: $K \times Q = \begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 19 \\ -20 \end{bmatrix} = \begin{bmatrix} -83 \\ 56 \end{bmatrix}$	1
23	Finds the determinant of matrix K as (-25).	1
	Finds the matrix used for decryption corresponding to K as: $K^{-1} = \frac{-1}{25} \begin{bmatrix} 1 & -7 \\ -4 & 3 \end{bmatrix}$	1
24	Finds the original coordinates of M as: $\frac{-1}{25} \begin{bmatrix} 1 & -7 \\ -4 & 3 \end{bmatrix} \times \begin{bmatrix} 160 \\ 105 \end{bmatrix} = \begin{bmatrix} 23 \\ 13 \end{bmatrix}$	1
25	Finds K_1 as: $\begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 3 & 7 \\ 7 & 8 \end{bmatrix}$	1

Chapter - 4
Determinants

Q: 1 $\begin{vmatrix} p & q & r \\ a & b & c \\ 2x & 2y & 2z \end{vmatrix} + 2 \begin{vmatrix} p & q & r \\ d & e & f \\ x & y & z \end{vmatrix}$

Which of the following is equal to the above sum?

A. $\begin{vmatrix} 3p & 3q & 3r \\ a+2d & b+2e & c+2f \\ 4x & 4y & 4z \end{vmatrix}$

B. $\begin{vmatrix} p & q & r \\ a+d & b+e & c+f \\ 2x & 2y & 2z \end{vmatrix}$

C. $\begin{vmatrix} 2p & 2q & 2r \\ a+2d & b+2e & c+2f \\ 3x & 3y & 3z \end{vmatrix}$

D. $\begin{vmatrix} p & q & r \\ a+2d & b+2e & c+2f \\ x & y & z \end{vmatrix}$

1 A
2 B
3 C
4 D

Q: 2 **P is a matrix of order *n*.**

How many minors are there in the determinant of P?

1 (n - 1)
2 n
3 n^2
4 n^n

Q: 3 **Which of the following matrices is nonsingular?**

1 $\begin{bmatrix} -6 & 3 & 5 \\ 0 & 0 & 0 \\ 4 & 9 & -5 \end{bmatrix}$
2 $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
3 $\begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 6 \\ 4 & 7 & 8 \end{bmatrix}$
4 $\begin{bmatrix} 0 & 2 & 5 \\ 2 & 0 & 7 \\ 5 & 7 & 0 \end{bmatrix}$

Q: 4 **S is a diagonal matrix with all the principal diagonal elements equal to p ($p \neq 0$).**

What is the determinant of S^{-1} ?

1 $\frac{1}{p}$
2 $\frac{1}{p^3}$
3 $\frac{1}{p^3}\mathrm{I}$
4 p^3

Q: 5

$$|A| = \begin{vmatrix} \sin x & \square & \sin 3x \\ 3\cos x & 6\sin 2x & 9\cos 3x \\ 2\sin x & 4\cos 2x & 2\sin 3x \end{vmatrix}$$

If |A| = 0, which of the following could be ☐?

1 $\sin 2x$ **2** $\cos 2x$ **3** $2\cos 2x$ **4** $4\cos 2x$

Q: 6 **Evaluate the determinant, |A|, of the matrix shown below. Justify your answer.** **[1]**

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 5 & 0 \end{bmatrix}$$

Q: 7 **Shown below is a matrix T and its cofactor matrix, where A_{ij} is the cofactor of the element a_{ij}.** **[1]**

$$T = \begin{bmatrix} 2 & -3 & 1 \\ a_{21} & 0 & a_{23} \\ 1 & a_{32} & a_{33} \end{bmatrix}$$

$$\text{Cofactor matrix of } T = \begin{bmatrix} 8 & -9 & -10 \\ A_{21} & -3 & A_{23} \\ -12 & A_{32} & A_{33} \end{bmatrix}$$

Find the determinant of T using the given information. Show your work.

Q: 8 **P and Q are square matrices of order 4. The determinants of P and Q are 5 and 4 respectively.** **[1]**

Find the determinant of the matrix $3P^2Q$. Show your work.

Q: 9 **Shown below are two matrices A and B such that det (A) = det (B).** **[1]**

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 6 \\ 3 & 8 & 1 \\ 1 & -7 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & 3 & x \\ 0 & 8 & -7 \\ 6 & 1 & 2 \end{bmatrix}$$

Without evaluating the determinants, find the value of x. Give a valid reason.

Q: 10 **Amit and Rohan are solving a question on determinants, where β is a non-zero real number. Shown below is a part of their working.** **[1]**

$$Amit: \left| \frac{1}{\beta} \begin{bmatrix} 1 & -2 \\ 0 & \beta \end{bmatrix} \right| = \frac{\beta}{\beta} = 1$$

$$Rohan: \left| \frac{1}{\beta} \begin{bmatrix} 1 & -2 \\ 0 & \beta \end{bmatrix} \right| = \frac{\beta}{\beta^2} = \frac{1}{\beta}$$

Whose working is correct? Give a reason.

Q: 11 **[2]**

$$|\mathbf{A}| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \qquad |\mathbf{B}| = \begin{vmatrix} kb_1 & kc_1 & ka_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$$

If |A| = k, where k is a real number, find |B| using the properties of determinants without expanding.

Q: 12 **If a, b and c are the first three terms of a geometric progression, find the value of ▲ without expanding the determinant.** **[2]**

$$\triangle = \begin{vmatrix} b & 2a & 3a \\ abc & abc & abc \\ c & 2b & 3b \end{vmatrix}$$

Show your steps.

Q: 13 Given below is the adjoint of matrix B. **[2]**

$$\begin{bmatrix} -1 & -1 & 2 \\ 1 & 4 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$

Find the determinant of matrix B. Show your work with valid reasons.

Q: 14 Shown below is a quadrilateral with A(1, 2), B(5, 1), C(4, 6) and D(2, 6) as its vertices. **[3]**

Find the area of ABCD using determinants. Show your steps.

Q: 15 In an arithmetic progression, the 11^{th}, 23^{rd} and 37^{th} terms are p, q and r respectively with a common difference of d. **[3]**

Without expanding the determinant, find ▲.

$$\triangle = \begin{vmatrix} p & q & r \\ 11 & 23 & 37 \\ d & d & d \end{vmatrix}$$

Show your work with valid reasons.

Q: 16 A theatre has two categories of tickets, one for adults and one for children. **[3]**

Bindu's family paid Rs 400 to buy 6 tickets for adults and 4 tickets for children.
Nalini's family paid Rs 325 to buy 5 tickets for adults and 3 tickets for children.

Find the cost of each category of the ticket by matrix method. Show your steps.

Q: 17 **[3]**

$$f(x) = \begin{vmatrix} (x+1) & x(x+1) & x(x-1)(x+1) \\ x & x^2 & 3x^3 \\ (x-1) & x(x-1) & x(x-1)(x+1) \end{vmatrix}$$

Find f (2022). Show your steps with valid reasons.

(Hint: You need not expand the determinant.)

Q: 18 **[5]**

Given A = $\begin{bmatrix} 5 & 3 \\ -2 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -3 \\ 4 & -8 \end{bmatrix}$.

Show that (adj A) × (adj B) = adj(BA).

Q: 19 **An insurance company agent has the following record of policies sold in the month of April, May and June 2022 for three different policies - Policy A, Policy B and Policy C. He is paid a fixed commission per policy sold but the commission varies for the policies A, B and C.** **[5]**

Months	Number of policies Sold			Total commission earned in the month (in Rs)
	Policy A	Policy B	Policy C	
April	8	4	6	7850
May	9	9	6	9600
June	12	9	12	15000

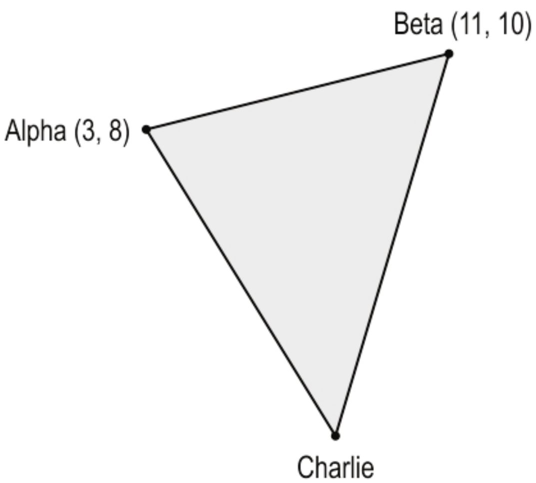
Find the fixed commission payable on policies A, B and C per unit using matrix method. Show your work.

Q: 20 **$y = ax^2 + bx + c$ passes through the points (-1, 0), (2, 12) and (3, 20).** **[5]**

Use the matrix method to find the quadratic equation. Show your steps.

Answer the questions based on the information given below.

In an army camp, three teams, Alpha, Beta and Charlie are located at the three corners of a triangular plot as shown below.



The area of the triangular plot is 37 sq units and the coordinates of team Charlie lie on the x-axis (positive direction). Team Charlie received a message from the camp head about a secret room as follows.

The coordinates of the secret room are (a_{12}, a_{21}) of the adjoint of the matrix $\begin{bmatrix} 3 & 8 \\ 11 & 10 \end{bmatrix}$.

Q: 21 **Find the *x* -coordinate of the location of team Charlie. Use the determinant method and show your steps.** **[2]**

Q: 22 **The head of the camp plans to have a medical centre on the line joining the coordinates of teams Alpha and Beta such that its *x* -coordinate is 5.** **[2]**

Find the *y* -coordinate of the medical centre using the determinant method. Show your steps and give a valid reason.

Q: 23 **Find the coordinates of the secret room. Show your steps.** **[1]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	3
3	4
4	2
5	3

Q.No	Teacher should award marks if students have done the following:	Marks
6	**Writes that the given determinant cannot be evaluated as only square matrices have determinants.**	1
7	**Finds the determinant of T as:** $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 2(8) + (-3)(-9) + 1(-10) = 33.$	1
8	**Finds** $\lvert 3P^2Q \rvert$ **as** $3^4 \times 5 \times 5 \times 4 = 3^4(100) = 8100.$	1
9	**Writes the value of** x **as 1.**	0.5
	Writes that the value of the determinant is the same when its rows and columns are interchanged.	0.5
10	**Writes that Rohan's working is correct and gives a reason. For example, If A =** k **B, where A and B are square matrices of order** n**, then** $\lvert A \rvert = k^n \lvert B \rvert$**, where** n **= 1, 2, 3, ...**	1
11	**Mentions the property that, if each element of a row (or a column) of a determinant is multiplied by a constant** k**, then the value of the determinant gets multiplied by** k **.** **Uses the above property and writes that,** $\lvert B \rvert = \begin{vmatrix} kb_1 & kc_1 & ka_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix} = k \begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$	0.5
	Mentions the property that, the value of the determinant remains unchanged if its rows and columns are interchanged. **Uses the above property and writes that,** $\lvert B \rvert = k \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Applies row transformations on the above determinant to bring it to the form of |A|. **For example,** $\lvert \boldsymbol{B} \rvert = k\begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix} \xrightarrow{R_2 \leftrightarrow R_3} k\begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_2} k\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$	**0.5**
	Mentions the property that, if any two rows of a determinant are interchanged, the sign of the determinant changes. **Uses the above property twice and concludes that |B| = k^2.**	**0.5**
12	**Substitutes *b* and *c* in the determinant with *ar* and ar^2 respectively, where *a* is the first term and *r* is the common ratio of the geometric progression and rewrites the given determinant as:** $\triangle = \begin{vmatrix} ar & 2a & 3a \\ a^3r^2 & a^3r^2 & a^3r^2 \\ ar^2 & 2ar & 3ar \end{vmatrix}$	**0.5**
	Uses the properties and evaluates the determinant as: $\triangle = (a)\,(a^3r^2)\,(ar)\begin{vmatrix} r & 2 & 3 \\ 1 & 1 & 1 \\ r & 2 & 3 \end{vmatrix}$ $\triangle = (a^5)\,(r^3)\,(0)$ $\triangle = 0$	**1.5**
13	**Writes that for a square matrix of order n, |adj B| = |B| $^{(n-1)}$.** **Applies the above result on (adj B) and writes |adj B| = |B|^2.**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds |adj B| as: **(-1)(-8 - 3) - 1(2 - 2) = 11**	**0.5**
	Uses the relation in step 1 and finds |B| as √11 or (-√11).	**0.5**
14	**Divides the quadrilateral into two triangles ABC and ACD and writes an expression for the area of the quadrilateral using determinants. For example:** $\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 5 & 1 & 1 \\ 4 & 6 & 1 \end{vmatrix} + \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 4 & 6 & 1 \\ 2 & 6 & 1 \end{vmatrix}$	**1**
	Simplifies the above expression by evaluating the determinants as $\frac{1}{2}$ [1(1 - 6) - 2(5 - 4) + 1(30 - 4)] + $\frac{1}{2}$ [1(6 - 6) - 2(4 - 2) + 1(24 - 12)].	**1**
	Finds the area of the quadrilateral as $\frac{27}{2}$ sq units.	**1**
15	**Takes the first term of the arithmetic progression as *a* and rewrites the given determinant ▲ as:** $\triangle = \begin{vmatrix} a + 10d & a + 22d & a + 36d \\ 11 & 23 & 37 \\ d & d & d \end{vmatrix}$	**0.5**
	Uses the properties of determinants and rewrites the above determinant as: $\triangle = \begin{vmatrix} a & a & a \\ 11 & 23 & 37 \\ d & d & d \end{vmatrix} + \begin{vmatrix} 10d & 22d & 36d \\ 11 & 23 & 37 \\ d & d & d \end{vmatrix}$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the properties of determinants and rewrites the above determinant as: $$\triangle = ad\begin{vmatrix} 1 & 1 & 1 \\ 11 & 23 & 37 \\ 1 & 1 & 1 \end{vmatrix} + d^2\begin{vmatrix} 10 & 22 & 36 \\ 11 & 23 & 37 \\ 1 & 1 & 1 \end{vmatrix}$$	**0.5**
	Writes that in the first determinant R_1 and R_3 are identical hence the value of the determinant is 0.	**0.5**
	Writes that in the second determinant $R_2 = R_1 + R_3$ hence the value of the determinant is 0.	**0.5**
	Concludes that $\triangle = ad\ (0) + d^2\ (0) = 0$.	**0.5**
16	**Assumes the cost of each adult ticket as x and each child's ticket as y. Writes the equations that represent the given scenario as follows:** **$6x + 4y = 400$** **$5x + 3y = 325$**	**0.5**
	Writes the above system of equations in the matrix form as AX = B where: $$A = \begin{bmatrix} 6 & 4 \\ 5 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 400 \\ 325 \end{bmatrix}$$	**0.5**
	Finds |A| as 18 - 20 = -2 ≠ 0. Hence writes that A^{-1} exists and the system has a unique solution.	**0.5**
	Finds A^{-1} as: **$A^{-1} = \frac{1}{\|A\|} \times \text{adj } A$** $$\Rightarrow \frac{1}{-2}\begin{bmatrix} 3 & -4 \\ -5 & 6 \end{bmatrix}$$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that X = A^{-1} B and finds X as: $X = \frac{1}{-2}\begin{bmatrix} 3 & -4 \\ -5 & 6 \end{bmatrix}\begin{bmatrix} 400 \\ 325 \end{bmatrix}$ $\Rightarrow X = \begin{bmatrix} 50 \\ 25 \end{bmatrix}$ **Concludes that the cost of each adult and child's ticket is Rs 50 and Rs 25 respectively.**	1
17	**Takes (*x* + 1), *x* and (*x* - 1) common from the first, second and third rows respectively to write:** $f(x) = x\,(x+1)\,(x-1)\begin{vmatrix} 1 & x & x\,(x-1) \\ 1 & x & 3\,x^2 \\ 1 & x & x\,(x+1) \end{vmatrix}$	1
	Takes *x* common from the second column to write: $f(x) = x^2\,(x+1)\,(x-1)\begin{vmatrix} 1 & 1 & (x-1)\,x \\ 1 & 1 & 3\,x^2 \\ 1 & 1 & x\,(x+1) \end{vmatrix}$	0.5
	Writes that the first and second columns are identical. Hence, concludes that *f* (*x*) = 0 for any value of *x* . **(Award only 0.5 marks if the reason is not mentioned.)**	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the above step to conclude that *f* (2022) = 0.	**0.5**
18	**Finds adj A as:** $\text{adj } A = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$	**1**
	Finds adj B as: $\text{adj } B = \begin{bmatrix} -8 & 3 \\ -4 & 2 \end{bmatrix}$	**1**
	Finds (adj A) × (adj B) as: $\text{adj } A \times \text{adj } B = \begin{bmatrix} -20 & 6 \\ -36 & 16 \end{bmatrix}$	**1**
	Finds BA as: $B \times A = \begin{bmatrix} 16 & -6 \\ 36 & -20 \end{bmatrix}$	**1**
	Finds adj(BA) as: $\text{adj } (BA) = \begin{bmatrix} -20 & 6 \\ -36 & 16 \end{bmatrix}$ **Uses step 3 and step 5 to conclude that (adj A) × (adj B) = adj(BA).**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
19	**Takes the fixed commission payable on policies A, B and C per unit as x, y and z respectively and frames the system of linear equations as:** $8x + 4y + 6z = 7850$ $9x + 9y + 6z = 9600$ $12x + 9y + 12z = 15000$ **(Award full marks if a student skips this step and writes the system in matrix form directly.)**	0.5
	Writes the system of equations in the form of a matrix equation as AX = B, where, $A = \begin{bmatrix} 8 & 4 & 6 \\ 9 & 9 & 6 \\ 12 & 9 & 12 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7850 \\ 9600 \\ 15000 \end{bmatrix}$	0.5
	Finds $\|A\| = 126 \neq 0$ and concludes that A^{-1} exists.	0.5
	Finds adjA as: $\text{adj } A = \begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}$	1.5
	Finds A^{-1} as: $A^{-1} = \frac{1}{\|A\|} \text{ adj } A = \frac{1}{126} \begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}$	0.5

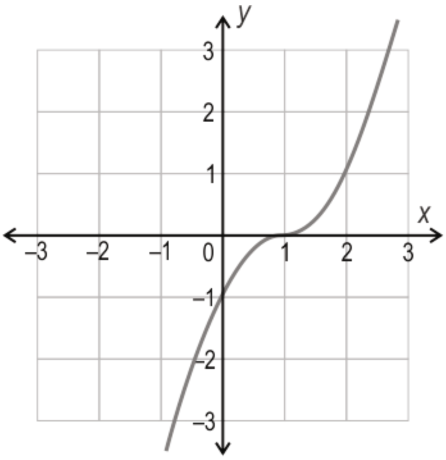
Q.No	Teacher should award marks if students have done the following:	Marks
	Rewrites the matrix equation in step 2 as X = A^{-1} B and solves the same to obtain the values of x, y and z as 250, 300 and 775 respectively as: $X = A^{-1}B = \frac{1}{126}\begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}\begin{bmatrix} 7850 \\ 9600 \\ 15000 \end{bmatrix} = \begin{bmatrix} 250 \\ 300 \\ 775 \end{bmatrix}$ **Hence concludes that the fixed commission payable on policies A, B and C per unit are Rs 250, Rs 300 and Rs 775 respectively.**	**1.5**
20	**Writes the system of equations as:** $a - b + c = 0$ $4a + 2b + c = 12$ $9a + 3b + c = 20$	**0.5**
	Writes the above system of equations in the form AX = B as: $\begin{bmatrix} 1 & -1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 20 \end{bmatrix}$	**0.5**
	Finds |A| as 1(2 - 3) + 1(4 - 9) + 1(12 - 18) = -12 and writes that A^{-1} exists as |A| ≠ 0.	**0.5**
	Finds A^{-1} as: $A^{-1} = \frac{-1}{12}\begin{bmatrix} -1 & 4 & -3 \\ 5 & -8 & 3 \\ -6 & -12 & 6 \end{bmatrix}$ **(Award 1 mark if only all the cofactors are found correctly.)**	**2**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes X as: $$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{-1}{12}\begin{bmatrix} -1 & 4 & -3 \\ 5 & -8 & 3 \\ -6 & -12 & 6 \end{bmatrix}\begin{bmatrix} 0 \\ 12 \\ 20 \end{bmatrix}$$	**0.5**
	Finds the values of a, b and c as 1, 3 and 2 and finds the quadratic equation as $x^2 + 3x + 2$.	**1**
21	**Uses the determinant method to write the expression for the area of a triangle as:** $$\frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ 11 & 10 & 1 \\ x & 0 & 1 \end{vmatrix} = 37$$	**0.5**
	Expands the LHS of the above equation and simplifies it to get: **$-58 - 2x = \pm 74$**	**1**
	Finds the value of x as (-66) or 8 and concludes that $x = 8$.	**0.5**
22	**Writes that the Alpha team, medical centre and the Beta team are on the same line and hence the area of the triangle formed by three collinear points will be zero.**	**0.5**
	Writes the relation as: $$\frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ 11 & 10 & 1 \\ 5 & y & 1 \end{vmatrix} = 0$$	**0.5**
	Simplifies the above equation to find the value of the y -coordinate as $\frac{68}{8}$ or 8.5 units.	**1**

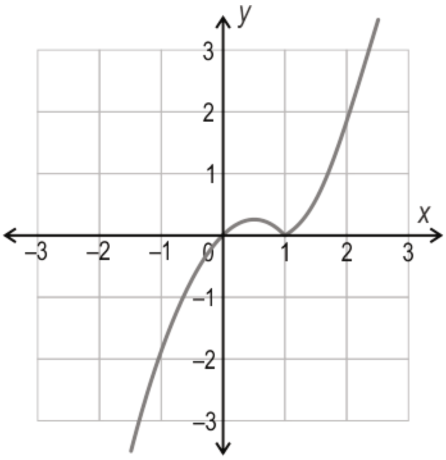
Q.No	Teacher should award marks if students have done the following:	Marks
23	**Finds the adjoint of the given matrix as:** $\begin{bmatrix} 10 & -8 \\ -11 & 3 \end{bmatrix}$	0.5
	Writes the coordinates of the location of the secret room as (-11, -8).	0.5

Chapter - 5
Continuity and Differentiability

Q: 1 **Shown below are the graphs of two functions.**



$g(x) = (x - 1)\,|x - 1|$



$f(x) = x\,|x - 1|$

What can one conclude from the above graphs?

1 Product of a differentiable function and a non-differentiable function is ALWAYS differentiable.
2 Product of a differentiable function and a non-differentiable function is ALWAYS NOT differentiable.
3 Product of a differentiable function and a non-differentiable function MAY BE differentiable.
4 (cannot conclude anything from the given graphs.)

Q: 2 **For what real value of α, is the function given below continuous for $x \in (-\infty, +\infty)$?**

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{\alpha x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

1 2
2 4
3 8
4 (such a value of α does not exist)

Q: 3 **Which of the following is INCORRECT about a function f : R -> R?**

1 If f is differentiable at $x = c$, then f is continuous at $x = c$.
2 If f is not differentiable at $x = c$, then f is not continuous at $x = c$.
3 If f is not continuous at $x = c$, then f is not differentiable at $x = c$.
4 If f is continuous at $x = c$, then f may or may not be differentiable at $x = c$.

Q: 4 **In which of these sets is the function $f(x) = x\,|x - 2|^2$ differentiable twice?**

1 R
2 R - {2}
3 R - {0, 2}
4 (the function cannot be differentiated twice in R)

Q: 5 **[1]**

If $f(x) = \sqrt{\cos^2 x - 25}$, then $f'(x) = \dfrac{1}{2\sqrt{\cos^2 x - 25}} g(x)$.

Find $g(x)$. Show your steps.

Q: 6 **A teacher asked her students for an example of a function whose first-order derivative is the same as its second-order derivative.** **[1]**

Shyama said, "there is no such function".

Is Shyama correct? Justify your answer.

Q: 7 **Differentiate $y = e^{\log \sin x}$, where $x \in (0, \pi)$, with respect to x. Show your steps.** **[1]**

Q: 8 **Is the following statement true or false? Justify your answer.** **[1]**

Statement: $\sin \frac{1}{x}$ is a continuous function in the domain of real numbers.

Q: 9 **[1]**

Dhriti was asked to check the continuity of the function $f(x) = \dfrac{x^2 - 2x}{x^2 - 4}$ at $x = 2$.
She simplified the limit as show below:

Left hand limit: $\lim\limits_{x \to 2^-} \dfrac{x^2 - 2x}{x^2 - 4} = \lim\limits_{x \to 2^-} \dfrac{x(x-2)}{(x+2)(x-2)} = \lim\limits_{x \to 2^-} \dfrac{x}{(x+2)} = \dfrac{2}{2+2} = \dfrac{1}{2}$

Right hand limit: $\lim\limits_{x \to 2^+} \dfrac{x^2 - 2x}{x^2 - 4} = \lim\limits_{x \to 2^+} \dfrac{x(x-2)}{(x+2)(x-2)} = \lim\limits_{x \to 2^+} \dfrac{x}{(x+2)} = \dfrac{2}{2+2} = \dfrac{1}{2}$

She then concluded that $f(x)$ is continuous at $x = 2$.

Is Dhriti's conclusion correct? Justify your answer.

Q: 10 **Examine if Rolle's theorem is applicable to the following function.** **[2]**

$$f(x) = \frac{(x-1)^3}{15} - \frac{(x-1)^2}{5} \; ; x \in [1, 4]$$

Show your steps and give a valid reason.

Q: 11 **Manu and Madhura were asked to examine if Rolle's theorem is applicable to the following function:** **[2]**

$$f(x) = 1 + \sqrt[3]{x^2}\text{, for } x \in [-1, 1]$$

Manu concluded that Rolle's theorem is applicable and Madhura concluded that Rolle's theorem is not applicable to the given function.

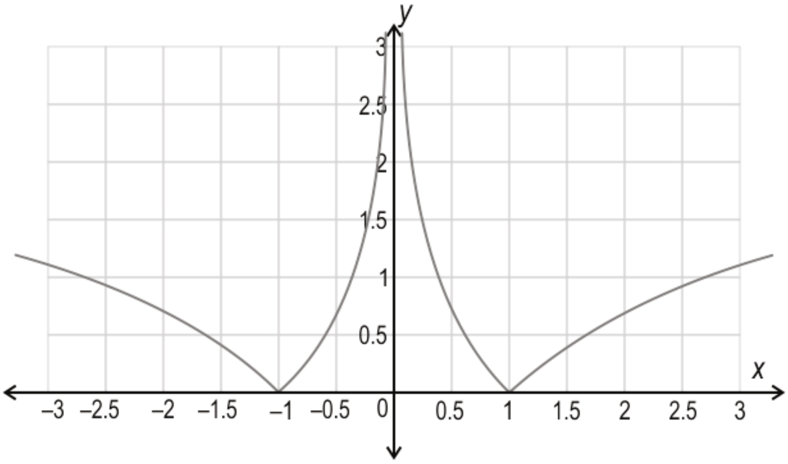
Who is correct? Justify your answer.

Q: 12 **Differentiate the following function with respect to *x*.** **[2]**

$$y = \frac{e^x}{\left(1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\right)\log x}$$

Show your steps.

Q: 13 **Shown below is the graph of a function discontinuous at *x* = 0.** **[2]**



Identify the point(s) in the domain [-3, 3], where the function is NOT differentiable. Give a reason for each point of non-differentiability.

Q: 14 **If $x = a(\log t)$ and $f(t) = a(\sin^{-1} t)$ where a is a constant, find $\frac{df(t)}{dx}$. Show your steps.** **[2]**

Q: 15 At 4:30 p.m., a boat was travelling at a velocity of 28 km/h. Three minutes later, the boat was travelling at a velocity of 92 km/h. **[3]**

Based on the above information, state whether the following assertion is true or false. Justify your answer.

"The acceleration was 1280 km/h^2 at some point during the 3 minute interval."

Q: 16 Find the subset of R in which the function $f(x) = x|x|$ is differentiable twice. Show your work with a valid reason. **[3]**

Q: 17 Differentiate: **[3]**

$$y = \cos^{-1} \frac{(1+3^x)(1-3^x)}{1+3^{2x}}$$

Show your steps.

Q: 18 Check if the mean value theorem holds for the function $f(x) = \sqrt{(4 - x)}$ in the interval [-5, 4]. Show your steps. **[5]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3
3	2
4	1

Q.No	Teacher should award marks if students have done the following:	Marks
5	Uses the chain rule to find $f'(x)$ as follows: $f'(x) = \frac{1}{2\sqrt{\cos^2 x - 25}} \times (-2 \sin x \cos x)$	0.5
	Writes that $g(x) = (-2 \sin x \cos x)$ or $(-\sin 2x)$.	0.5
6	Writes that Shyama is wrong.	0.5
	Justifies by giving an example of a function whose first-order derivative is the same as its second-order derivative. For example, $f(x) = e^x$.	0.5
7	Takes logarithm to the base *e* on both sides and simplifies the given equation as: $y = \sin x$	0.5
	Differentiates the above equation with respect to *x* as: $\frac{dy}{dx} = \cos x$ (Award full marks if $y(\cot x)$ is obtained instead of $\cos x$.)	0.5
8	Writes False(F).	0.5
	Writes that the function is not continuous in the domain of real numbers since $\frac{1}{x}$ is not defined at $x = 0$.	0.5
9	Writes no.	0.5
	Writes that $f(x)$ is not continuous at $x = 2$ because $f(2) = \frac{0}{0}$ which is not defined.	0.5
10	Writes that $f(x)$ is continuous and differentiable everywhere as $f(x)$ is a polynomial.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds *f* (1) and *f* (4) as 0 and writes: **$f(1) = f(4) = 0$**	**1**
	Uses steps 1 and 2 to conclude that Rolle's theorem is applicable to the given function.	**0.5**
11	**Writes that Madhura is correct.**	**0.5**
	Differentiates the function as: $f^{\prime}(x) = \frac{2}{3} \times \frac{1}{\sqrt[3]{x}}$	**0.5**
	Writes that the function is not differentiable at $x = 0$ with 0 belonging to [-1, 1]. Hence, concludes that Rolle's theorem cannot be applied to the given function.	**1**
12	**Rewrites the function as:** $y = \frac{e^{x}}{e.\log x}$	**0.5**
	Differentiates the function using the quotient rule to get: $\frac{dy}{dx} = e^{x-1}\left(\frac{x(\log x) - 1}{x(\log x)^{2}}\right)$	**1.5**
13	**Identifies the points in the domain [-3, 3] where the function is not differentiable as x = 0, 1 and (-1).**	**1**
	Writes that, as the function is not continuous at $x = 0$, it is not differentiable at x = 0.	**0.5**
	Writes that at $x = 1$ and (-1), the graph is pointed/not smooth, hence not differentiable.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
14	**Finds the derivative of *x* with respect to *t* as, $\frac{dx}{dt} = \frac{a}{t}$.**	**0.5**
	Finds the derivative of *f* (*t*) with respect to *t* as follows: $$\frac{df(t)}{dt} = \frac{a}{\sqrt{(1-t^2)}}$$	**0.5**
	Uses above steps to find $\frac{df(t)}{dx}$ as follows: $$\frac{df(t)}{dx} = \frac{t}{\sqrt{(1-t^2)}}$$	**1**
15	**Converts 3 mins to hours as:** **3 mins = $\frac{3}{60}$ hours = $\frac{1}{20}$ hours**	**0.5**
	Writes that the mean value theorem guarantees that at some time (*c*) during this 3-minute interval, the boat's acceleration, *v* '(*c*), was: $$v'(C) = \frac{v\left(\frac{1}{20}\right) - v(0)}{\frac{1}{20} - 0}, \text{ where } C \in \left(0, \frac{1}{20}\right)$$	**1**
	Substitutes the values in the above equation to get *v* '(*c*) as: **$v'(c)$ = 20 [92 - 28] = 1280 km/h^2**	**1**
	Concludes that, as the derivative of velocity at a point is nothing but acceleration at that point, the acceleration was 1280 km/h^2 at some point during the 3 minute interval.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
16	**Rewrites the given function as:** $$f(x)=\begin{cases}-x^2 & x<0\\ 0 & x=0\\ x^2 & x>0\end{cases}$$	1
	Finds the first derivative of the given function as: $$f'(x)=\begin{cases}-2x & x<0\\ 0 & x=0\\ 2x & x>0\end{cases}$$	0.5
	Finds the second derivative of the given function as: $$f''(x)=\begin{cases}-2 & x<0\\ 0 & x=0\\ 2 & x>0\end{cases}$$	0.5
	Writes that, since $f''(0^-) \neq f''(0^+)$, $f''(0)$ does not exist.	0.5
	Finds the subset of R in which the function $f(x) = x\|x\|$ is differentiable twice as R - {0}.	0.5
17	**Rewrites the expression as:** $$y=\cos^{-1}\frac{1-3^{2x}}{1+3^{2x}}$$ **and puts 3^x as tan t.**	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Write the expression as: $y = \cos^{-1} \frac{1 - \tan^2 t}{1 + \tan^2 t}$ or $\cos^{-1}(\cos 2t)$	0.5
	Writes that, $y = 2t$ or $y = 2\tan^{-1} 3^x$.	0.5
	Finds $\frac{dy}{dx}$ as $\frac{2}{1 + (3^x)^2} \times (3^x \ln(3))$.	1
18	**Writes that $f(x) = \sqrt{(4 - x)}$ is well defined for every $x \in [-5, 4]$.** **Writes that, since $f(x)$ is a root of a polynomial, it is continuous in [-5, 4].**	1
	Finds the derivative of $f(x)$ as: $f'(x) = \frac{-1}{2\sqrt{(4-x)}}$	1
	Writes that $f'(x)$ is defined in the interval (-5, 4) and hence $f(x)$ is differentiable in (-5, 4).	0.5
	Applies the mean value theorem to obtain $f'(c)$ as: $f'(c) = \frac{f(4) - f(-5)}{4 - (-5)} = \frac{0 - 3}{9} = \frac{-1}{3}$, where $c \in (-5, 4)$	1
	Equates $f'(x)$ at $x = c$ with $f'(c)$ to obtain c as: $\frac{-1}{2\sqrt{(4-c)}} = \frac{-1}{3}$ $\Rightarrow c = \frac{7}{4}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Concludes that, at $c = \frac{7}{4} \in (-5, 4)$, we have $f'(c) = \frac{-1}{3}$. Hence, the mean value theorem holds for the function $f(x) = \sqrt{4 - x}$ in the interval [-5, 4].	**0.5**

Chapter - 6
Application of Derivatives

Q: 1 **The diagonal of a square, of side 3√2 cm, is increasing at a rate of 2 cm/s.**

Which of the following is the rate at which its area is increasing?

1. √2 cm^2/s
2. 6√2 cm^2/s
3. 12 cm^2/s
4. 24 cm^2/s

Q: 2 **Sameer wants to find the area of his garden. He measures the side of his square garden as 10 m with an error of 0.05 m.**

Which of the following is the approximate error that Sam will have in calculating its area?

1. 0.05 m^2
2. 0.5 m^2
3. 1 m^2
4. 10.05 m^2

Q: 3 **Gagan wants to calculate the capacity of a cylindrical water tank. He precisely measures the height of the tank as 7 m. Next, he measures the radius of the tank as 2 m, with an error of 0.05 m. The error occured while measuring the radius results in an error in calculating the capacity.**

What is the approximate error in calculating the capacity of the water tank?

(Note: Take π = $\frac{22}{7}$.)

1. 2.2 m^3
2. 4.4 m^3
3. 44 m^3
4. 88 m^3

Q: 4 **Study the function below.**

$$f(x) = \frac{2x^3}{3} - 18x + k, \text{ where } k \text{ is a constant}$$

Which of the following is true about the nature of the function in the interval [-3, 3]?

1. $f(x)$ is increasing
2. $f(x)$ is decreasing
3. $f(x)$ is neither increasing nor decreasing
4. (cannot say without knowing the value of k)

Q: 5 **The normal to the curve $y = f(x)$ at the point (5, 7) makes an angle of $\frac{\pi}{4}$ with the x -axis in the positive direction.** [1]

Find $f'(5)$. Show your work.

Q: 6 **Aditi is making a circular dosa. She is spreading the dosa batter such that its radius is increasing at a rate of 2 cm/s.** **[1]**

Find the rate of change of the area of the dosa, in terms of π, when its radius is 9 cm. Show your steps.

(Note: Assume that the dosa has negligible thickness.)

Q: 7 **Find the critical points of the function $y = \tan^{-1}(\sec x)$ where $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. Show your steps.** **[2]**

Q: 8 **The maximum value of the function $f(x) = x^{\frac{1}{x}}$, $x > 0$, is obtained at $x = e$.** **[2]**

Use the above fact and show that $e^{\pi} > \pi^{e}$.

Q: 9 **A circular metallic plate is expanding such that its area is constantly increasing with respect to time.** **[2]**

Milind claims that the rate of increase of its perimeter with respect to time is inversely proportional to its radius.

Is Milind's claim correct? Justify your answer.

Q: 10 **A particle is moving such that its distance s from a fixed point at any time t is given by $s = A\sin t + B\cos t$, where A, B ∈ R.** **[2]**

Show that the particle's acceleration is always numerically equal to its distance from the fixed point.

Q: 11 **A solid sphere of gold, of radius 5 cm, is being melted in a furnace such that the radius is decreasing uniformly.** **[2]**

When its radius is 1 cm, show that the rate at which its surface area is decreasing with respect to time is twice the rate at which its volume is decreasing with respect to time.

Q: 12 **A basketball has a tiny hole that leads to it getting deflated while maintaining a spherical shape.** **[3]**

Find the ratio of the rate of loss of the volume of air to the rate of loss of the surface area of the ball when the radius of the ball is 8 cm. Show your steps.

Q: 13 **A cylindrical disk of radius R and height H is pressed by a hydraulic press. During the process, the radius and the height of the disk change such that the cylindrical shape is retained and the volume of the disk remains constant.** **[3]**

What is the ratio of the rate of change of height to the rate of change of radius in terms of R? Show your steps and give valid reasons.

Q: 14 **Companies issue shares as a means to raise money. This may be to finance company expansion, a new development, or to move into overseas markets. When you buy shares, you effectively become a part owner of the company.** **[3]**

Kabir observes the price of his share in the stock market for 16 months. He comes up with a function $p(t) = 16t - t^2 + 8$ where p is the maximum price of the share (in Rs) during a month and t is the number of months since he started observing it.

Find the maximum price of the share (in Rs) during the period Kabir observed it. Show your steps and give valid reasons.

Q: 15 **Simran cuts a metallic wire of length a m into two pieces. She uses both pieces to create two squares of different side lengths.** **[3]**

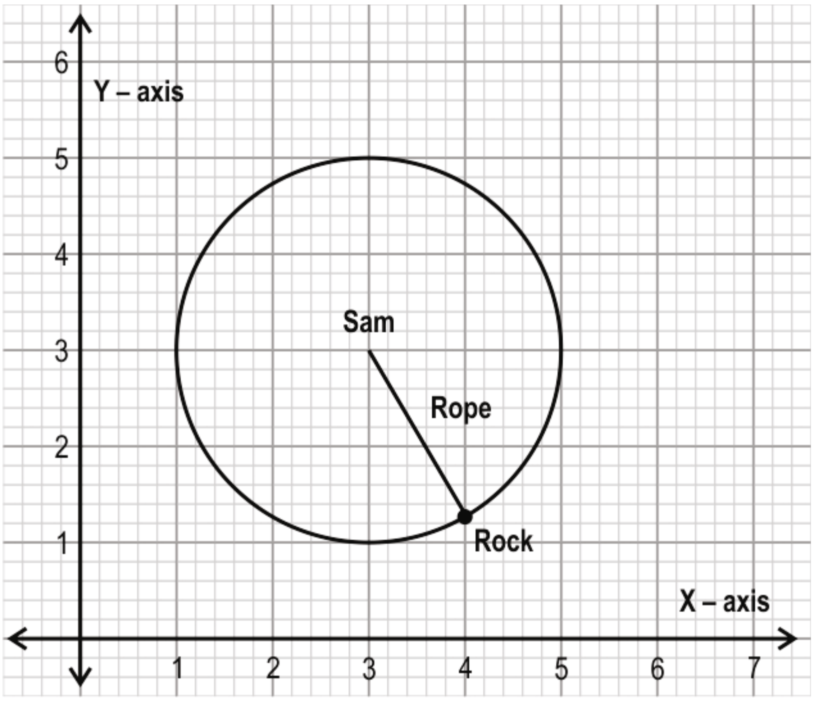
Find the side lengths of the squares (in terms of a) for which the combined area will be MINIMUM? Show your steps and give reasons.

Q: 16 **The function shown below is increasing in the interval ($-\frac{\pi}{2}, \frac{\pi}{4}$]:** **[3]**

$$f(x) = \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$$

What is the nature of the function in the interval ($\frac{\pi}{2}, \pi$)? Show your work.

Q: 17 **Sameer has a piece of rock tied with a rope. He holds the other end of the rope and starts rotating the rock in a circular motion. The equation of the circular path traced by the rock is given by $x^2 + y^2 - 6x - 6y + 14 = 0$.** **[3]**



When the tension reaches its maximum, the rope snaps and the rock starts moving along a straight-line path. The rock is at (4,1.27) at this point.

Find the equation of the straight-line path that the rock follows. Show your steps with valid reasons.

Q: 18 **$y = x^2 + bx - b$, where b is a real number, represents a curve.** **[5]**

The tangent to the curve y, at the point (1, 1), forms a triangle with the coordinate axes in the first quadrant. The area of the triangle is 2 sq units.

Find the value of b. Show your work.

Q: 19 **The life span of a certain flowering plant is around 6 years. *t* years after the sapling is planted, the plant produces *r* grams of flowers each day.** **[5]**

The relation between *r* and *t* can be approximated as:

$$r = \frac{t^3}{3} - 6t^2 + 32t,\ 0 \leq t \leq 6$$

i) What will be the yield per day, 3 years after planting the sapling?
ii) When will the maximum yield per day be obtained?

Show your steps with valid reasons.

Q: 20 **Find the equation of the tangent to the curve represented by x = 3 sec θ, y = 3 tan θ where 0 < θ < $\frac{\pi}{2}$ such that the line represented by -2 y = x + 1 is normal to the tangent at the point of contact with the curve. Show your steps.** **[5]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3
3	2
4	2

Q.No	Teacher should award marks if students have done the following:	Marks
5	**Finds the slope of the normal to the curve *y* = *f* (*x*) at the point (5, 7) as tan $\frac{\pi}{4}$ = 1.**	0.5
	Finds the slope of the tangent to the curve *y* = *f* (*x*) at the point (5, 7) as ($\frac{-1}{1}$) = (-1) and concludes that *f* '(5) = (-1).	0.5
6	**Finds the expression for the rate of change of the area of the dosa as:** $\frac{dA}{dt} = \pi \times 2r \times \frac{dr}{dt}$ cm^2/s	0.5
	Uses the given information and finds the rate of change of the area of the dosa when its radius is 9 cm as $\frac{dA}{dt} = \pi \times 2 \times 9 \times 2 = 36\pi$ cm^2/s.	0.5
7	**Differentiates the function as:** $\frac{dy}{dx} = \frac{\sec x \tan x}{1+\sec^2 x}$	1
	Writes that 0 is a critical point as $\frac{dy}{dx}$ is 0 when *x* = 0.	0.5
	Writes that $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ are critical points as $\frac{dy}{dx}$ is not differentiable at these two points.	0.5
8	**Writes that, as the maximum value of *f* (*x*) is obtained at *x* = *e*, *f* (*e*) > *f* (*x*), for every *x* > 0.**	0.5
	Uses the above step to write: **For x = π, f(e) > f(π)** $\Rightarrow e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Raises the power on both sides of the above inequality by (*e* π) and simplifies the same to get: $\left(e^{\frac{1}{e}}\right)^{e\pi} > \left(\pi^{\frac{1}{\pi}}\right)^{e\pi}$ $\Rightarrow e^{\pi} > \pi^{e}$	**1**
9	**Finds the rate of change of area of the circular plate with respect to time and equates it to a constant *k* as:** $\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt} = k$ **cm^2/s, where *k* is a positive real number, *t* is the time, *A* and *r* are area and radius of the circular plate respectively.**	**0.5**
	Finds the rate of change of perimeter of the circular plate with respect to time as: $\frac{dP}{dt} = 2\pi \times \frac{dr}{dt}$ **, where *P* is the perimeter of the circular plate.**	**0.5**
	Uses steps 1 and 2 to write: $\frac{dP}{dt} = 2\pi \times \frac{k}{2\pi r} = \frac{k}{r}$	**0.5**
	Concludes that Milind's claim is correct.	**0.5**
10	**Differentiates *s* to find velocity as:** $s' = A\cos t - B\sin t$	**0.5**
	Differentiates *s*' to find acceleration as: $s'' = -A\sin t - B\cos t$	**0.5**
	Rewrites acceleration in terms of distance as $s'' = (-s)$ and writes that the magnitudes of distance and acceleration are the same. **Hence, concludes that acceleration is always numerically equal to the distance of the particle from the fixed point.**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
11	Writes that the volume of a sphere, V, is given by $\frac{4}{3}\pi r^3$, where r is the radius of the sphere and differentiates the same with respect to time as: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$	0.5
	Writes that the surface area of a sphere, A, is given by $4\pi r^2$ and differentiates the same with respect to time as: $\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$	0.5
	Uses steps 1 and 2 to find the relation between $\frac{dV}{dt}$ and $\frac{dA}{dt}$ as: $\frac{dV}{dt} = \frac{r}{2} \times \frac{dA}{dt}$	0.5
	Substitutes $r = 1$ in the above equation to show that $2\frac{dV}{dt} = \frac{dA}{dt}$.	0.5
12	Writes that the volume of the air in the basketball is $\frac{4}{3}\pi r^3$ and finds the rate of loss of volume of air $\frac{dV}{dt}$ as $4\pi r^2 \frac{dr}{dt}$.	1
	Writes that the surface area of the basketball is $4\pi r^2$ and finds the rate of loss of the surface area $\frac{dA}{dt}$ as $8\pi r \frac{dr}{dt}$.	1
	Uses steps 1 and 2 to find the ratio as $r : 2$ and evaluates the same at $r = 8$ cm as 8 : 2 or 4 : 1.	1
13	Writes the volume function of the disk as $V = \pi \times R^2 \times H$.	0.5
	Writes that since volume remains constant, the derivative of V should be zero. $\frac{dV}{dt} = 0$	0.5
	Applies chain rule to get the equation as: $\pi \times R^2 \times \frac{dH}{dt} + \pi \times H \times 2R \times \frac{dR}{dt} = 0$	1
	Simplifies the equation in step 3 to get the ratio as: $\frac{dH}{dt} : \frac{dR}{dt} = \frac{-2H}{R}$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes $H = \frac{V}{\pi R^2}$ from step 1 to get the ratio in terms of R as: $\frac{dH}{dt} : \frac{dR}{dt} = \frac{-2V}{\pi R^3}$	0.5
14	**Finds the first derivative of the function as $\frac{dp}{dt}$ = 16 - 2 t.**	0.5
	Calculates the critical point as t = 8 by equating 16 - 2 t to 0.	0.5
	Finds the second derivative as: $\frac{d^2p}{dt^2} = -2$ **Writes that this means that the function will be at its maximum at t = 8.**	1
	Finds the maximum price of the share as p (8) as 16 × 8 - 8^2 + 8 = Rs 72.	1
15	**Assumes the perimeter of one square as x m and the perimeter of the other square as (a - x) m.**	0.5
	Finds the combined area of the two squares as: $A = (\frac{x}{4})^2 + (\frac{a-x}{4})^2$ m^2	0.5
	Differentiates the combined area as: $\frac{dA}{dx} = \frac{(4x-2a)}{16}$	0.5
	Equates $\frac{dA}{dx}$ to 0 to find the critical point as $x = \frac{a}{2}$ m.	0.5

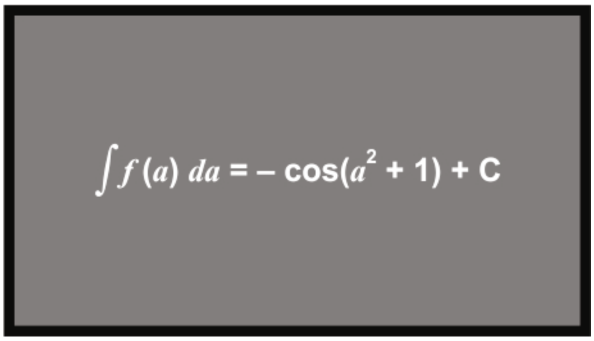
Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the second derivative as: $\frac{d^2A}{dx^2} = \frac{1}{4}$ **Concludes that $x = \frac{a}{2}$ m is a minima.**	**0.5**
	Writes that the combined area of the two squares will be minimum when the side lengths of both the squares is $\frac{a}{8}$ m.	**0.5**
16	**Simplifies the given function as:** $f(x) = \sin^{-1}\left(\sin x\left(-\cos\frac{3\pi}{4}\right) + \cos x \sin\frac{3\pi}{4}\right)$ **as sin $x = \frac{1}{\sqrt{2}}$ and -cos $x = \frac{1}{\sqrt{2}}$ only when $x = \frac{3\pi}{4}$ in the interval ($\frac{\pi}{2}$, π).**	**1**
	Simplifies the above step as: $f(x) = \sin^{-1}\left(\sin\left(\frac{3\pi}{4} - x\right)\right) = \left(\frac{3\pi}{4} - x\right)$	**1**
	Differentiates the above function to get $f\,'(x) = -1$ and concludes that the function is decreasing in the interval ($\frac{\pi}{2}$, π).	**1**
17	**Differentiates the equation of the circle to get $\frac{dy}{dx}$ as $\frac{(6-2x)}{(2y-6)}$ or $\frac{(3-x)}{(y-3)}$.**	**1**
	Calculates the slope of the curve at (4, 1.27) as $\frac{dy}{dx} = \frac{100}{173}$.	**1**
	Writes the equation of the path of the rock as $y - 1.27 = \frac{100}{173}(x - 4)$.	**1**
18	**Differentiates the given function as $y' = 2x + b$.**	**0.5**
	Finds the derivative of the given function at (1, 1) as $y' = 2 + b$.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes the equation of the tangent to the given curve at (1, 1) as $y - 1 = (2 + b)(x - 1)$.	0.5
	Assumes the point of intersection of the tangent with the x- axis as A. Finds the length of OA, where O is the origin, by substituting $y = 0$ in the above equation as $\frac{(1+b)}{(2+b)}$ units.	0.5
	Assumes the point of intersection of the tangent with the y- axis as B. Finds the length of OB by substituting $x = 0$ in the above equation as $-(1 + b)$ units.	0.5
	Mentions that ▲AOB is a right triangle. Hence, writes an expression to find the area of ▲AOB and equates it to 2 sq units as: $\frac{1}{2} \times \frac{(1+b)}{(2+b)} \times [-(1 + b)] = 2$	1
	Simplifies the above to get the quadratic equation: $b^2 + 6b + 9 = 0$	1
	Solves the above quadratic equation and finds the value of b as -3.	0.5
19	i) Finds the yield(r) per day 3 years after planting the sapling as: $r = \frac{3^3}{3} - 6(3^2) + 32(3)$ $\Rightarrow r = 51$ grams	1
	ii) Finds the first derivative of the given function with respect to time as: $r'(t) = t^2 - 12t + 32$	0.5
	Writes $r'(t) = 0$ and finds the critical points as $t = 4$ and $t = 8$.	1
	Writes that, since the life span of the plant is around 6 years, only $t = 4$ years is applicable here.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the second derivative of the given function with respect to time as: $r''(t) = 2t - 12$	0.5
	Finds the value of $r''(t)$ at $t = 4$ years as: $r''(4) = (-4)$	0.5
	Writes that, by the second derivative test, $t = 4$ years is the point of maxima. Hence, concludes that the maximum yield per day is obtained 4 years after planting the saplings.	1
20	**Differentiates x with respect to θ as:** $\frac{dx}{d\theta} = 3 \sec \theta \tan \theta$	0.5
	Differentiates y with respect to θ as: $\frac{dy}{d\theta} = 3 \sec^2 \theta$	0.5
	Finds $\frac{dy}{dx}$ as: $\frac{dy}{dx} = \frac{3 \sec^2 \theta}{3 \sec \theta \tan \theta} = \operatorname{cosec} \theta$	0.5
	Finds the slope of the given normal line as $\frac{-1}{2}$. Concludes that the slope of the tangent is 2.	1
	Equates $\frac{dy}{dx}$ with the slope of the tangent to find the value of θ as $\frac{\pi}{6}$.	0.5
	Finds the point of contact between the tangent and curve as: $x = 3 \sec \frac{\pi}{6} = \frac{6}{\sqrt{3}}$ or $2\sqrt{3}$ $y = 3 \tan \frac{\pi}{6} = \frac{3}{\sqrt{3}}$ or $\sqrt{3}$	1
	Writes the equation of the tangent as $y - \sqrt{3} = 2(x - 2\sqrt{3})$ or $2x - y = 3\sqrt{3}$.	1

Chapter - 7
Integrals

Q: 1 **Mr. Dinesh writes the following expression on the blackboard:**



He asked his students to determine the function that he integrated.

Sameer says, "sin(a^2 + 1)".
Danish says, "-sin(a^2 + 1)".
Meera says, "2 a sin(a^2 + 1)".
Deepa says, "-2 a sin(a^2 + 1)".

Who is correct?

1 Sameer **2** Danish **3** Meera **4** Deepa

Q: 2 **Read the statements independently and carefully and then choose the option that correctly describes them.**

$$P = \int_a^b f(x)$$

Statement 1 : If the function $f(x)$ is well defined and continuous in the interval (a, b), then P is always positive.

Statement 2: **If P exists, then the function $f(x)$ is always continuous in the interval (a, b).**

1 Statement 1 is true but Statement 2 is false
2 Statement 1 is false but Statement 2 is true
3 Both Statement 1 and Statement 2 are true
4 Both Statement 1 and Statement 2 are false

Q: 3 **$g(x)$ + C is an integral of $h(x)$, where C is an arbitrary constant.**

$\log_e(2x + g^2(x))$ + C is an integral of which of the following?

1 $\frac{2h(x)g(x) + 2}{2x + h^2(x)}$ **2** $\frac{2g(x)h(x) + 2}{2x + g^2(x)}$ **3** $\frac{2g(x) + 2}{2x + g^2(x)}$ **4** $\frac{1}{2x + g^2(x)}$

Q: 4 Look at the integral given below and find *f* (*x*). Show your steps. **[1]**

$$\int f(x)\,dx = \log\left|\log x\right| + C$$

where C is an arbitrary constant.

Q: 5 If *g* (*x*) is a polynomial function, find: **[1]**

$$\int_3^3 g(x)\,dx$$

Justify your answer.

Q: 6 Solve: **[2]**

$$\int \frac{6x}{x^2+2}\,dx + \int \frac{4}{x\,(x^2+2)}\,dx$$

Show your steps.

Q: 7 Shown below is the first step of solving an integral. **[2]**

$$\int \cot x\, g(x)\,dx = \cot x \sin x + \int \csc^2 x \sin x\,dx$$

Find *g* (*x*). Show your steps.

Q: 8 Integrate: **[2]**

$$\int \frac{\sin 2x\left(\frac{3}{2}\sin x - 1\right)}{e^{(\cos^2 x + \sin^3 x)}}\,dx$$

Show your steps.

Q: 9 **[2]**

$$\int \frac{1}{p^2\left(\sqrt{1+p^2}\right)}\,dp$$

Lalitha integrates the above problem as shown below. She made an error in one of the steps.

Step 1: Let $p = \tan\theta$

$\Rightarrow dp = \sec^2\theta\, d\theta$

Step 2: $\int \dfrac{\sec^2\theta}{\tan^2\theta \sec\theta}\, d\theta$

Step 3 : $\Rightarrow \int \operatorname{cosec}\theta \cot\theta\, d\theta$

Step 4: $\Rightarrow$ cosec θ + C, where C is an arbitrary constant.

Step 5: $\Rightarrow \dfrac{\sqrt{p^2+1}}{p} + C$

In which step did she go wrong? Explain the error.

Q: 10 **[3]**

$f(x)$ is a function such that:

$$f(x) + C = \int (g(x) \times h(x))\, dx$$

where C is an arbitrary constant.

If $g'(x) = \dfrac{1}{x}$ and $\int h(x)\, dx = x^2$, find $f(x)$. Show your steps.

Q: 11 **Evaluate:** **[3]**

$$\int_0^1 e^x (\sec x + \sec x \tan x)\, dx$$

Show your steps.

Q: 12 **In the equation below, p, q, r and s are constants.** **[3]**

$$-9\int x\, e^{-3x} dx = e^{-3x}(\mathbf{p}x^3 + \mathbf{q}x + \mathbf{r}) + \mathbf{s}$$

Find the values of p, q, r and s. Show your steps.

Q: 13 **If *u* and *v* are two functions in *x*, integrate:** **[3]**

$$\int x^2[u(x^3)v''(x^3) - u''(x^3)v(x^3)]dx$$

Show your steps.

Q: 14 **Evaluate:** **[5]**

$$\int_2^5 \frac{x^2 + 3}{(x^2 + 2)(x^2 - 5)}\, dx$$

Show your steps.

Q: 15 **Integrate the function given below.** **[5]**

$$\int \sqrt{\sqrt{x^2 + 4} + x}\, dx$$

Show your steps.

Use the information given below to answer the questions that follow.

The derivative of the velocity function, $v(t)$, with respect to time, t, gives us the acceleration of the object. This can be written as $\frac{dv}{dt} = a(t)$ (in m/s^2).

The derivative of the displacement function, $x(t)$, with respect to time, t, gives us the velocity of the object. This can be written as $\frac{dx}{dt} = v(t)$ (in m/s).

A car is moving at a constant velocity of 15 m/s. It starts decelerating when it is about to reach its destination with acceleration, $a(t) = -\frac{t}{3}$ m/s^2 where t is the time (in seconds) for which it decelerated before coming to rest.

Q: 16 **Find the velocity function of the car after it started decelerating. Show your steps.** **[2]**

Q: 17 **Find the time it takes the car to stop after it starts decelerating. Show your work and give a valid reason.** **[1]**

Q: 18 **Find the displacement function of the car and calculate its displacement from the moment it starts decelerating till it stops. Show your steps.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	2
3	2

Q.No	Teacher should award marks if students have done the following:	Marks
4	**Differentiates log |log x | + C using the chain rule to get $f(x)$ as $\frac{1}{x.\log x}$.**	1
5	**Writes the value of the definite integral as 0.**	0.5
	Writes that, since we are integrating the function from $x = 3$ to $x = 3$, the area under the function will be zero. **(Award full marks for any other valid justification.)**	0.5
6	**Rewrites the integral as:** $2\int \frac{3x^2 + 2}{x^3 + 2x}\, dx$	0.5
	Substitutes $x^3 + 2x$ as t to get: $dx = \frac{dt}{3x^2 + 2}$	0.5
	Rewrites the integral as: $2\int \frac{1}{t}\, dt$	0.5
	Integrates the above expression and gets 2log | t | + C as the solution where C is an arbitrary constant. **Substitutes t as $x^3 + 2x$ to get 2log | $x^3 + 2x$ | + C as the solution.**	0.5
7	**Compares the integral with the standard form of integration by parts to conclude:** $\int g(x)\, dx = \sin x$	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the above step to find $g(x)$ as $\cos x$ or $\cos(2n\pi + x)$ where n is a whole number.	**0.5**
8	**Substitutes ($\cos^2 x + \sin^3 x$) as t and finds dt as follows:** $dt = \sin 2x\,(\frac{3}{2}\sin x - 1)\,dx$	**1**
	Rewrites the given integral as: $\int e^{(-t)}\,dt$	**0.5**
	Integrates the above expression to get $-e^{(-t)} + C$, where C is an arbitrary constant. **Substitutes t as $\cos^2 x + \sin^3 x$ to get:** $-e^{-(\cos^2 x + \sin^3 x)} + C$	**0.5**
9	**Writes that Lalitha made an error in step 4.**	**1**
	Writes that Lalitha has integrated the expression given in step 3 as cosec θ instead of -cosec θ.	**1**
10	**Writes that:** $f(x) + C = g(x)\int h(x)\,dx - \int[g'(x)\int h(x)\,dx]\,dx$	**1**
	It is given that $g'(x) = \frac{1}{x}$. **Integrates both sides of the above equation to get $g(x) = \log x + A$ where A is an arbitrary constant.**	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes the values of *g* (*x*), *g* '(*x*) and integral of *h* (*x*) in the expression obtained in step 1 to get: $$f(x) + \mathrm{C} = x^2 (\log x + \mathrm{A}) - \int \frac{x^2}{x}\, dx$$	**0.5**
	Simplifies the above equation to get the following, where B is an arbitrary constant: $$f(x) + \mathrm{C} = x^2 \log x + \mathrm{A}x^2 - \frac{x^2}{2} + \mathrm{B}$$	**0.5**
	Writes that $f(x) = x^2 \log x + \mathrm{K}x^2 + \mathrm{D}$ where $\mathrm{K} = \mathrm{A} - \frac{1}{2}$ and D = B - C.	**0.5**
11	**Rewrites the integral as:** $$\int_0^1 e^x \sec x \, dx + \int_0^1 e^x \sec x \tan x \, dx$$	**0.5**
	Applies integration by parts on the first term to get: $$[e^x \sec x]_0^1 - \int_0^1 \sec x \tan x \, e^x \, dx + \int_0^1 e^x \sec x \tan x \, dx$$	**1**
	Simplifies the above expression to get: $$[e^x \sec x]_0^1$$	**0.5**
	Finds the value of the integral as e.sec 1° - e^0.sec 0° = e.sec 1° - 1.	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
12	**Differentiates both sides of the given equation with respect to x to get:** $-9x(e^{-3x}) = (e^{-3x})(3px^2 + q) + (px^3 + qx + r)(-3e^{-3x})$	1
	Simplifies the above equation as: $-9x(e^{-3x}) = (e^{-3x})(-3px^3 + 3px^2 - 3qx + q - 3r)$	0.5
	Equates the coefficients on both sides of the equation to find the values of p, q and r as 0, 3 and 1 respectively.	1
	Writes that the constant, s cannot be uniquely determined from the given information. **(Award full marks if the problem is solved by applying integration by parts to the LHS and then equating the coefficients on both sides.)**	0.5
13	**Substitutes x^3 as t and finds dt as $3x^2 dx$.**	0.5
	Rewrites the given expression as: $\frac{1}{3}[u(t)v''(t) - u''(t)v(t)]dt$	0.5
	Uses integration by parts for each term of the above expression as: $\frac{1}{3}[u(t)v'(t) - \int u'(t)v'(t)dt - v(t)u'(t) + \int v'(t)u'(t)dt] + C$ $\Rightarrow \frac{1}{3}[u(t)v'(t) - v(t)u'(t)] + C$ **where C is an arbitrary constant.**	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes t as x^3 and writes the final expression as: $$\frac{1}{3}[u(x^3)v'(x^3) - v(x^3)u'(x^3)] + C$$	0.5
14	**Assumes $x^2 = y$ and rewrites the given expression as follows:** $$\frac{x^2 + 3}{(x^2 + 2)(x^2 - 5)} = \frac{y + 3}{(y + 2)(y - 5)}$$	0.5
	Rewrites the above expression using the partial fraction method as: $$\frac{-1}{7(y + 2)} + \frac{8}{7(y - 5)}$$	1.5
	Writes the integral as: $$-\frac{1}{7}\int_2^5 \frac{1}{x^2 + 2}\,dx + \frac{8}{7}\int_2^5 \frac{1}{x^2 - 5}\,dx$$	0.5
	Solves the integral as: $$-\frac{1}{7}\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right]_2^5 + \frac{8}{7}\left[\frac{1}{2\sqrt{5}}\log\left\|\frac{x - \sqrt{5}}{x + \sqrt{5}}\right\|\right]_2^5$$	1
	Evaluates the above integral as: $$\frac{1}{7\sqrt{2}}\left(\tan^{-1}\sqrt{2} - \tan^{-1}\frac{5}{\sqrt{2}}\right) + \frac{4}{7\sqrt{5}}\left(\log\left\|\frac{5 - \sqrt{5}}{5 + \sqrt{5}}\right\| - \log\left\|\frac{2 - \sqrt{5}}{2 + \sqrt{5}}\right\|\right)$$	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
15	**Substitutes $\sqrt{x^2 + 4} + x$ as t and finds x as:** $\sqrt{x^2 + 4} = t - x$ **Squares both sides to get:** $2tx = t^2 - 4$ $x = \frac{t}{2} - \frac{2}{t}$	1.5
	Finds dx as $\left(\frac{1}{2} + \frac{2}{t^2}\right) dt$.	1
	Rewrites the integral as: $\int \sqrt{t}\left(\frac{1}{2} + \frac{2}{t^2}\right) dt$ $\Rightarrow \frac{1}{2}\int t^{\frac{1}{2}}\, dt + 2\int t^{\frac{-3}{2}} dt$	1
	Integrates the above expression as: $\frac{(t)^{\frac{3}{2}}}{3} - 4\,(t)^{-\frac{1}{2}} + C$ **where C is an arbitrary constant.**	1
	Substitutes t as $\sqrt{x^2 + 4} + x$ to get: $\frac{(\sqrt{x^2 + 4} + x)^{\frac{3}{2}}}{3} - \frac{4}{(\sqrt{x^2 + 4} + x)^{\frac{1}{2}}} + C$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
16	**Writes the velocity function of the car (in m/s) as:** $v(t) = \int a(t)\,dt$ or $-\frac{1}{3}\int t\,dt$	0.5
	Integrates the above expression to get *v* (*t*) as follows: $v(t) = -\frac{1}{3} \times \frac{t^2}{2} + C$ **where C is an arbitrary constant.**	1
	Writes that, at *t* = 0, *v* (0) = 15 m/s. **Uses the above steps to find C as 15 and writes that:** $v(t) = -\frac{t^2}{6} + 15$	0.5
17	**Writes that the velocity of the car will be zero when the car stops. Hence, gets the equation:** $0 = 15 - \frac{t^2}{6}$	0.5
	Solves the above equation to find *t* as √90 or 3√10 seconds.	0.5
18	**Writes the displacement function of the car (in m) as:** $x(t) = \int v(t)\,dt$ or $\int\left(15 - \frac{t^2}{6}\right)dt$	0.5
	Integrates the above expression to get: $x(t) = 15t - \frac{t^3}{18} + C$ **where C is an arbitrary constant.**	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that, at t = 0, the car starts decelerating until it stops and x (0) = 0. Substitutes these values in the above expression to find C as 0 and rewrites the displacement function x (t) as: $x(t) = 15t - \frac{t^3}{18}$	0.5
	Finds the displacement from the moment it starts decelerating until it stops as: $x\,(3\sqrt{10}) = 15 \times 3\sqrt{10} - \frac{3\sqrt{10}\times3\sqrt{10}\times3\sqrt{10}}{18}$ $= 45\sqrt{10} - 15\sqrt{10}$ $= 30\sqrt{10}$ m	0.5

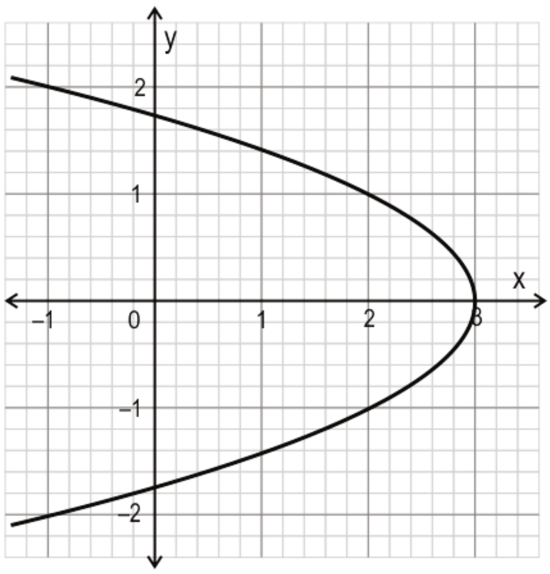
Chapter - 8
Application of Integrals

Q: 1 **If the temperature of an electric oven is increasing at the rate of $R(t)$ °C per minute, then what does the following expression represent?**

$$\int_0^{15} R'(t)\,dt$$

1. Rate of increase of temperature from the 0-minute mark to the 15-minute mark
2. Average increase in the temperature from the 0-minute mark to the 15-minute mark
3. Average rate of increase of temperature from the 0-minute mark to the 15-minute mark
4. Difference in the rate of increase of temperature at the 15-minute mark and the 0-minute mark

Q: 2 **Shown below is the graph of ($-x = y^2 - 3$).**



Ravi says that the area under the curve in the first quadrant can be found as:

$$\int_0^{3} \sqrt{3-x}\,dx$$

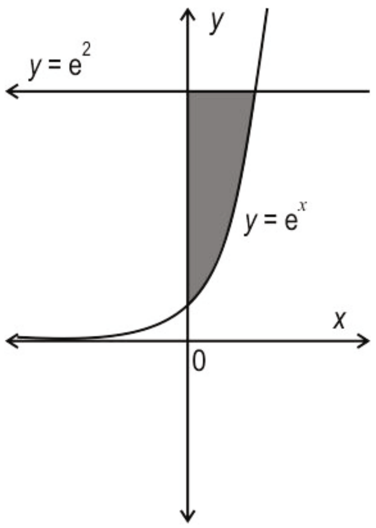
Kanika says that the area under the curve in the first quadrant can be found as:

$$\int_0^{\sqrt{3}} (3-y^2)\,dy$$

Who is correct?

1. only Ravi
2. only Kanika
3. both Ravi and Kanika
4. neither Ravi nor Kanika

Q: 3 **The shaded region shown below is bounded by the curve $y = e^x$, the y -axis and the line $y = e^2$.**



Which of the following is the area of the shaded region?

1. $\int_{1}^{e^2} (e^2 - e^x)\, dx$
2. $\int_{1}^{2} (e^2 - e^x)\, dx$
3. $\int_{1}^{e^2} e^x\, dx$
4. $\int_{0}^{2} e^x\, dx$

Q: 4 **The area under the curve $y = x^2$ between the line $x = 0$ and $x = k$ is 9 square units.**

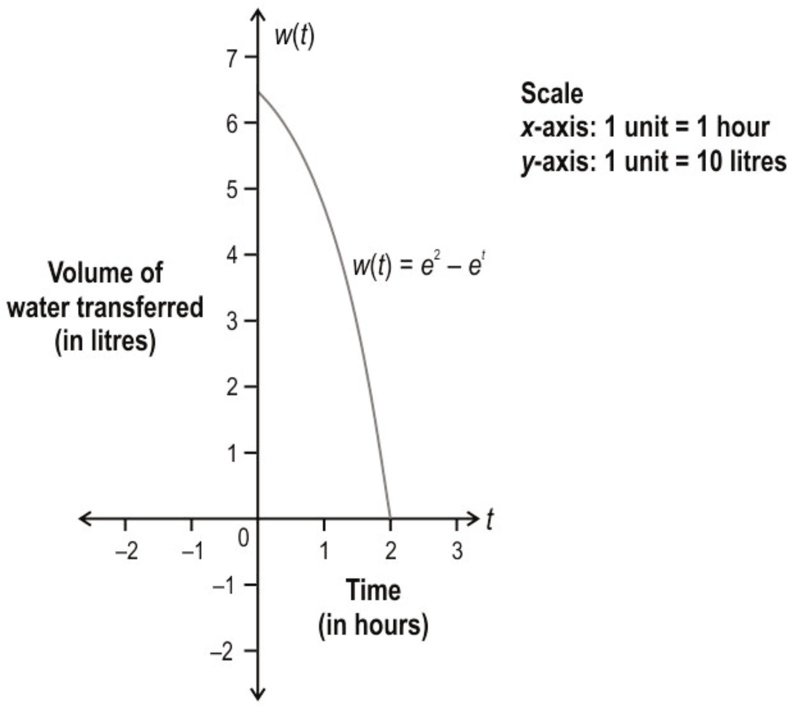
Which of the following could be the value of k?

1. 3
2. 4.5
3. 9
4. 27

Q: 5 **Water from an overhead tank is flowing down through different pipes. The volume of water (in litres) transferred at time *t* (in hours) is given by the function:**

$w(t) = e^2 - e^t$

Shown below is the graph of *w* (*t*).

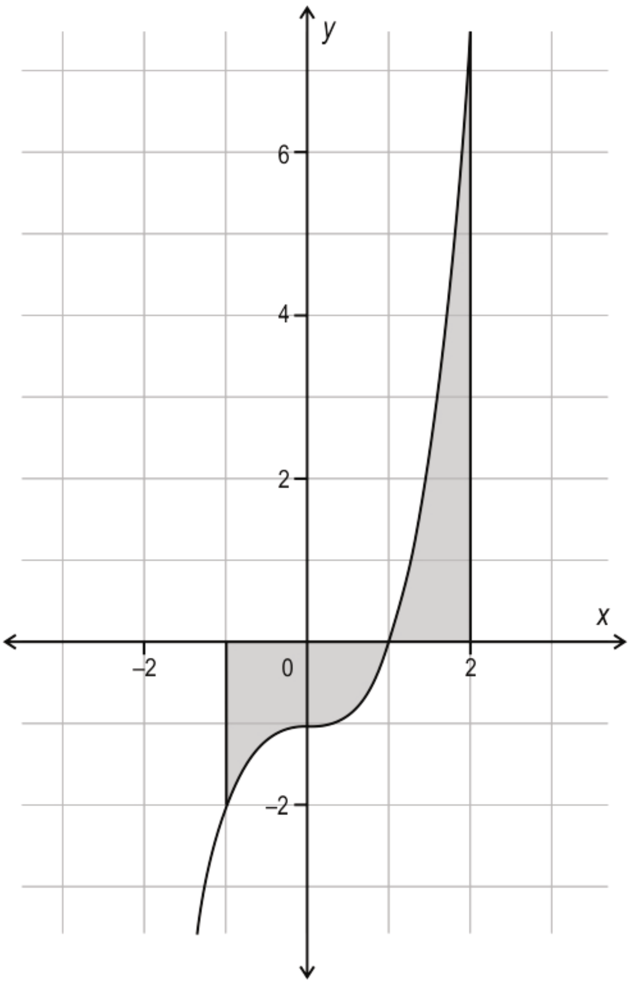


Approximately, what is the total volume of water transferred from the overhead tank in 2 hours?

(Note: Take e **^{2} = 7.389.)**

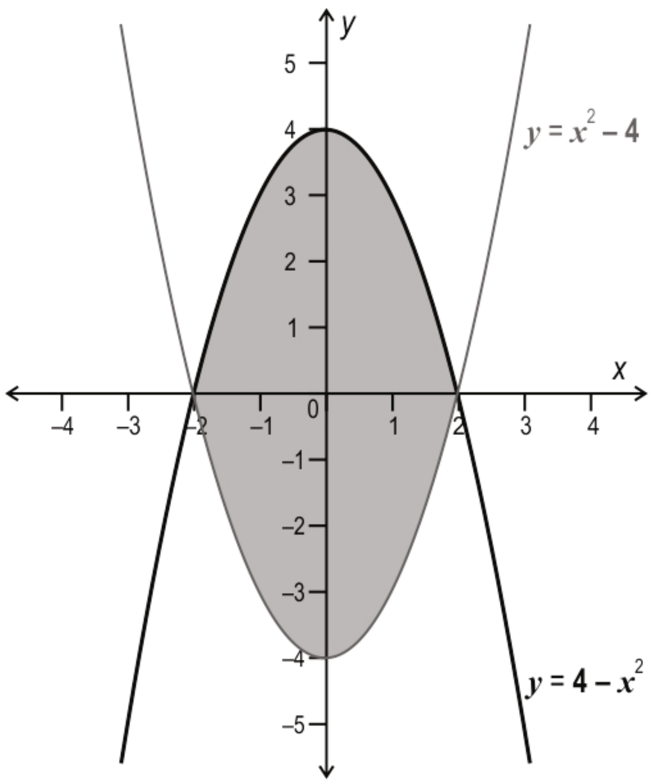
1 6.4 litres
2 8.4 litres
3 64 litres
4 84 litres

Q: 6 **Shown below is the graph of the function $y = x^3 - 1$.** **[1]**



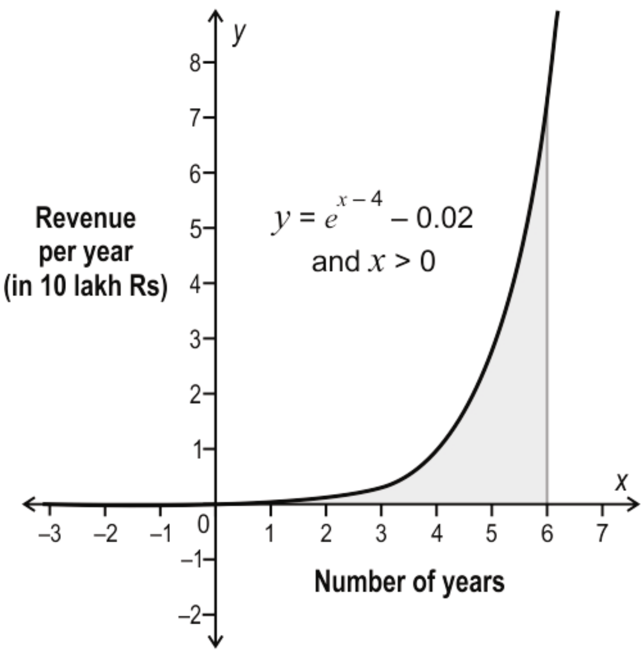
Write an expression to find the area of the shaded region by integrating with respect to the x -axis.

Q: 7 **Shown below are two curves.** **[2]**



Find the area of the shaded region. Show your steps.

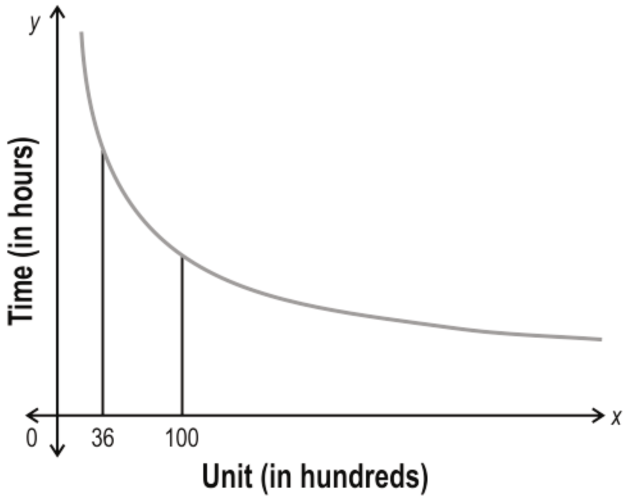
Q: 8 **A taxi company projected its revenue for 6 years using the graph shown below. The shaded region represents the total revenue that can be generated over 6 years.** **[2]**



Find the total revenue that will be generated over the last two years based on the projection. Show your steps.

(Note: Take $e^2 = 7.39$.)

Q: 9 **In economics, a learning curve is defined as a curve that shows the relation between the number of units produced and the time taken to produce them over a given period. Shown below is the learning curve of Hunar Cosmetics Private Limited.** **[2]**

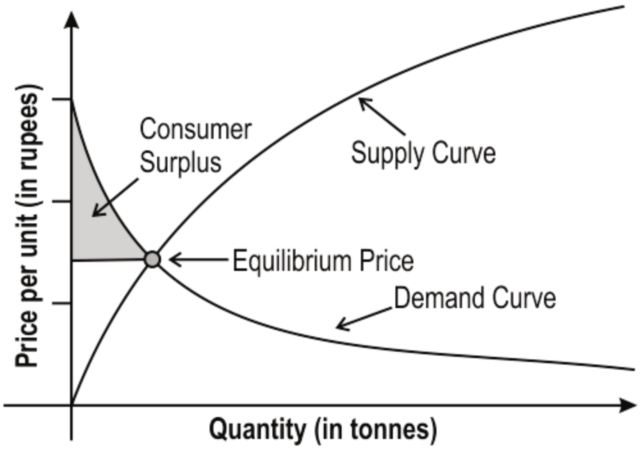


(Note: The figure is not to scale.)

The learning curve is represented by $y = 1272\, x^{\frac{-1}{2}}$.

Find the total hours required to produce the units from the 3600 th to the 10000 th unit. Show your steps.

Q: 10 **In economics, the equilibrium price is the point of intersection of the supply curve and demand curve. Consumer surplus is defined as the area covered between the demand curve and the equilibrium price line (see the shaded region in the image below).** **[3]**



For rice, assume that the demand curve is given by $\frac{180}{r+2}$ and supply curve is given by $\frac{56r}{r+1}$ where *r* is the quantity of rice (in tonnes).

Determine

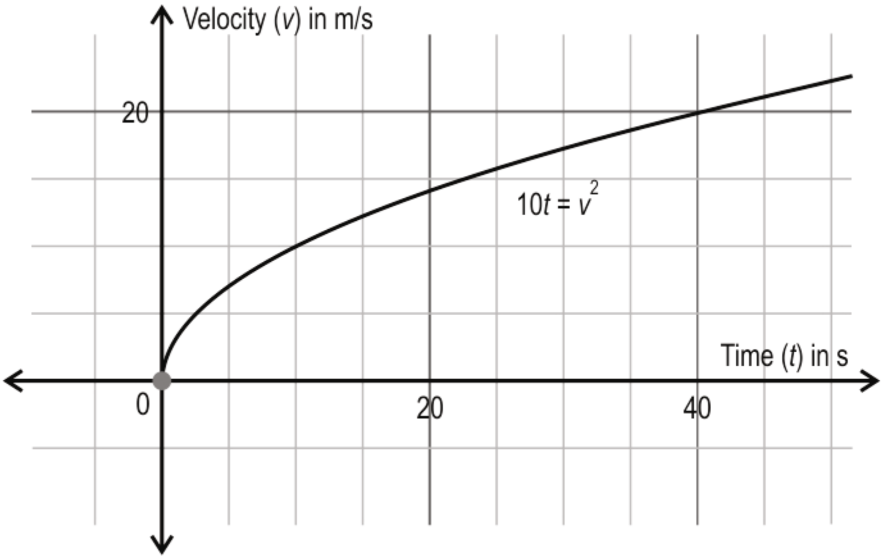
i) the equilibrium price for rice.
ii) the consumer surplus.

Show your steps.

(Note: Use ln(2) = 0.69, ln(2.25) = 0.81, ln(4.5) = 1.50)

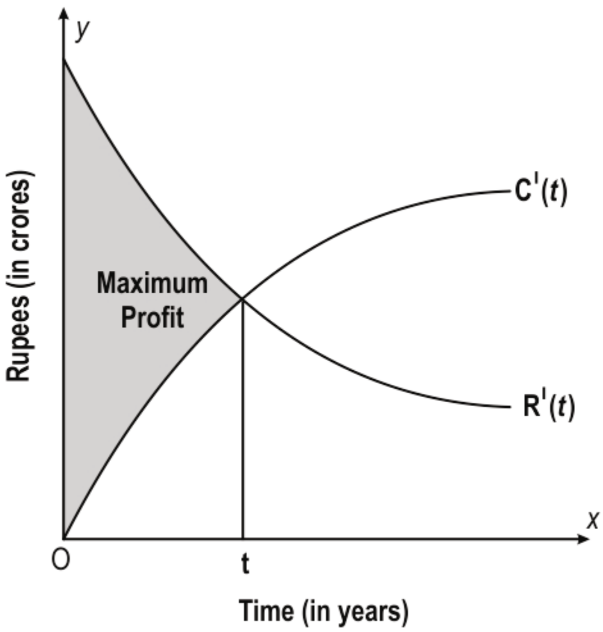
Q: 11 **The area under the velocity-time graph gives the displacement of a moving object.** **[3]**

Shown below is the velocity-time graph of a bird's flight.



Find the displacement of the bird in the first 40 seconds. Show your work.

Q: 12 **In economics, the number of years for which any company will be profitable can be found at the point of intersection of the marginal revenue (*R*') curve and the marginal cost (*C*') curve. The maximum profit a company can earn during this time is defined as the area between the *R*' curve, the *C*' curve and the *y* -axis as shown in the figure.** **[3]**



For Indian Collection Limited,

Marginal revenue function is given by $R'(t) = 10 - t^{\frac{1}{3}}$

Marginal cost function is given by $C'(t) = 2 + 3t^{\frac{1}{3}}$

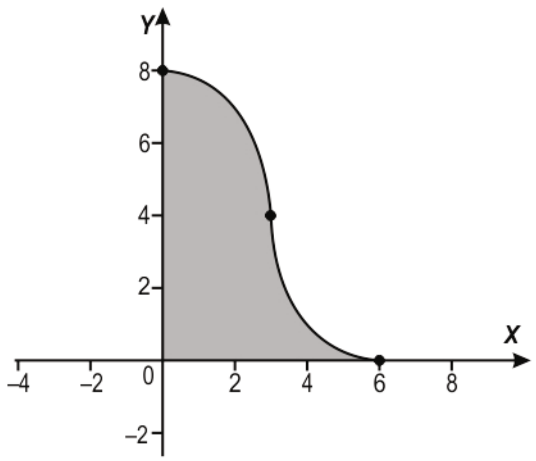
where *t* is the number of years and *R*' and *C*' are in crores of rupees.

Find:

i) the number of years for which Indian Collection Limited will be profitable.
ii) the maximum profit that Indian Collection Limited can earn during that period.

Show your steps.

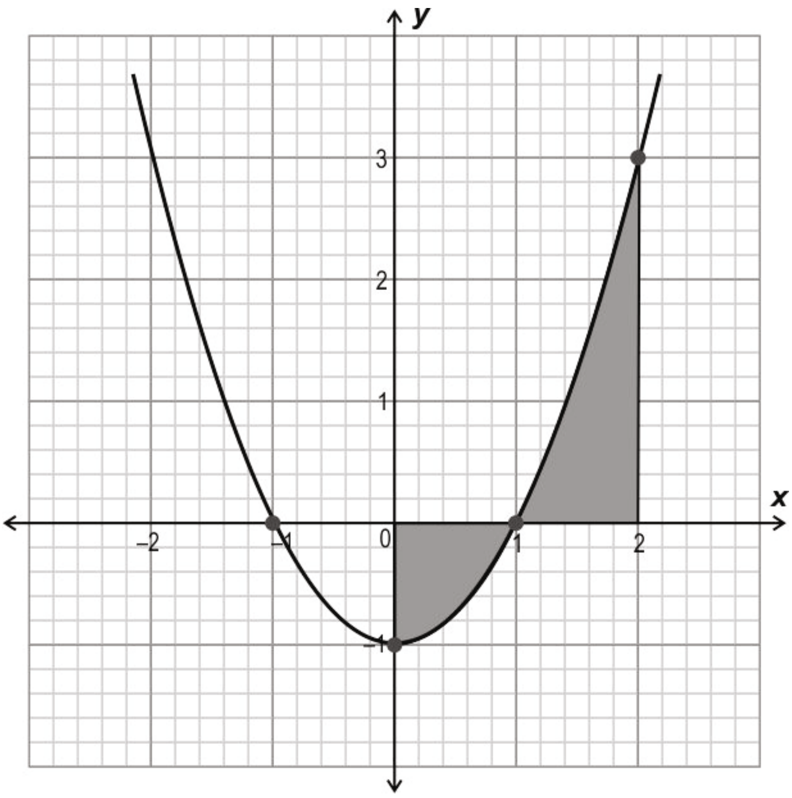
Q: 13 **A function $f(x)$ is graphed and defined below for $x \in [0, 6]$.** **[5]**



$$f(x) = \begin{cases} 4 + 4\sqrt{1 - \dfrac{x^2}{9}} & \text{if } x \in [0, 3) \\ 4 - 4\sqrt{1 - \dfrac{(x-6)^2}{9}} & \text{if } x \in [3, 6] \end{cases}$$

Find the area of the shaded region. Show your steps.

Q: 14 **Shown below is the graph of the function $y = x^2 - 1$.** **[5]**



i) Compute the area of the shaded region. Show your work.

ii) Write an expression to find the area of the shaded region in the fourth quadrant by integrating with respect to the y -axis.

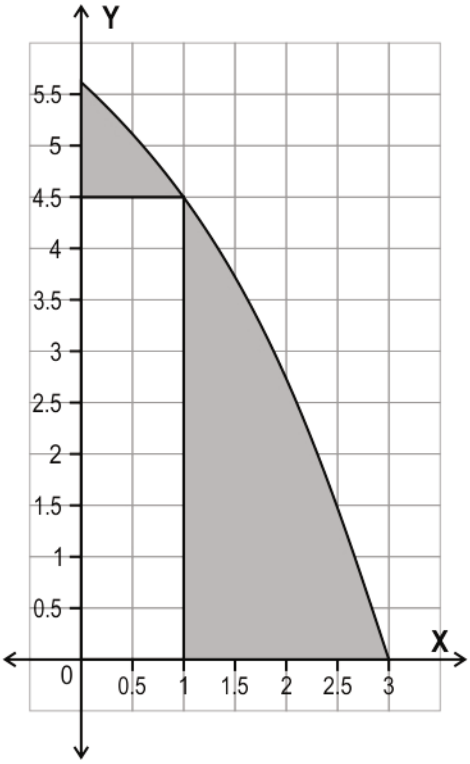
Q: 15 **i) Find the area under the curve of $f(x)$ in the first quadrant from 0 to π, where $f(x)$ is given by:** **[5]**

$$f(x) = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$

ii) Frame an expression to find the area of sin 2 x in the first quadrant from 0 to π.

Show your steps and give valid reasons.

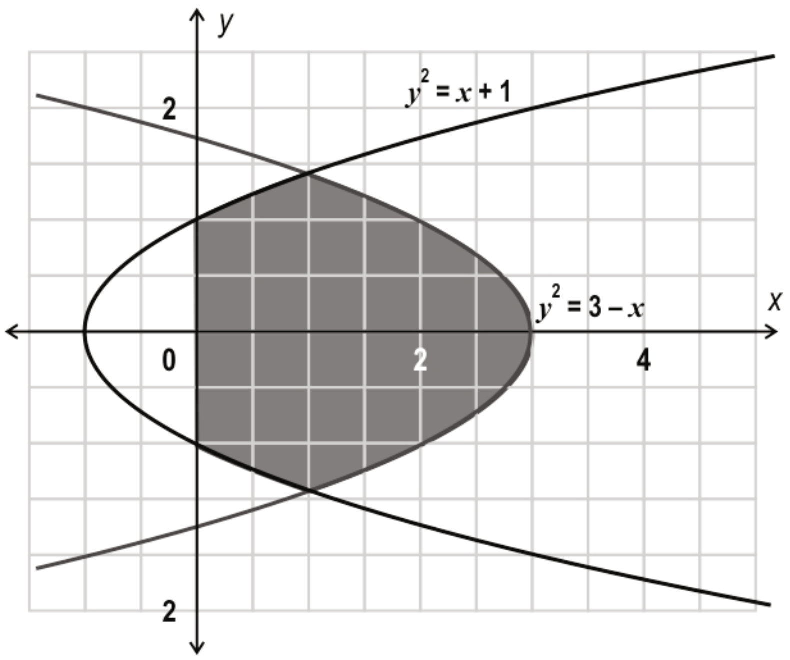
Q: 16 **Shown below is the graph of $f(x) = \frac{-3}{8}(x + 1)^2 + 6$ in the first quadrant.** **[5]**



i) Find the area of the shaded region. Show your steps.

ii) In the second quadrant, if an identical unshaded rectangle is drawn under the graph of $f(x)$, would the area of the shaded area be the same as part i)? Justify your answer.

Q: 17 **Shown below are the graphs of two parabolas.** **[5]**



Find the area of the shaded region. Show your work and give your answer to 2 decimal places.

(Note: Use √2 = 1.4, √3 = 1.7 and √5 = 2.2.)

Answer the questions based on the given information.

Marginal revenue refers to the rate of change of total revenue with respect to the number of units sold at an instant and marginal cost refers to the rate of change of total cost with respect to the number of units produced at an instant.

The marginal revenue of DuPoint Limited from selling x units of personal protective equipment (PPE) kits in a day is given by (2 - 6 x) and its marginal cost of producing the same is given by (16 x - 1582). The fixed cost of producing PPE kits is Rs 1800 per day, where the fixed cost refers to the cost incurred at zero level of production. The company earns zero revenue at zero level of production.

(Revenue and cost are in Rupees.)

Q: 18 **Find the total cost function. Show your work.** **[1]**

Q: 19 **Find the profit function. Show your work.** **[2]**

(Note: Profit function = Total revenue function - Total cost function)

Q: 20 **Find the number of units of PPE kits that can be sold in a day to earn maximum profit. Show your work.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	3
3	2
4	1
5	4

Q.No	Teacher should award marks if students have done the following:	Marks
6	**Writes the expression to find the area of the shaded region by integrating with respect to the *x* -axis as:** $\left\lvert \int_{-1}^{1} (x^3 - 1)\, dx \right\rvert + \int_{1}^{2} (x^3 - 1)\, dx$ **(Award 0.5 marks if only the correct expression with correct limits is written without the modulus symbol.)**	1
7	**Writes the expression for the area of the shaded region as:** $2\int_{-2}^{2} (4 - x^2)\, dx$	1
	Simplifies the above integral as: $2\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 21.33$ **Concludes that the area of the shaded region is 21.33 sq units.** **(Award full marks if the area in the first quadrant is found and then multiplied by 4.)**	1
8	**Writes the expression for the total revenue that will be generated over the last two years based on the projection as:** $\int_{4}^{6} (e^{x-4} - 0.02)\, dx$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Solves the integral to get: $\left[e^{x-4} - 0.02\,x\right]_4^6$	**0.5**
	Simplifies the above expression as [e^2 - 0.02 × 6) - (e^0 - 0.02 × 4) = 7.39 - 0.12 - 1 + 0.08 = 6.35.	**0.5**
	Finds the total revenue that will be generated over the last two years based on the projection as 6.35 × 1000000 = Rs 6350000.	**0.5**
9	**Writes the expression for the total hours required to produce the units from 3600 th to 10000 th unit as:** $\int_{3600}^{10000} 1272\,x^{-\frac{1}{2}}\,dx$	**0.5**
	Solves the above integral as: $= 2544\left[x^{\frac{1}{2}}\right]_{3600}^{10000}$	**1**
	Solves the above integral to find the hours required to produce the units from the 3600 th to the 10000 th unit as 101760 hours.	**0.5**
10	**i) Writes the equation to calculate equilibrium price as:** $\frac{180}{r+2} = \frac{56r}{r+1}$ **=> 14 r^2 - 17 r - 45 = 0**	**0.5**
	Solves the equation in step 1 and finds r as 2.5 tonnes and equilibrium price as 40 rupees.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Frames the consumer surplus as the area between the demand curve, the *y* -axis and *y* = 40 as: $$\text{Consumer Surplus} = \int_0^{2.5} \left(\frac{180}{r+2} - 40\right) dr$$ $$\text{Consumer Surplus} = \int_0^{2.5} \frac{180}{r+2}\, dr - 40 \times 2.5$$ (Note: Award full marks for this step if the student directly writes the second equation.)	1
	Finds the integral value as 180 × ln(2.25) - 100 = Rs 45.8.	1
11	Writes the integral to find the displacement of the bird in the first 40 seconds as: $$\sqrt{10}\int_0^{40} \sqrt{t}\ dt$$	1
	Integrates the above expression to get: $$\sqrt{10}\left.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right\|_0^{40}$$	1
	Finds the displacement of the bird in the first 40 seconds as $\frac{1600}{3}$ m or 533.33 m.	1
12	i) Writes the equation to find the number of years for which Indian Collection Limited will be profitable as: $10 - t^{\frac{1}{3}} = 2 + 3\,t^{\frac{1}{3}}$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Solves the above equation for *t* to find the total number of years for which Indian Collection Limited will be profitable as 8 years.	**0.5**
	ii) Writes the expression for the maximum profit that Indian Collection Limited can earn during that period as: $$\int_0^8 [(10 - t^{\frac{1}{3}}) - (2 + 3t^{\frac{1}{3}})]\, dt$$ $$= \int_0^8 [8 - 4t^{\frac{1}{3}})]\, dt$$	**0.5**
	Solves the above integral as: $$= [8t - 3t^{\frac{4}{3}}]_0^8$$	**1**
	Solves the above integral to find the maximum profit that Indian Collection Limited can earn during that period as 16 crore rupees.	**0.5**
13	**Writes the total area to be a sum of two integrals as:** $$I = I_1 + I_2 = \int_0^3 \left(4 + 4\sqrt{1 - \frac{x^2}{9}}\right) dx + \int_3^6 \left(4 - 4\sqrt{1 - \frac{(x-6)^2}{9}}\right) dx$$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Applies integration by parts and writes the first integral as: $I_1 = 4\int_0^3 dx + \frac{4}{3}\int_0^3 \sqrt{9-x^2}\,dx$ $I_1 = 4\,[x]_0^3 + \frac{4}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$	**1**
	Simplifies the integral and finds the value as: $I_1 = 4 \times (3-0) - \frac{4}{3}\left[0 + \frac{9\pi}{4} - 0 - 0\right] = 12 + \frac{4}{3}\left[\frac{9\pi}{4}\right]$ $I_1 = 12 + 3\pi$ sq units	**1**
	Applies integration by parts and writes the second integral as: $I_2 = 4\int_3^6 dx + \frac{4}{3}\int_3^6 \sqrt{9-(x-6)^2}\,dx$ $I_2 = 4\,[x]_3^6 + \frac{4}{3}\left[\frac{(x-6)}{2}\sqrt{9-(x-6)^2} + \frac{9}{2}\sin^{-1}\frac{(x-6)}{2}\right]_3^6$	**1**
	Simplifies the integral and finds the value as: $I_2 = 4 \times (6-3) - \frac{4}{3}\left[0 + 0 - 0 + \frac{9\pi}{4}\right] = 12 - \frac{4}{3}\left[\frac{9\pi}{4}\right]$ $I_2 = 12 - 3\pi$ sq units	**1**
	Adds the two integrals to find the area under the curve as: **$I = I_1 + I_2 = 24$ sq units**	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
14	**i) Considers the area of the shaded region in the first quadrant as A_1.** **Writes an expression to find A_1 as:** $A_1 = \int_1^2 (x^2 - 1)\, dx$	0.5
	Considers the area of the shaded region in the fourth quadrant as A_2. **Writes an expression to find A_2 as:** $A_2 = \left\| \int_0^1 (x^2 - 1)\, dx \right\|$ **(Award 0.5 marks if only the correct expression is written without the modulus symbol.)**	1
	Integrates and computes A_1 as: $A_1 = \left[\frac{x^2}{3} - x\right]_1^2 = \frac{4}{3}$ sq units	1
	Integrates and computes A_2 as: $A_2 = \left\|\left[\frac{x^2}{3} - x\right]_1^2\right\| = \left\|\frac{-2}{3}\right\| = \frac{2}{3}$ sq units	1
	Finds the area of the shaded region as $A_1 + A_2 = \frac{4}{3} + \frac{2}{3} = 2$ sq units.	0.5

<table>
<tr><th>Q.No</th><th>Teacher should award marks if students have done the following:</th><th>Marks</th></tr>
<tr><td></td><td>ii) Writes an expression to find the area of the shaded region in the fourth quadrant by integrating with respect to the y -axis as:

$$\left| \int_{-1}^{0} \sqrt{y+1}\, dy \right|$$
(Award 0.5 marks if only the correct expression is written without the modulus symbol.)</td><td>1</td></tr>
<tr><td>15</td><td>i) Writes that $f(x) = (\cos^2 x - \sin^2 x)$ or $\cos 2x$.</td><td>0.5</td></tr>
<tr><td></td><td>Writes that the curve $\cos 2x$ lies in the first quadrant for $x \in (0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \pi]$ and justifies the limits. For example, draws a table as shown below:

<table><tr><td>x</td><td>0</td><td>π/4</td><td>3π/4</td><td>π</td></tr><tr><td>cos 2 x</td><td>1</td><td>0</td><td>0</td><td>1</td></tr></table>
(Note: The students might include more values for x .)</td><td>1</td></tr>
<tr><td></td><td>Writes the expression for area as:

$$\int_{0}^{\frac{\pi}{4}} \cos 2x\, dx + \int_{\frac{3\pi}{4}}^{\pi} \cos 2x\, dx$$</td><td>0.5</td></tr>
<tr><td></td><td>Solves the integral to get:

$$\left[\frac{\sin 2x}{2}\right]_{0}^{\frac{\pi}{4}} + \left[\frac{\sin 2x}{2}\right]_{\frac{3\pi}{4}}^{\pi}$$</td><td>0.5</td></tr>
<tr><td></td><td>Finds the area as 1 sq unit.</td><td>1</td></tr>
</table>

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Writes that the curve sin 2 *x* lies in the first quadrant for $x \in (0, \frac{\pi}{2})$ and justifies the limits. For example, draws a table as shown below: x: 0, π/4, π/2, 3π/4, π sin 2 x: 0, 1, 0, -1, 0 **(Note: The students might include more values for *x* .)**	1
	Frames an expression for the area under the curve in the first quadrant as: $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$	0.5
16	**i) Writes the expression to find the area under the curve in the first quadrant as:** $\text{Area} = \int_0^3 \left(\frac{-3}{8}(x+1)^2 + 6\right) dx$	0.5
	Writes the solution of the above integral as: $\text{Area} = \left[\frac{-x^3}{8} - \frac{3x^2}{8} + \frac{45x}{8}\right]_0^3$	1.5
	Finds the area under the curve in the first quadrant as $\frac{81}{8}$ sq units.	1
	Finds the area of the unshaded rectangular region as $1 \times \frac{9}{2} = \frac{9}{2}$ sq units.	0.5
	Finds the area of the shaded region as $\frac{81}{8} - \frac{9}{2} = \frac{45}{8}$ sq units.	0.5
	ii) Writes that the area of the shaded region in the second quadrant will not be the same as the first quadrant as the graph is not symmetrical about *x* -axis.	1

Q.No	Teacher should award marks if students have done the following:	Marks
17	**Writes the expression for the area enclosed between the parabolas as:** $$2\int_0^1 \sqrt{x+1}\,dx + 2\int_1^3 \sqrt{3-x}\,dx$$	1.5
	Integrates the above expression as: $$4\left[\frac{(x+1)^{\frac{3}{2}}}{3}\right]_0^1 - 4\left[\frac{(x+1)^{\frac{3}{2}}}{3}\right]_1^3$$	1.5
	Simplifies the above expression as $\frac{8\sqrt{2}}{3} - \frac{4}{3} + \frac{8\sqrt{2}}{3}$.	1.5
	Finds the area enclosed between the two parabolas as $\frac{16\sqrt{2}-4}{3}$ **= 6.13 sq units.**	0.5
18	**Integrates the given marginal cost function, 16 x - 1582, with respect to x to get the total cost function as TC(x) = 8 x^2 - 1582 x + k, where k is an arbitrary constant.**	0.5
	At x = 0, the total cost is equal to the fixed cost which is Rs 1800. Hence, k = 1800. Thus, the total cost function is 8 x^2 - 1582 x + 1800.	0.5
19	**Integrates the given marginal revenue function, 2 - 6 x, with respect to x to get the total revenue function as TR(x) = 2 x - 3 x^2 + k, where k is an arbitrary constant.**	0.5
	At x = 0, the total revenue is equal to Rs 0. Hence, k = 0. **Thus, the total revenue function is 2 x - 3 x^2.**	0.5
	Uses the total cost function found in the previous question and the above steps to find the profit function as: **2 x - 3 x^2 - (8 x^2 - 1582 x + 1800)** **= -11 x^2 + 1584 x - 1800**	1

Q.No	Teacher should award marks if students have done the following:	Marks
20	**Finds the first derivative of the profit function found in the previous question as $\frac{dx}{dP}$ = -22 *x* + 1584.**	**0.5**
	Calculates the critical point as *x* = 72 by equating -22 *x* + 1584 to 0.	**0.5**
	Finds the second derivative as: $$\frac{d^2x}{dP^2} = -22$$	**0.5**
	Concludes that the maximum profit that can be earned in one day is by selling 72 units of PPE kits.	**0.5**

Chapter - 9
Differential Equations

Q: 1 **A differential equation has an order of 3 and a degree of 2. Which of the following could this differential equation be?**

$$\frac{d^3y}{dx^3} - \left(\frac{d^2y}{dx^2}\right)^2 = 0$$

A

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 = 0$$

B

$$\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3 = 0$$

C

$$\left(\frac{d^3y}{dx^3}\right)^2 + \tan\left(\frac{d^2y}{dx^2}\right) = 0$$

D

1 A
2 B
3 C
4 D

Q: 2 **Look at the differential equation given below. What is its degree?**

$$y^3 = e^{4\frac{d^2y}{dx^2}}$$

1 1
2 2
3 3
4 4

Q: 3 **Shown below is a differential equation.**

$\sin x \frac{dy}{dx} + y \cos x = \tan x$

Which of the following is the integrating factor for the above differential equation ?

$e^{\int \sin(x)dx}$

Expression 1

$e^{\int \cos(x)dx}$

Expression 2

$e^{\int \cot(x)dx}$

Expression 3

1 Expression 1
2 Expression 2
3 Expression 3
4 (the equation has no integrating factor as it is not a linear differential equation)

Q: 4 **Which of the following differential equations will give $y = \sin x + \cos x + C$ as the general solution where C is an arbitrary constant?**

$$\frac{d^2y}{dx^2} = \cos x + \sin x \qquad \frac{d^2y}{dx^2} = \cos x - \sin x \qquad \frac{d^2y}{dx^2} = -\cos x - \sin x$$

Equation 1 Equation 2 Equation 3

1. Equation 1
2. Equation 2
3. Equation 3
4. (cannot say without knowing the value of C)

Q: 5 **Three friends - Bulbul, Ipsita and Sagarika were asked to find a particular solution of the following differential equation.**

$$\frac{d^2y}{dx^2} + y = 0$$

Shown below are their solutions.

Bulbul: $y = \sin x$
Ipsita: $y = \cos x$
Sagarika: $y = \sin x + \cos x$

Whose answer is correct?

1. only Bulbul
2. only Sagarika
3. only Bulbul and Ipsita
4. All - Bulbul, Ipsita and Sagarika

Q: 6 **Read the statements carefully and choose the option that correctly describes them.**

Statement 1 : The differential equation that represents the family of straight lines with slope 7 is given by $y = 7x + c$, where c is an arbitrary constant.

Statement 2 **: $\frac{dy}{dx} = 7$ is a differential equation of order 0 as it contains no arbitrary constant.**

1. Statement 1 is true but Statement 2 is false
2. Statement 1 is false but Statement 2 is true
3. Both Statement 1 and Statement 2 are true
4. Both Statement 1 and Statement 2 are false

Q: 7 What is the order of the differential equation whose solution is given below? **[1]**

y = (R - S)cosec (x + T) - Ue$^{2x^{-V}}$, where R, S, T, U and V are some constants.

Show your steps and give a valid reason.

Q: 8 Three students were discussing the nature of the differential equation given below. **[2]**

$x + y\frac{dy}{dx} = b$, where b is a constant.

Samreen said, "This equation represents a family of straight lines".

Jamal said, "This equation represents a family of circles".

Kavya said, "This equation represents a family of ellipses."

Who is correct? Justify your answer.

Q: 9 The first derivative of a function is given below: **[2]**

$$\frac{-1}{x^2(1+x^2)}$$

Find the function. Show your steps.

Q: 10 The rate of decay of a radioactive isotope is given by $\frac{dN}{dt}$ = -kN, where k is a positive constant and N is the quantity (in grams) of the radioactive material available at time t (in days). k is equal to $\frac{0.693}{T}$ where T is the half-life (in days) of the isotope. **[3]**

If the initial quantity of the radioactive material is 45 g and its half-life is 3 days, find the quantity of the radioactive material left after 10 days. Show your steps.

(Note: Take the value of e $^{-2.31}$ as 0.099.)

Q: 11 In a research experiment, the population of fruit flies grows at the rate of 20% every day. **[3]**

i) Frame a differential equation depicting the above experiment.
ii) Find the time taken for the population to triple.

Show your steps.

Q: 12 **$y = f(x)$ is a curve such that, at every point on the curve, the slope is half the product of the coordinates of that point.** **[3]**

i) Frame a differential equation that represents the above curve.
ii) Find the general solution of the differential equation obtained in part i). Show your work.

Q: 13 **Find the general solution of the given differential equation. Show your steps.** **[3]**

$$\frac{dy}{dx} = ye^{\ln\frac{1}{x}} + 1$$

Q: 14 **Ankita bought a car for Rs y. The price of the car, P(t) after t years depreciates as per the equation $\frac{dP}{dt} = -a(T - t)$ where T is the total life of the car (in years) and a is an arbitrary constant.** **[3]**

Find the value of the car when the car has been used for T years, P(T), in terms of y. Show your steps.

Q: 15 **Find the general solution of the following differential equation.** **[5]**

$$\sin x \frac{dy}{dx} = \left(y + \sqrt{\sin^2 x - y^2}\right) \cos x$$

Show your steps.

Q: 16 **Show that the given differential equation is homogeneous and solve it.** **[5]**

$$\frac{dy}{dx} = \frac{y}{x} + \cos\frac{y}{x}$$

Show your steps.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	1
3	3
4	3
5	4
6	4

Q.No	Teacher should award marks if students have done the following:	Marks
7	**Rewrites the given equation in terms of arbitrary constants as:** $y = C_1 \text{ cosec } (x + C_2) - C_3 e^{2x}$ **where,** $C_1 = (R - S)$, $C_2 = T$ **and** $C_3 = Ue^{-V}$	0.5
	Writes that since the order of a differential equation is same as the number of arbitrary constants, the order of the given differential equation is 3.	0.5
8	**Writes that Joseph is correct.**	0.5
	Rewrites the given differential equation as: $y dy = (b - x)\, dx$	0.5
	Integrates both sides of the above equation to get: $\frac{y^2}{2} = bx - \frac{x^2}{2} + K$ $\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} - bx = K$ **where K is an arbitrary constant.** **Concludes that the above equation represents a family of circles.**	1
9	**Writes the derivative as:** $\frac{dy}{dx} = \frac{1}{1+x^2} - \frac{1}{x^2}$	0.5
	Rewrites both sides of the equation as: $\int dy = \int \frac{1}{(1+x^2)} dx - \int \frac{1}{x^2} dx$	0.5
	Integrates the above equation to get the function as: $y = \tan^{-1}(x) + \frac{1}{x} + C$	1

Q.No	Teacher should award marks if students have done the following:	Marks
10	Solves the differential equation: $\int \frac{dN}{N} = -k \int dt$ and writes $N = Ce^{-k\,t}$ as the solution where C is an arbitrary constant.	1
	Writes that, at $t = 0$, N = 45 g and hence finds C as 45.	0.5
	Calculates k as $\frac{0.693}{3} = 0.231$	0.5
	Calculates the quantity of the radioactive material left after 10 days as: $N = 45e^{-0.231 \times 10} = 45 \times 0.099 = 4.45$ g	1
11	i) Frames the differential equation depicting the given experiment as: $\frac{dP}{dt} = 0.2P$ $\Rightarrow \frac{dP}{P} = 0.2\,dt$ where P is the population at time t (in days).	0.5
	Integrates the above equation to get: $\log_e P = 0.2\,t + C$, where C is an arbitrary constant.	0.5
	Assumes that, at $t = 0$, $P = P_0$ and gets $C = \log_e P_0$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses steps 2 and 3 and writes: $log_e P = 0.2t + log_e P_0$ $\Rightarrow log_e \frac{P}{P_0} = 0.2t$ **(Award full marks if the equation is derived in exponential form, P = P_0 e $^{(0.2t)}$.)**	0.5
	Finds the time taken for the population to triple as: $log_e \frac{3P_0}{P_0} = 0.2t$ $\Rightarrow t = (5log_e 3)$ days	1
12	**i) Frames the differential equation representing the given curve as:** $\frac{dy}{dx} = \frac{1}{2}(xy)$	1
	ii) Separates the variables of the differential equation obtained in step 1 as: $\frac{2}{y} dy = x\, dx$	0.5
	Integrates the equation obtained in step 2 to get: $2logy = \frac{x^2}{2} + c,$ where c is an arbitrary constant **Simplifies the above equation to get the general solution as:** $y = Ce^{(\frac{x^2}{4})},$ where $C = e^{(\frac{c}{2})}$ is an arbitrary constant. **(Award full marks if the final answer is written as 2log y = x^2/2 + c.)**	1.5

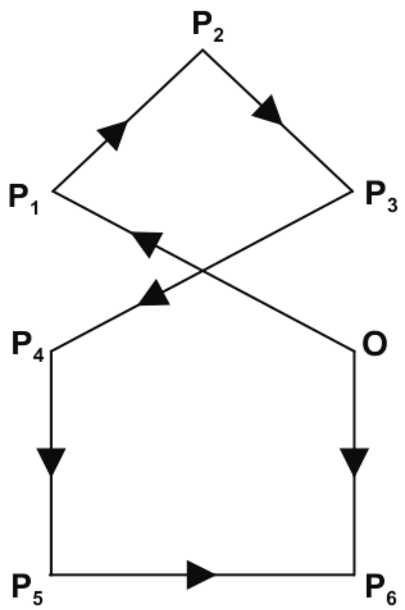
Q.No	Teacher should award marks if students have done the following:	Marks
13	Simplifies and writes the linear differential equation as $\frac{dy}{dx} - \frac{y}{x} = 1$.	0.5
	Finds the integrating factor as: $e^{-1\int \frac{1}{x}dx} = e^{\ln x^{-1}}$ or $\frac{1}{x}$	1
	Writes the solution as: $y \times \frac{1}{x} = \int 1 \times \frac{1}{x}\, dx$	1
	Simplifies the solution as: $\frac{y}{x} = \ln x + C$ (Award full marks if any other form of this equation is written.)	0.5
14	Rewrites the given differential equation as: $\int dP = -a\int (T-t)\, dt$	0.5
	Integrates the above equation as: $P(t) = \frac{at^2}{2} - atT + C$ where C is an arbitrary constant.	1
	Writes that, at $t = 0$, P(t) = y so C = y.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the value of the car as: $P(T) = \frac{aT^2}{2} - aT^2 + y = y - \frac{aT^2}{2}$	**1**
15	**Substitutes sin *x* as *t* and finds $\frac{dx}{dt} = \frac{1}{\cos x}$.**	**0.5**
	Writes that from the given equation and $\frac{dx}{dt} = \frac{1}{\cos x}$ we can find: $\frac{dy}{dt} = \frac{y + \sqrt{t^2 - y^2}}{t}$ **where *t* = sin *x*.**	**1.5**
	Substitutes *y* = *vt* and gets $\frac{dy}{dt} = v + t\frac{dv}{dt}$.	**0.5**
	Rewrites the equation as: $v + t\frac{dv}{dt} = v + \sqrt{(1 - v^2)}$ **where** $v = \frac{y}{t}$ **and reduces it to:** $\frac{dv}{\sqrt{1 - v^2}} = \frac{dt}{t}$	**1**
	Solves the differential equation as: $\sin^{-1}(v) = \ln\|t\| + C$ **where C is an arbitrary constant.**	**0.5**
	Substitutes the value of *v* as $\frac{y}{t}$ and gets the solution as: $\sin^{-1}\left(\frac{y}{t}\right) = \ln\|t\| + C$	**0.5**
	Substitutes the value of *t* as sin *x* and gets the final solution as: $\sin^{-1}\left(\frac{y}{\sin x}\right) = \ln\|\sin x\| + C$	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
16	Takes $F(x, y) = \frac{y}{x} + \cos\frac{y}{x}$ and finds $F(\lambda x, \lambda y)$ as: $F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + \cos\frac{\lambda y}{\lambda x}$ => $F(\lambda x, \lambda y) = \lambda^0 F(x, y)$ Hence, concludes that the given differential equation is homogeneous.	1
	Takes $y = vx$ and differentiates it to get $\frac{dy}{dx} = v + x\frac{dv}{dx}$.	1
	Rewrites the given differential equation using the above step as: $\sec v \, dv = \frac{1}{x} dx$	1
	Integrates the above equation as: $\log(\sec v + \tan v) + \log C = \log x$ where, C is an arbitrary constant.	1
	Simplifies the above equation as: $C(\sec v + \tan v) = x$	0.5
	Replaces $y = vx$ in the above equation to find the general solution of the given differential equation as: $x = C\left[\sec\frac{y}{x} + \tan\frac{y}{x}\right]$	0.5

Chapter - 10
Vector Algebra

Q: 1 **Observe the vector diagram given below.**



Which of the following is/are equal to

$\overrightarrow{OP_1} + \overrightarrow{P_1P_2} + \overrightarrow{P_2P_3} + \overrightarrow{P_3P_4} + \overrightarrow{P_4P_5} + \overrightarrow{P_5P_6}$?

i) $\overrightarrow{P_6O}$
ii) $\overrightarrow{OP_6}$
iii) $-\overrightarrow{OP_6}$
iv) $-\overrightarrow{P_6O}$

1 only i) **2** only ii) **3** only i) and iii) **4** only ii) and iv)

Q: 2

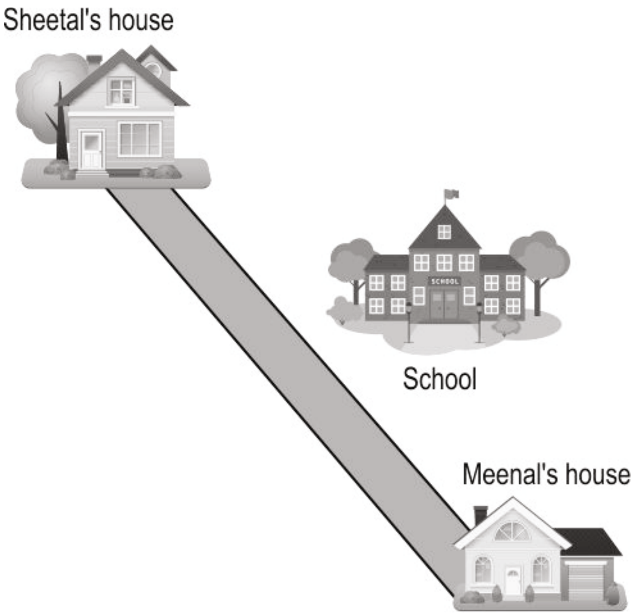
$\vec{u} = \hat{i}$, $\vec{v} = \hat{j}$ and $\vec{w} = \hat{k}$ are unit vectors. Which of the following is the angle between $(\vec{v} \times \vec{u})$ and $\vec{w}$?

1 0°
2 90°
3 180°
4 (cannot be found without knowing the angle between the vectors *v* and *u*)

Q: 3 Under which of the following conditions will the magnitude of the cross product of two vectors, $\vec{a}$ and $\vec{b}$, attain its maximum value?

1. When $\vec{a}$ is parallel to $\vec{b}$
2. When $\vec{a}$ is perpendicular to $\vec{b}$
3. When $\vec{a}$ is equal to and parallel to $\vec{b}$
4. When the magnitudes of $\vec{a}$ and $\vec{b}$ are equal but they are in opposite direction

Q: 4 **Sheetal and Meenal walk to school from two ends of a street in a straight line path.**



(Note: The figure is not to scale.)

At a certain time, Sheetal's velocity vector is $2\hat{i} - 3\hat{j}$ m/s.

Which of the following could be Meenal's velocity vector (in m/s) at that time?

1. $4\hat{i} - 6\hat{j}$
2. $\hat{i} + \frac{3}{2}\hat{j}$
3. $-\hat{i} + \frac{3}{2}\hat{j}$
4. (cannot say without knowing Meenal's velocity)

Q: 5 The following question was given in a class test. **[1]**

"If $\vec{p} = 5\hat{i}$, $\vec{q} = -2\hat{j}$ and $\vec{r} = 3\hat{k}$, then find $(\vec{p} \times \vec{q}) + (\vec{q} \times \vec{r}) + (\vec{r} \times \vec{p})$."

Aditi wrote the answer as 0.

Karn wrote the answer as $-10\hat{i} - 6\hat{j} + 15\hat{k}$.

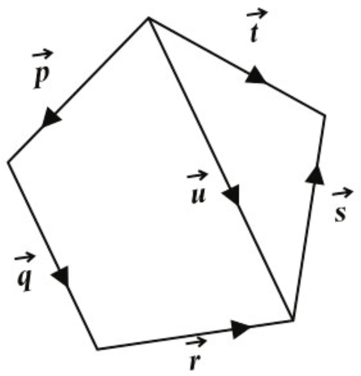
Manisha wrote the answer as $-6\hat{i} + 15\hat{j} - 10\hat{k}$.

Jamal wrote the answer as $6\hat{i} - 15\hat{j} + 10\hat{k}$.

Who was right? Justify your answer.

Q: 6 **[1]**

Two vertices of a regular pentagon are joined by a vector u (as shown below), followed by two statements. Identify if the statements are true or false. Give valid reasons for your answer.



i) $\vec{p} = \vec{q} = \vec{r} = \vec{s} = \vec{t}$

ii) There are three coinitial vectors.

Q: 7 **State whether the following statement is true or false. If true, give a reason. If false, give an example.** **[1]**

Two collinear vectors are always equal.

Q: 8 **[1]**

Vectors $\overrightarrow{OA} = 2\hat{i} + 6\hat{k}$ and $\overrightarrow{OB} = 4\hat{i} - 2\hat{j} + 4\hat{k}$ are two sides of a $\triangle OAB$, where O is the origin. Find the length of the median $\overrightarrow{OC}$.

Show your work with a valid reason.

Q: 9 A parallelogram, ABCD, is constructed such that its adjacent sides, AB and AD, are $3\vec{a} - 5\vec{b}$ and $\vec{a} - 2\vec{b}$ respectively. $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = \sqrt{8}$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. **[2]**

Find the length of the diagonal BD. Show your steps.

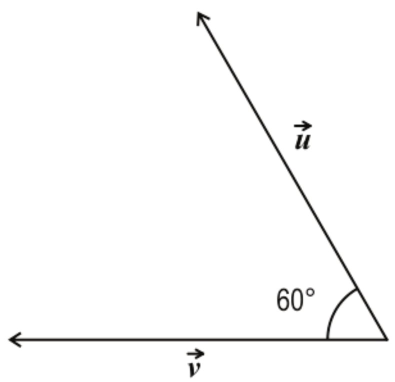
Q: 10 Two vectors are orthogonal if they are perpendicular to each other. **[2]**

Determine whether $\vec{u} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 5\hat{i} - 2\hat{j} + \hat{k}$ are orthogonal vectors. Show your steps and give valid reasons.

Q: 11 If $\vec{u} = (t-1)\hat{i} + 2\hat{j} + (t-3)\hat{k}$ is rotated about the origin in a clockwise direction to obtain $\vec{v} = \hat{i} + (t-1)\hat{j} + 2\hat{k}$, where t is a real number, find the possible values of t. **[2]**

Show your steps and give valid reasons.

Q: 12 In the figure below, the magnitudes of vectors *u* and *v* are 5 and 3 respectively. **[2]**



(Note: The figure is not to scale.)

Using the dot product, find the magnitude of the vector (*u* + *v*). Show your steps.

Q: 13 The projection of vector $\vec{a} = 3\hat{i} + q\hat{j} - \hat{k}$ on vector $\vec{b} = \hat{i} + \sqrt{2}\hat{j} + p\hat{k}$ is 1, where p and q are natural numbers. **[2]**

If $|\vec{b}| = \sqrt{12}$, find:

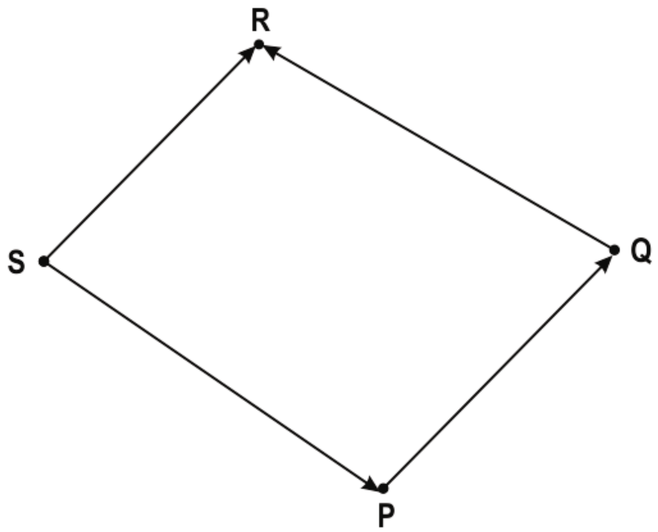
i) p

ii) q

Show your steps.

Q: 14 Find the angles made by the vector, $2\hat{i} - 3\hat{j} + \hat{k}$, with the coordinate axes. Show your steps. **[2]**

Q: 15 In the figure below, point S is at the origin. The position vectors of points P, Q and R are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. **[3]**



Prove that $\overrightarrow{PQ} \times \overrightarrow{RS} + \overrightarrow{QR} \times \overrightarrow{PS} + \overrightarrow{RP} \times \overrightarrow{QS} = 2(\overrightarrow{SR} \times \overrightarrow{SQ} + \overrightarrow{SP} \times \overrightarrow{SR} + \overrightarrow{SQ} \times \overrightarrow{SP})$.

Q: 16 Using vector operations, find the area of a parallelogram RSTU with vertices R(-1, 2, 3), S(-2, 5, -1) and U(0, 0, 3). Show your steps. **[3]**

Q: 17 A vector $\vec{v}$ has a magnitude of 2, makes an angle of $\frac{\pi}{6}$ with $\hat{i}$, an angle of θ with $\hat{j}$ and an angle of $\frac{\pi}{3}$ with $\hat{k}$. Find: **[3]**

i) θ, where $0 \le \theta \le 90°$

ii) $\vec{v}$ in its component form

Show your steps.

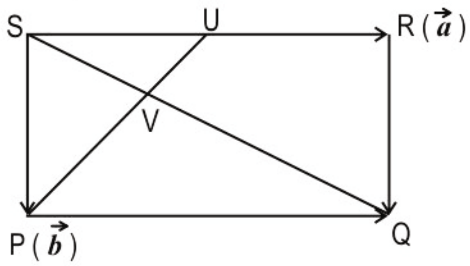
Q: 18 **PQRS is a rhombus. Points X and Y are the midpoints of the sides SP and PQ respectively.** **[3]**

Draw a vector diagram and prove that:

$$\overrightarrow{RX} + \overrightarrow{RY} = \frac{3}{2}\overrightarrow{RP}$$

Q: 19 **[5]**

In the rectangle PQRS below, S is at the origin and the position vectors of the points P and R are $\vec{b}$ and $\vec{a}$ respectively.



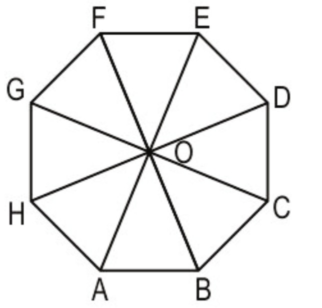
(Note: The figure is not to scale.)

i) Point U bisects SR. Using the vector method, prove that V divides PU and QS in the same ratio.

ii) Find the ratio in which point V divides QS.

Show your steps and give valid reasons.

Q: 20 **Shown below is a regular octagon, ABCDEFGH, with centre at O.** **[5]**



Show that $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AD} - \overrightarrow{BC})$.

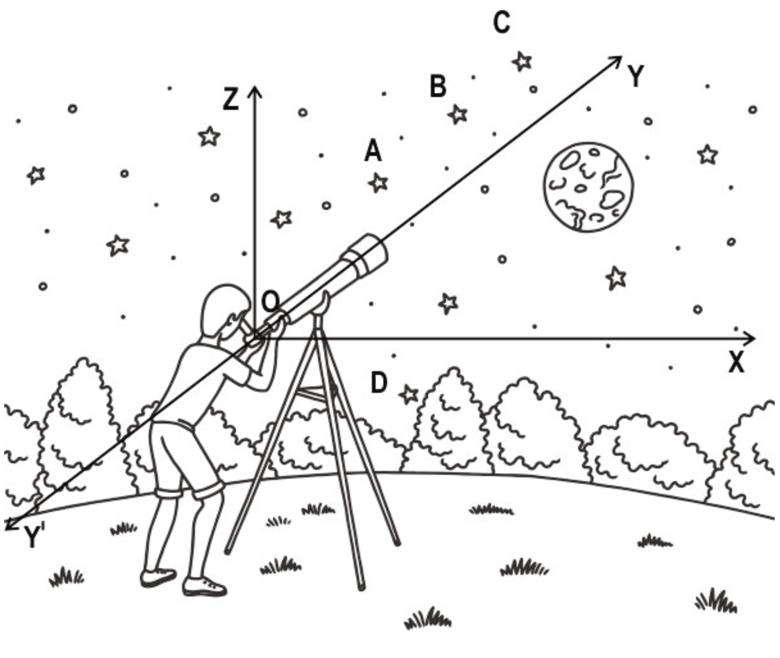
Q: 21 **PQR is a right-angled isosceles triangle with ∠Q = 90°. O is a point on the hypotenuse of ▲PQR and S is a fixed point outside the triangle such that:** **[5]**

$$\overrightarrow{OS} = \overrightarrow{QO} + \overrightarrow{OR} + \overrightarrow{OP}$$

Draw a vector diagram and prove that PQRS is a square.

Read the given information and answer the questions that follow.

On a certain night, Ziad was observing stars using his telescope.



(Note: Image is purely for representation purposes.)

Considering the eyepiece of the telescope as the origin, the position vectors of some of the stars are given below:

A $(2\hat{i} + 3\hat{j} + 4\hat{k})$

B $(5\hat{i} + 6\hat{j} + 8\hat{k})$

C $(8\hat{i} + 9\hat{j} + 12\hat{k})$

D $(2\hat{i} + 3\hat{j} - 6\hat{k})$

Q: 22 **Show that the stars with position vectors A, B and C are collinear.** **[2]**

Q: 23 Express the vector $\overrightarrow{AD}$ in terms of its components and find the distance between the stars A and D. **[1]**

(*Note: 1 unit = 1 light year.*)

Q: 24 **A star at a certain position is inclined at angles of 45° and 60° with the positive direction of the *x* -axis and *y* -axis respectively.** **[2]**

If the star is 2 light years away from O, find its position vector.

(Note: 1 unit = 1 light year.)

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	3
3	2
4	3

Q.No	Teacher should award marks if students have done the following:	Marks
5	**Writes that Manisha was right.**	0.5
	Justifies the above answer as follows: $(\vec{p} \times \vec{q}) + (\vec{q} \times \vec{r}) + (\vec{r} \times \vec{p})$ $= (5\hat{i} \times -2\hat{j}) + (-2\hat{j} \times 3\hat{k}) + (3\hat{k} \times 5\hat{i})$ $= -10(\hat{i} \times \hat{j}) - 6(\hat{j} \times \hat{k}) + 15(\hat{k} \times \hat{i})$ $= -6\hat{i} + 15\hat{j} - 10\hat{k}$	0.5
6	**i) Writes False(F). Gives the reason that all the five vectors mentioned in i) have different directions.**	0.5
	ii) Writes True(T). Gives the reason that the following three vectors have the same initial point. $\vec{p}$, $\vec{u}$ and $\vec{t}$	0.5
7	**Writes false.**	0.5
	Gives an example: $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 6\hat{i} + 9\hat{j} + 12\hat{k}$ are collinear vectors that are not equal.	0.5
8	**Writes that, since $\overrightarrow{OC}$ is the median, C is the midpoint of $\overrightarrow{AB}$. Hence, writes $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}) = \frac{1}{2}(6\hat{i} - 2\hat{j} + 10\hat{k}) = 3\hat{i} - \hat{j} + 5\hat{k}$.**	0.5

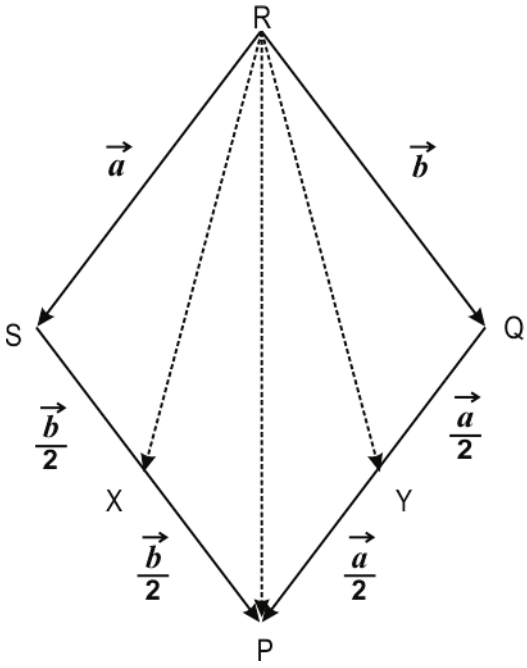
Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the length of the median as $\lvert\overrightarrow{OC}\rvert = \sqrt{(9 + 1 + 25)} = \sqrt{35}$ units.	0.5
9	**Writes that the length of the diagonal, BD is represented by:** $\lvert\overrightarrow{BD}\rvert = \lvert 3\vec{a} - 5\vec{b} - (\vec{a} - 2\vec{b})\rvert = \lvert 2\vec{a} - 3\vec{b}\rvert$	0.5
	Finds $\left\lvert\vec{BD}\right\rvert^2$ as follows: $(2\vec{a} - 3\vec{b}).(2\vec{a} - 3\vec{b})$ $= 4\left\lvert\vec{a}\right\rvert^2 - 12(\vec{a}.\vec{b}) + 9\left\lvert\vec{b}\right\rvert^2$ $= 8 - 12(\sqrt{2} \times \sqrt{8} \times \cos\frac{\pi}{3}) + 72$ $= 80 - 24$ $= 56$	1
	Finds the length of the diagonal, BD as √56 units.	0.5
10	**Finds the dot product of the given two vectors as:** $\vec{u}\,.\,\vec{v} = (1)(5) + (3)(-2) + (1)(1) = 0$	0.5
	Finds the angle between $\vec{u}$ and $\vec{v}$ as follows: $\vec{u}\,.\,\vec{v} = 0$ $\Rightarrow \lvert\vec{u}\rvert\,\lvert\vec{v}\rvert \cos\theta = 0$ As $\lvert\vec{u}\rvert \neq 0$ and $\lvert\vec{v}\rvert \neq 0$, $\cos\theta = 0$. $\Rightarrow \theta = 90°$	1
	Concludes that $\vec{u}$ and $\vec{v}$ are orthogonal vectors.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
11	**Writes that the magnitudes of the two vectors will be the same even after rotation.** **Hence, writes that:** $\lvert\vec{u}\rvert = \lvert\vec{v}\rvert \Rightarrow \lvert\vec{u}\rvert^2 = \lvert\vec{v}\rvert^2$	0.5
	Uses the above step and writes: $(t - 1)^2 + 2^2 + (t - 3)^2 = 1^2 + (t - 1)^2 + 2^2$	0.5
	Simplifies the above equation as: $(t - 3) = \pm 1$	0.5
	Simplifies the above equation to find the values of *t* as 2 and 4.	0.5
12	Finds $\lvert\vec{u} + \vec{v}\rvert^2$ as follows : $= (\vec{u} + \vec{v}).(\vec{u} + \vec{v})$ $= \lvert\vec{u}\rvert^2 + 2(\vec{u}.\vec{v}) + \lvert\vec{v}\rvert^2$ $= 25 + 2(5 \times 3 \times \cos 60^\circ) + 9$ $= 49$	1.5
	Finds the magnitude of the vector $(\vec{u} + \vec{v})$ as $\sqrt{49}$ or 7 units.	0.5
13	**i) Uses magnitude of vector *b* to find *p* as:** $\lvert\vec{b}\rvert = \sqrt{1 + (\sqrt{2})^2 + p^2} = \sqrt{12}$ $\Rightarrow p^2 = 9$ $\Rightarrow p = 3$ as $p > 0$ **(Award only 0.5 marks if the formula to find the magnitude of the vector is correctly written.)**	1

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Uses the value of *p* found in step 1 and the formula for the projection of a vector to find *q* as: $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\lvert\vec{b}\rvert}$ $\Rightarrow \frac{(3)(1) + (q)(\sqrt{2}) + (-1)(3)}{\sqrt{12}} = 1$ $\Rightarrow q = \frac{\sqrt{12}}{\sqrt{2}} = \sqrt{6}$ **(Award only 0.5 marks if the formula to find the projection of a vector on another vector is correctly written.)**	1
14	**Finds the magnitude of the vector as √(4 + 9 + 1) or √14 units.**	0.5
	Finds the direction cosines of the vector as $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$.	0.5
	Writes that the given vector makes angles: $\cos^{-1}\frac{2}{\sqrt{14}}$ **with *x* -axis** $\cos^{-1}\frac{-3}{\sqrt{14}}$ **with *y* -axis** $\cos^{-1}\frac{1}{\sqrt{14}}$ **with *z* -axis**	1
15	Writes that $\overrightarrow{PQ} \times \overrightarrow{RS} = (\vec{b} - \vec{a}) \times (-\vec{c})$.	0.5
	Writes that $\overrightarrow{QR} \times \overrightarrow{PS} = (\vec{c} - \vec{b}) \times (-\vec{a})$.	0.5
	Writes that $\overrightarrow{RP} \times \overrightarrow{QS} = (\vec{a} - \vec{c}) \times (-\vec{b})$.	0.5

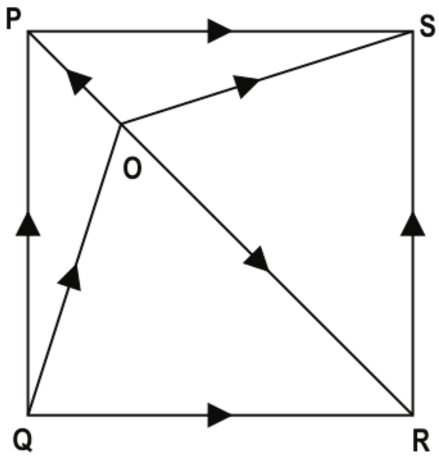
Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes the above expression in the LHS to prove the given RHS as: $= (\vec{PQ} \times \vec{RS}) + (\vec{QR} \times \vec{PS}) + (\vec{RP} \times \vec{QS})$ $= \{(\vec{b} - \vec{a}) \times (-\vec{c})\} + \{(\vec{c} - \vec{b}) \times (-\vec{a})\} + \{(\vec{a} - \vec{c}) \times (-\vec{b})\}$ $= (\vec{b} \times -\vec{c}) + (-\vec{a} \times -\vec{c}) + (\vec{c} \times -\vec{a}) + (-\vec{b} \times -\vec{a}) + (\vec{a} \times -\vec{b}) + (-\vec{c} \times -\vec{b})$ $= 2(\vec{c} \times \vec{b}) + 2(\vec{a} \times \vec{c}) + 2(\vec{b} \times \vec{a})$ $= 2(\vec{SR} \times \vec{SQ} + \vec{SP} \times \vec{SR} + \vec{SQ} \times \vec{SP})$ $= RHS$	1.5
16	**Finds $\vec{RS}$ as:** $(-2 + 1)\hat{i} + (5 - 2)\hat{j} + (-1 - 3)\hat{k} = -\hat{i} + 3\hat{j} - 4\hat{k}$	0.5
	Finds $\vec{RU}$ as: $(0 + 1)\hat{i} + (0 - 2)\hat{j} + (3 - 3)\hat{k} = \hat{i} - 2\hat{j}$	0.5
	Uses the cross product of the above two vectors as follows: $\vec{RS} \times \vec{RU} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -4 \\ 1 & -2 & 0 \end{bmatrix} = -8\hat{i} - 4\hat{j} - \hat{k}$	1.5
	Finds the area of the parallelogram RSTU as: $\lvert \vec{RS} \times \vec{RU} \rvert = \sqrt{64 + 16 + 1} = 9$ sq units	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
17	i) Finds v_1, the scalar component along the x-axis as follows: $\cos\frac{\pi}{6} = \frac{v_1}{\lvert\vec{v}\rvert}$ $\Rightarrow v_1 = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$	0.5
	Finds v_2, the scalar component along the y-axis as follows: $\cos\theta = \frac{v_2}{\lvert\vec{v}\rvert}$ $\Rightarrow v_2 = 2\cos\theta$	0.5
	Finds v_3, the scalar component along the z-axis as follows: $\cos\frac{\pi}{3} = \frac{v_3}{\lvert\vec{v}\rvert}$ $\Rightarrow v_3 = \frac{1}{2} \times 2 = 1$	0.5
	Uses above three steps and $\lvert\vec{v}\rvert = 2$ to find θ as: $\sqrt{v_1^2 + v_2^2 + v_3^2} = 2$ $\Rightarrow (\sqrt{3})^2 + 4\cos^2\theta + 1 = 4$ $\Rightarrow \cos^2\theta = 0$ $\Rightarrow \theta = 90° \text{ or } \frac{\pi}{2}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	ii) Writes $\vec{v}$ in its component form as: $= v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ $= \sqrt{3}\,\hat{i} + 2\cos\frac{\pi}{2}\,\hat{j} + \hat{k}$ $= \sqrt{3}\,\hat{i} + \hat{k}$	0.5
18	**Draws the vector representation of rhombus PQRS by taking RS and QP as vector *a*, SP and RQ as vector *b*. The diagram may look as follows:**  	0.5
	Finds $\overrightarrow{RX}$ and $\overrightarrow{RY}$ as: $\overrightarrow{RX} = \vec{a} + \frac{\vec{b}}{2}$ $\overrightarrow{RY} = \vec{b} + \frac{\vec{a}}{2}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds $\overrightarrow{RX}$ and $\overrightarrow{RY}$ as: $\vec{a} + \frac{\vec{b}}{2} + \vec{b} + \frac{\vec{a}}{2} = \frac{3}{2}(\vec{a} + \vec{b})$	0.5
	Finds $\overrightarrow{RP}$ as $\overrightarrow{RS} + \overrightarrow{SP} = (\vec{a} + \vec{b})$	0.5
	Uses steps 3 and 4 to conclude that: $\overrightarrow{RX} + \overrightarrow{RY} = \frac{3}{2}\overrightarrow{RP}$	0.5
19	**i) Uses the addition law of vectors to find the position vector of the point Q as:** $\vec{a} + \vec{b}$	0.5
	Finds the position vector of the point U as: $\frac{\vec{a}}{2}$	0.5
	Assumes that V divides PU in the ratio *m* :1 and uses the section formula to write the position vector of the point V in the ratio *m* :1 as: $\overrightarrow{SV} = \frac{m\left(\frac{\vec{a}}{2}\right) + 1(\vec{b})}{m+1} = \frac{m\vec{a} + 2\vec{b}}{2(m+1)}$ **(Award 0.5 marks if only the section formula in vector form is correctly written.)**	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Assumes that V divides QS in the ratio *n* :1 and uses the section formula to write the position vector of the point V in the ratio *n* :1 as: $\overrightarrow{SV} = \dfrac{n(\vec{0}) + 1(\vec{a} + \vec{b})}{n+1} = \dfrac{\vec{a} + \vec{b}}{(n+1)}$	1
	Argues that the above two vectors represent the position vector of the same point and equates the coefficients of vectors *a* and *b* to get: **1)** $\frac{m}{2(m+1)} = \frac{1}{n+1}$ **and** **2)** $\frac{1}{m+1} = \frac{1}{n+1}$ **Writes that *m* = *n* and concludes that V divides PU and QS in the same ratio.**	1
	ii) Substitutes *m* = *n* in equation 1) of the above step to get *m* = *n* = 2. **Concludes that the ratio is 2:1.**	1
20	Writes that, $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2\overrightarrow{AO} + 2\overrightarrow{OB} + 2\overrightarrow{CO} + 2\overrightarrow{OD}$.	1
	Simplifies the above equation as: $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AO} + \overrightarrow{OB}) + 2(\overrightarrow{CO} + \overrightarrow{OD})$ $= 2(\overrightarrow{AB}) + 2(\overrightarrow{CD})$ $= 2(\overrightarrow{AB} + \overrightarrow{CD})$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Adds and subtracts $\overrightarrow{BC}$ on the RHS of the above equation to get: $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} - \overrightarrow{BC})$	1
	Simplifies the above equation using the triangular law to get: $\overrightarrow{AE} + \overrightarrow{FB} + \overrightarrow{CG} + \overrightarrow{HD} = 2(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} - \overrightarrow{BC})$ $= 2(\overrightarrow{AC} + \overrightarrow{CD} - \overrightarrow{BC})$ $= 2(\overrightarrow{AD} - \overrightarrow{BC})$	2
21	**Draws the vector diagram that represents the given scenario. The figure may look as follows:**  	1
	Rearranges the given equation as: $\overrightarrow{OS} - \overrightarrow{OP} = \overrightarrow{QO} + \overrightarrow{OR}$ $\Rightarrow \overrightarrow{OS} + \overrightarrow{PO} = \overrightarrow{QO} + \overrightarrow{OR}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses vector addition and writes: $\overrightarrow{PS} = \overrightarrow{QR}$ $\Rightarrow$ PS = QR and PS || QR	1
	Similarly, concludes that: $\overrightarrow{SR} = \overrightarrow{PQ}$ $\Rightarrow$ SR = PQ and SR || PQ	1
	Uses steps 3, 4 and ∠Q = 90° to write ∠Q = ∠P = ∠R = ∠S = 90°.	0.5
	Uses steps 3, 4 and 5 to conclude that PQRS is a square.	0.5
22	Finds $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 3\hat{i} + 3\hat{j} + 4\hat{k}$.	0.5
	Finds $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 6\hat{i} + 6\hat{j} + 8\hat{k}$.	0.5
	Writes that $\frac{3}{6} = \frac{3}{6} = \frac{4}{8} = \frac{1}{2}$. Hence, $\overrightarrow{AC} = 2\overrightarrow{AB}$. Concludes that the stars with position vectors A, B and C are collinear.	1

Q.No	Teacher should award marks if students have done the following:	Marks
23	Expresses the vector $\overrightarrow{AD}$ in terms of its components as: $\overrightarrow{OD} - \overrightarrow{OA} = 0\hat{i} + 0\hat{j} - 10\hat{k}$	0.5
	Finds the distance between the stars A and D as √100 = 10 units = 10 light years.	0.5
24	**Finds the direction cosines of the star as:** $l = \cos 45° = \frac{1}{\sqrt{2}}$ $m = \cos 60° = \frac{1}{2}$ **where, *l* and *m* denotes the direction cosines with respect to the *x* -axis and *y* -axis respectively.**	0.5
	Uses the relation $l^2 + m^2 + n^2 = 1$ and finds the direction cosine with respect to the *z* -axis, *n*, as: $\frac{1}{2} + \frac{1}{4} + n^2 = 1$ $\Rightarrow n = \frac{1}{2}$ or $\frac{-1}{2}$	0.5
	Uses the given information along with the above steps and finds the position vector of the required star as: $\overrightarrow{OP} = \left\|\overrightarrow{OP}\right\|(l\hat{i} + m\hat{j} + n\hat{k})$ $= 2\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} \pm \frac{1}{2}\hat{k}\right)$ $= (\sqrt{2}\hat{i} + \hat{j} \pm \hat{k})$	1

Chapter - 11
Three Dimensional Geometry

Q: 1 **A line passes through (2, -1, -3) and (0, 2, 3).**

Which of the following are the direction cosines of a line parallel to the given line at a distance of 3 units from the given line?

1. $\frac{2}{7}$, $\frac{1}{7}$ and $\frac{0}{7}$
2. $\frac{-2}{7}$, $\frac{3}{7}$ and $\frac{6}{7}$
3. $\frac{2}{49}$, $\frac{1}{49}$ and $\frac{0}{49}$
4. $\frac{-2}{7}$ + 3, $\frac{3}{7}$ + 3 and $\frac{6}{7}$ + 3

Q: 2 **The direction ratios of two parallel lines are 3, λ, (-10) and (-5), 7, δ respectively. Which of the following are the values of λ and δ ?**

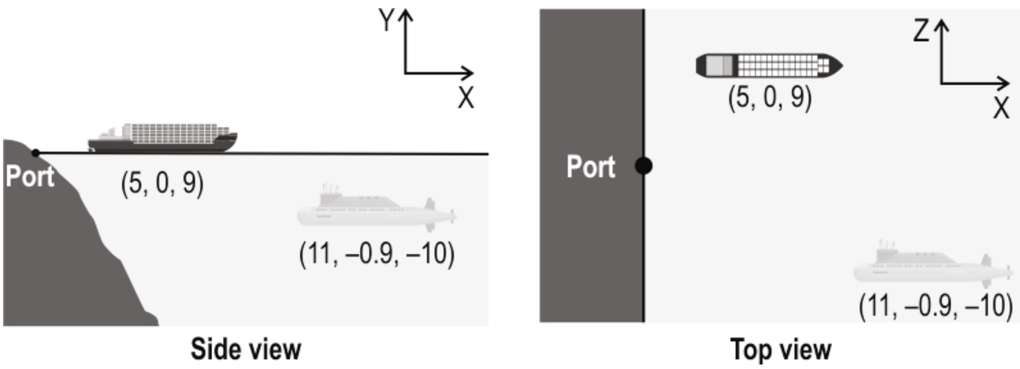
1. $\lambda = \frac{-35}{3}, \delta = 6$
2. $\lambda = \frac{3}{-10}, \delta = \frac{-5}{7}$
3. $\lambda = \frac{-21}{5}, \delta = \frac{50}{3}$
4. (cannot be found without knowing the distance between the two lines.)

Q: 3 **Which of the following is the sum of the direction cosines of the *y* -axis?**

1. 0
2. 1
3. 180°
4. (cannot say without being given two points on the *y* -axis)

Read the information given below and answer the questions that follow.

A supply ship left a port to replenish food and equipment for the Indian navy. Enemies used a submarine to track the ship and scout the port.



(Note: The figure is not to scale.)
Assuming the port to be at the origin, the position of the ship is (5 km, 0 km, 9 km) and the position of the submarine is (11 km, -0.9 km, -10 km).

Q: 4 **What angle does the line joining the ship and the port make with the *z* -axis?**

1. $\cos^{-1} 0$
2. $\cos^{-1} \pm 1$
3. $\cos^{-1} \pm(\frac{9}{\sqrt{106}})$
4. $\cos^{-1} \pm(\frac{10}{\sqrt{302}})$

Q: 5 **Which of the following is the cartesian equation of the line joining the ship to the submarine?**

1 $\frac{x}{6} = \frac{y}{-0.9} = \frac{z}{-19}$

2 $\frac{x-5}{6} = \frac{y}{-0.9} = \frac{z-9}{-19}$

3 $\frac{x-11}{6} = \frac{y+0.9}{-0.9} = \frac{z+10}{-19}$

4 $\frac{x-5}{16} = \frac{y}{-0.9} = \frac{z-9}{-1}$

Q: 6 **The submarine squad wants to find the direction cosines of the line joining the submarine to the port.**

The captain of the submarine says that the direction cosines are:

$\frac{-11}{\sqrt{221.81}}, \frac{0.9}{\sqrt{221.81}}, \frac{10}{\sqrt{221.81}}$

His subordinate says that the direction cosines are:

$\frac{11}{\sqrt{221.81}}, \frac{-0.9}{\sqrt{221.81}}, \frac{-10}{\sqrt{221.81}}$

Who is correct?

1 The captain **2** The subordinate **3** Both of them **4** Neither of them

Q: 7 **The distance between the port and the ship is √106 km, the distance between the submarine and the port is √221.81 km and the distance between the ship and the submarine is √397.81 km.**

Which of the following represents the angle between the lines joining the submarine to the ship and the line joining the submarine to the port?

1 $\cos^{-1}\left|\frac{-35}{\sqrt{106} \times \sqrt{221.81}}\right|$

2 $\cos^{-1}\left|\frac{201}{\sqrt{397.81} \times \sqrt{106}}\right|$

3 $\cos^{-1}\left|\frac{256.81}{\sqrt{397.81} \times \sqrt{221.81}}\right|$

4 $\cos^{-1}\left|\frac{2040}{\sqrt{397.81} \times \sqrt{106} \times \sqrt{221.81}}\right|$

Q: 8 **Which of the following is the equation of the line joining the submarine and the port?**

Equation 1 : $11\hat{i} - 0.9\hat{j} - 10\hat{k} + \lambda(-11\hat{i} + 0.9\hat{j} + 10\hat{k})$

Equation 2 : $\lambda(-11\hat{i} - 0.9\hat{j} - 10\hat{k})$

1 Equation 1
2 Equation 2
3 Both equation 1 and equation 2
4 Neither equation 1 nor equation 2

Q: 9 Given below are two parallel lines. **[1]**

$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+1}{-j}$ and $\frac{x+3}{k} = \frac{y+1}{-2} = \frac{z-3}{-2}$

Find the values of *k* and *j*. Show your work.

Q: 10 The direction cosines of a line are $\frac{3}{\sqrt{98}}, \frac{-5}{\sqrt{98}}, \alpha$. **[2]**

Find the angle between the line and the *z* -axis. Show your steps and give a valid reason.

Q: 11 The pair of lines given below are perpendicular to each other. **[2]**

$$\vec{r} = 2\hat{i} + 3\hat{j} + 7\hat{k} + \lambda(-\hat{i} + \hat{j} + \hat{k})$$
$$\vec{s} = 5\hat{i} - 3\hat{k} + \beta(2\hat{i} - \hat{k} + G\hat{j})$$

Find the value of G. Show your steps.

Q: 12 The distance between a point P and (-1, 2, 0) is 6√11 units. P also lies on the following line $\frac{x+1}{-3} = y - 2 = z$. **[3]**

Find the coordinates of P. Show your steps.

Q: 13 **[3]**

Two helicopters flying to Kedar Hills are moving in straight lines

represented by $2\hat{i} + 3\hat{j} + 2\hat{k} + (3\hat{i} + \hat{j} + 2\hat{k})$ and $\hat{i} + 2\hat{j} + 3\hat{k} + (3\hat{i} + \hat{j} + 2\hat{k})$ respectively.

Find the shortest possible distance between the helicopters during the flight. Show your steps and give a valid reason.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	3
3	2
4	3
5	2
6	3
7	3
8	1

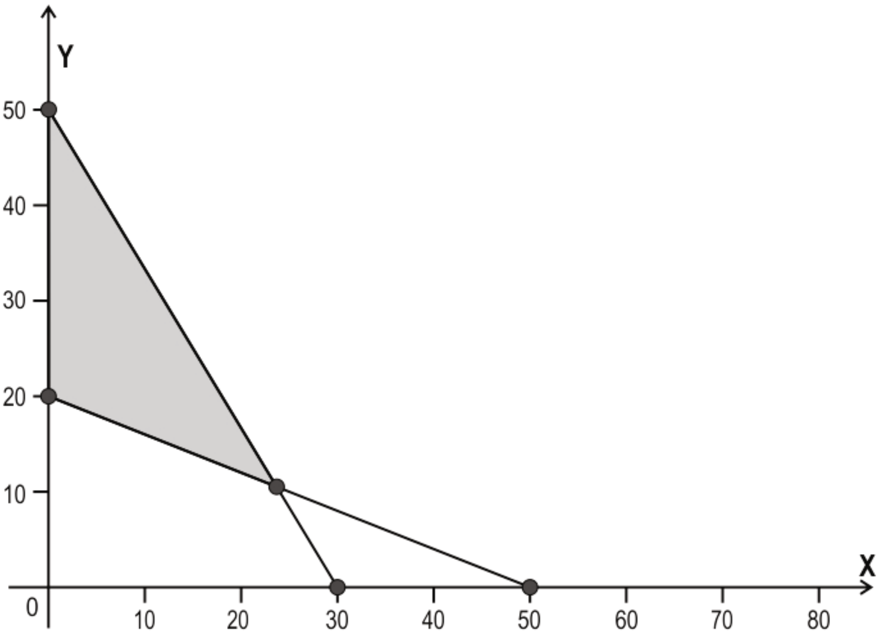
Q.No		Marks
9	**Writes that the lines are parallel and so $\frac{2}{k} = \frac{3}{-2} = \frac{-j}{-2}$.**	**0.5**
	Finds *k* as $\frac{-4}{3}$ and *j* as (-3).	**0.5**
10	**Writes that the sum of the squares of the direction cosines is equal to 1 and finds α as $\sqrt{1 - (\frac{9}{98} + \frac{25}{98})} = \frac{-8}{\sqrt{98}}$ or $\frac{8}{\sqrt{98}}$.** **(Award 1 mark if only the value of α is found correctly.)**	**1.5**
	Writes that the angle between the line and the *z* -axis is: $\cos^{-1}\frac{8}{\sqrt{98}}$ **or** $\cos^{-1}\frac{-8}{\sqrt{98}}$	**0.5**
11	**Assumes the angle between the two lines as θ and writes:** $\cos\theta = \left\lvert \frac{\vec{b_1}.\vec{b_2}}{\lvert\vec{b_1}\rvert\lvert\vec{b_2}\rvert} \right\rvert$ where, $\vec{b_1} = (-\hat{i} + \hat{j} + \hat{k})$ and $\vec{b_1} = (2\hat{i} + G\hat{j} - \hat{k})$.	**1**
	Substitutes θ as 90° and finds G as: $cos\ 90^\circ = 0 = \left\lvert \frac{(-1)(2)+(1)(G)+(1)(-1)}{\sqrt{3}\sqrt{5+G^2}} \right\rvert$ $\Rightarrow G = 3$	**1**
12	**Writes that the direction ratios of the given line are (*a* , *b* , *c*) = (-3, 1, 1).** **Writes that any point on the given line is:** **(-1 + λ *a* , 2 + λ *b* , 0 + λ *c*) = (-3λ - 1, λ + 2, λ), where λ is a parameter.**	**1**
	Uses the distance formula between (-1, 2, 0) and (-3λ - 1, λ + 2, λ) to find λ as: $(-3\lambda)^2 + \lambda^2 + \lambda^2 = (6\sqrt{11})^2$ $\Rightarrow \lambda = \pm 6$	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes λ = 6 and (-6) in (-3λ - 1, λ + 2, λ) to get (-19, 8, 6) and (17, -4, -6) respectively. **Concludes that the coordinates of P are (-19, 8, 6) or (17, -4, -6).**	**1**
13	**Writes that the helicopters are flying parallel to one another and the shortest distance, *d*, between them is:** $d = \left\lvert \frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{\lvert\vec{b}\rvert} \right\rvert$ Where $\vec{a}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{a}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$	**1**
	Simplifies the above expression as: $\left\lvert \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix}}{\sqrt{9+1+4}} \right\rvert$	**1**
	Evaluates the above expression as $\frac{\sqrt{19}}{\sqrt{7}}$ units.	**1**

Chapter - 12
Linear Programming

Q: 1 **Opticare Pvt. Ltd. is conducting an analysis of its operational costs, where labour cost is represented as *x* and the raw material is represented as *y*. The cost optimization was framed as a linear programming problem (LPP).**

Observe the graph of the feasible region of the cost optimization LPP shaded below.



Which of the following inequalities is one of the constraints of the LPP?

1. $5x + 3y \geq 150$
2. $5y + 3x \leq 150$
3. $2x + 5y \geq 100$
4. $2y + 5x \leq 100$

Q: 2 **A linear programming problem (LPP) along with its constraints is given below.**

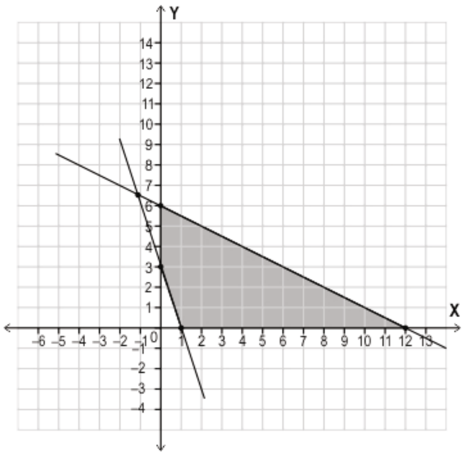
Maximise Z = 2 x + y

subject to the constraints:

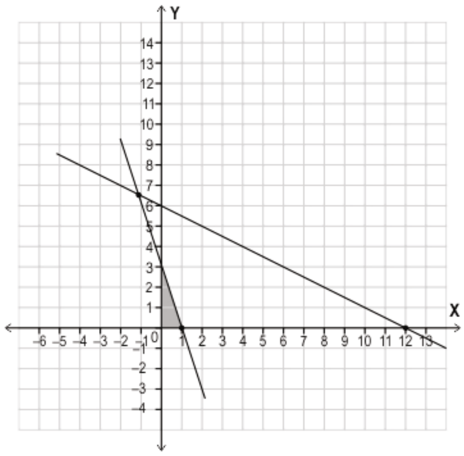
$x + 2y \leq 12$
$3x + y \geq 3$
$x \geq 0$
$y \geq 0$

Which option represents the feasible region of the above LPP?

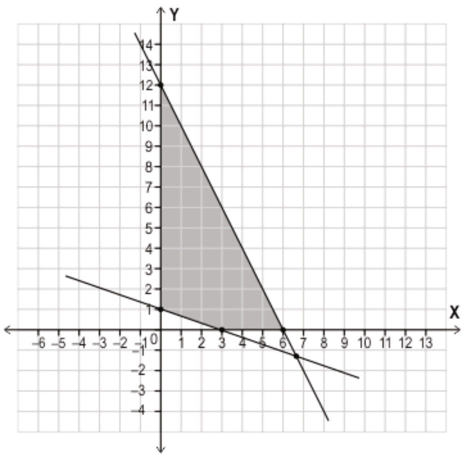
1



2



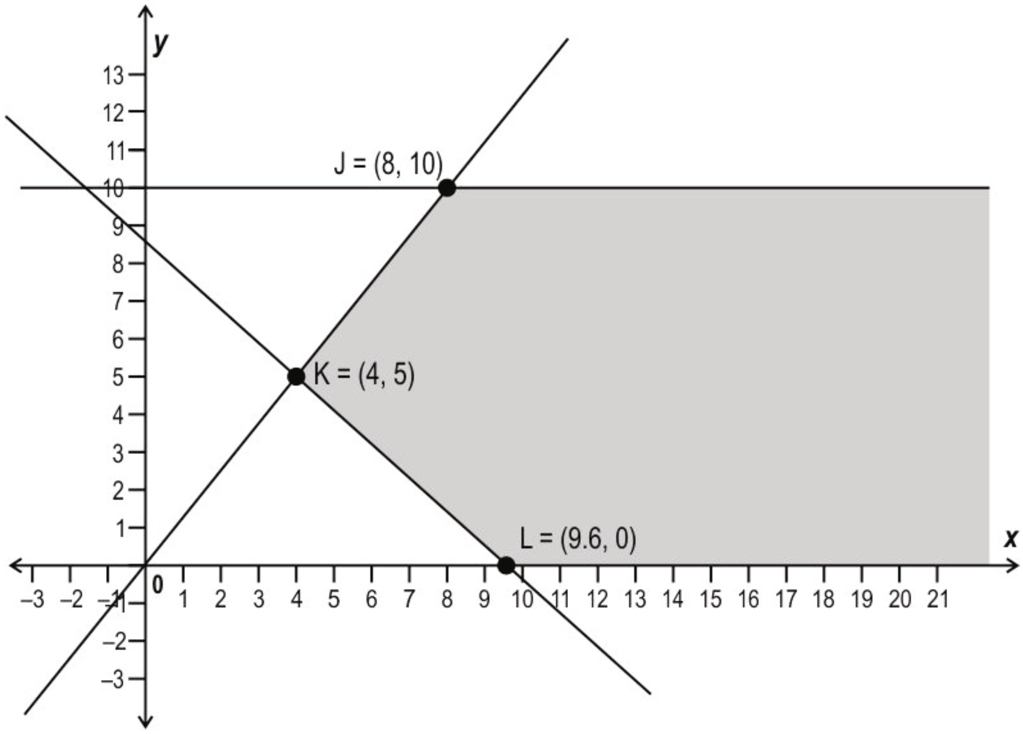
3



4 (there exists no feasible region for the given LPP)

Q: 3 **The objective function of a linear programming problem, along with its feasible region given below.**

Z = 10 *x* + 3 *y*

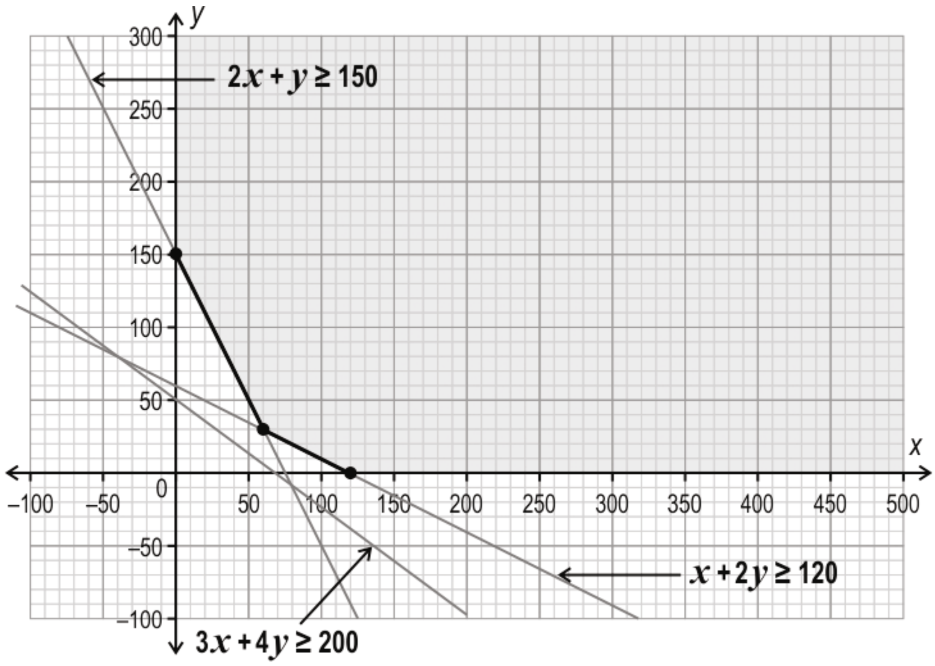


If J, K and L are the corner points, at which point does Z attain its minimum value?

1. Point J
2. Point K
3. Point L
4. (there exists no point at which Z attains its minimum value)

Q: 4 **The objective function of a linear programming problem (LPP), Z = 4x + 3y, has to be minimised.**

The feasible region of this LPP, along with its constraints, is shown in the graph below.



Which constraint, if removed, will not affect the feasible region?

1. $x + 2y \geq 120$
2. $2x + y \geq 150$
3. $3x + 4y \geq 200$
4. (Any of the given constraints, if removed, will affect the feasible region.)

Q: 5 **State whether the following statement is true or false. Justify your answer.** **[1]**

A linear programming problem can have infinitely many optimal solutions.

Q: 6 **A packaging company has the capacity to produce rectangular boxes and circular boxes. Each rectangular box takes 2 minutes to make and it sells for a profit of Rs 4. Each circular box takes 3 minutes to make and it sells for a profit of Rs 5.** **[1]**

Their client requests for 25 boxes to be ready in one hour. The packaging company wants to maximise their profit from this order.

Frame this optimisation problem as a linear programming problem.

Q: 7 **State whether the following statement is true or false. Justify your answer.** **[1]**

A linear programming problem whose feasible region is unbounded does not have an optimal solution.

Q: 8 **The objective function of a linear programming problem is given by Z = a x + b y ; where a and b are constants. If the minimum of Z occurs at two points, (50, 30) and (20, 40), find the relationship between a and b. Show your work.** **[2]**

Q: 9 **Sameer framed the following linear programming problem to minimise the monthly operational cost in running his bakery.** **[2]**

Minimise Z = 200 x + 300 y

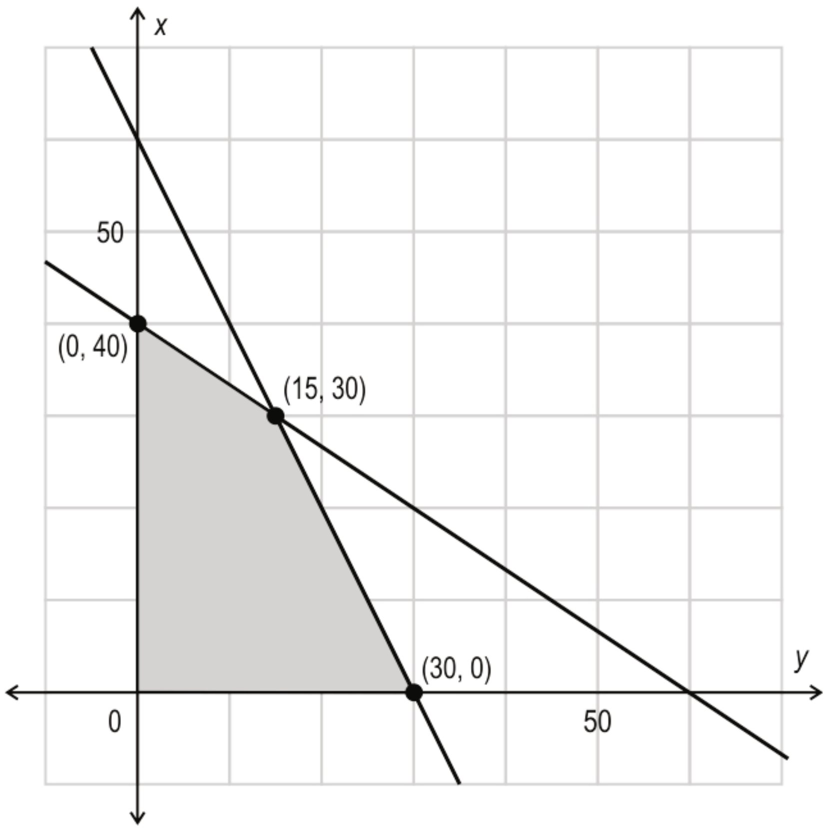
subject to the constraints:

$2x + 3y \geq 1200$
$1.5y + 2x \geq 900$
$x + y \leq 400$
$x, y \geq 0$

where x is the number of orders for bread loaves and y is the number of orders for cakes. Graph the feasible region for Sameer's LPP and find the optimal solution.

Q: 10 **The shaded region in the graph shown below represents the feasible region for a linear programming problem. The objective function of the LPP is:** **[2]**

Maximize Z = 5 *x* + 3 *y*

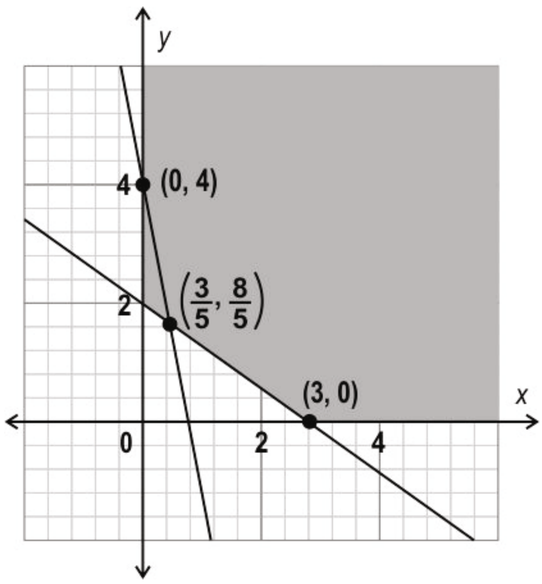


Find all the constraints for the above LPP. Show your work.

Q: 11 **A linear programming problem (LPP) along with the graph of its constraints is shown below. The shaded portion represents the feasible region.** **[2]**

Maximise: Z = 10 x + 20 y

Subject to:
$2x + 3y \geq 6$
$4x + y \geq 4$
$x \geq 0, y \geq 0$



What can you conclude about the existence of optimal solution for the above LPP? Justify your answer.

Q: 12 **A consultancy firm is preparing an office trip for its 400 employees. The bus rental company has 2 options for the buses that they can offer along with a maximum of 9 drivers. The capacity and rental costs are shown below.** **[3]**

Bus type	Maximum capacity	Rental cost (in Rs.)
Large	50	3000
Medium	40	2000

Determine the number of buses that the consultancy firm should book to minimise the total rental cost. Sketch the feasible region and show your steps.

Q: 13 **Ritvik owns two wholesale stores located in Noida and Gurgaon that supply shoes to retail stores in Dwarka, Saket and Greater Kailash in Delhi. The retail stores require 250, 350 and 400 pairs of shoes respectively in a month.** **[3]**

The distribution capacities of the wholesale stores at Noida and Gurgaon are 450 and 500 pairs respectively per month. The cost of transportation per pair is given below:

From	To Dwarka (in Rs)	To Saket (in Rs)	To Greater Kailash (in Rs)
Noida	10	15	12
Gurgaon	12	15	10

Assuming that the wholesale stores operate at their maximum distribution capacities, frame an objective function to minimise the cost of transportation of the shoes from the two wholesale stores to the three retail stores.

Q: 14 **Shaanta is a wholesale dry fruit trader who deals with figs and cashews.** **[5]**

The maximum capital available with her in a certain month is Rs 24,00,000. She has a warehouse with a maximum capacity to store 50 quintals of dry fruits at a time. Figs cost her Rs 40,000 per quintal and cashews Rs 60,000 per quintal. She earns a profit of Rs 2,000 per quintal on figs and Rs 3,000 per quintal on cashews.

How many quintals of figs and cashews should she purchase that month to make a maximum profit? Show your steps.

Q: 15 **A seller wants to pack Ajwa dates and Omani dates such that, in each packet, the combined weight is at most 15 kg and the weight of Omani dates is at most twice the weight of Ajwa dates. The seller anticipates a profit of Rs 55 per kg on Ajwa dates and Rs 70 per kg on Omani dates.** **[5]**

How many kilograms of Ajwa dates and Omani dates should be packed in each packet to attain maximum profit? Show your steps.

Answer the questions based on the given information.

Star Auto Private Limited (SAPL), an automobile company, manufactures and supplies precision machine components. Among other products, two products manufactured by SAPL are - Roller Bush and Hydraulic Valve shown below. These products are manufactured and packed in boxes of 25.


ROLLER BUSH


HYDRAULIC VALVE

Each box of Roller Bush requires 2 kg of aluminium and 1 kg of steel. Each box of Hydraulic Valve requires 14 kg of aluminium and 2 kg of steel. SAPL has an available supply of 70 kg of aluminium and 20 kg of steel per day to manufacture these two products. The plant makes a profit of Rs 20 on each box of Roller Bush and Rs 50 on each box of Hydraulic Valve.

Q: 16 **Write the mathematical formulation of the given scenario that gives the number of units of Roller Bush and Hydraulic Valve that SAPL must manufacture in order to maximize its profit.** **[1]**

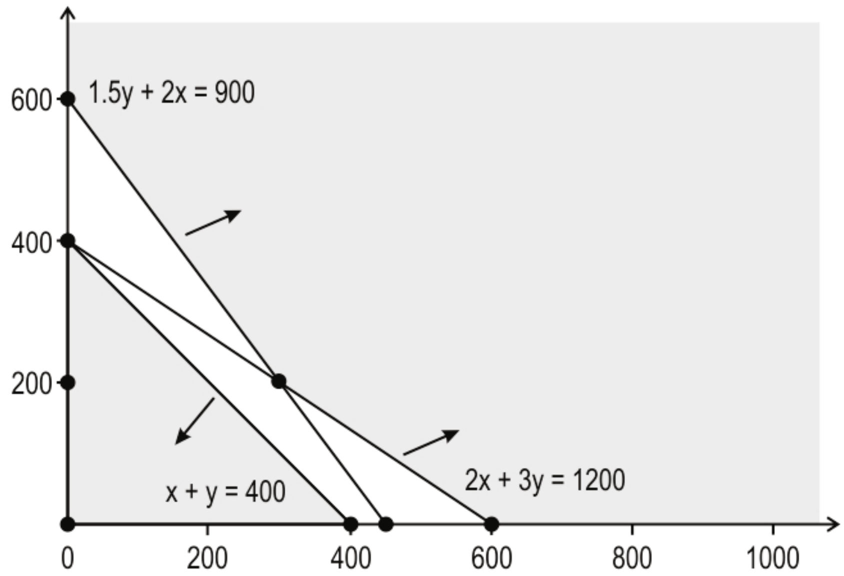
Q: 17 **Determine the feasible region of the linear programming problem defined by the above scenario graphically.** **[2]**

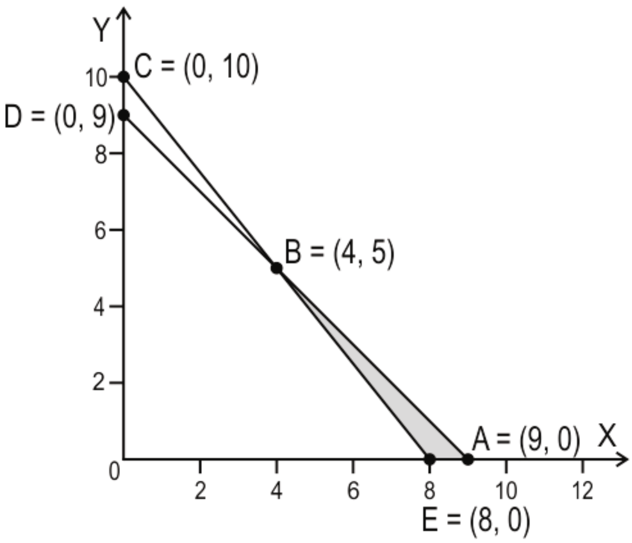
Q: 18 **Determine the number of units of Roller Bush and Hydraulic Valve that SAPL must manufacture in order to maximise its profit. Show your work.** **[2]**

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	1
3	2
4	3

Q.No	Teacher should award marks if students have done the following:	Marks
5	**Writes true.**	**0.5**
	Justifies that in certain linear programming problems, the maximum/minimum value is obtained at 2 distinct corner points of the feasible region. In such cases, every point on the line joining those two points will be an optimal solution. Hence, a linear programming problem can have infinitely many optimal solutions. **(Award full marks for any equivalent justification.)**	**0.5**
6	**Writes the entire LPP as:** **Maximise** $Z = 4r + 5c$ **subject to the constraints:** $r + c = 25$ $2r + 3c \leq 60$ $r \geq 0$ $c \geq 0$ **where,** r **= number of rectangular boxes** c **= number of circular boxes** **(Award 0.5 mark if either the objective function or constraints are correct, but not both.)**	**1**
7	**Writes False(F).**	**0.5**
	Writes that, if on graphing the objective function, the open half-plane does not have any points common with the feasible region then the linear programming problem has an optimal solution.	**0.5**
8	**Writes that, since the minimum of Z occurs at both (50, 30) and (20, 40), 50a + 30b = 20a + 40b.**	**1**
	Solves the above equation to find the relationship between a and b as 3a = b.	**1**

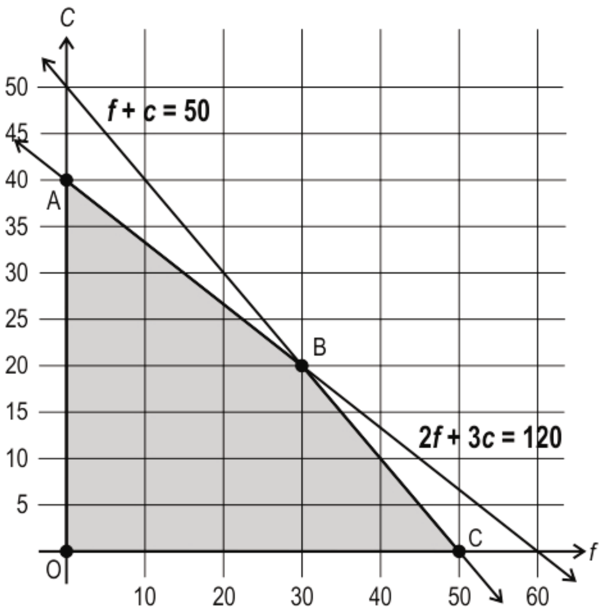
Q.No	Teacher should award marks if students have done the following:	Marks
9	Graphs the constraints of the LPP as: 	1
	Argues that, since no overlapping region exists for the LPP, there is no optimal solution in this case.	1
10	Uses the given points on the graph to find the following constraints using any suitable method: $2x + 3y \leq 120$ $2x + y \leq 60$	1.5
	Writes the remaining constraints by observing the graph of the feasible region as: $x \geq 0$ $y \geq 0$	0.5
11	Writes that the given LPP has no maximum value/no optimal solution.	0.5
	Justifies the answer by evaluating the objective function Z = 10 x + 20 y at each corner point and finding the largest value of Z as 80, which is obtained at the point (0, 4).	1

Q.No		Marks
	Writes that the open plane determined by 10 x + 20 y > 80 has points in common with the feasible region and hence the given LPP has no maximum value.	0.5
12	Assumes the number of large buses as x and medium buses as y. Writes the LPP as: Minimise Z = 3000 x + 2000 y subject to the constraints: 50 x + 40 y ≥ 400 x + y ≤ 9 x ≥ 0 y ≥ 0	0.5
	Sketches the feasible region as:  	1
	Finds the objective function value at the 3 corners of the feasible region as: **Vertex** | **Objective function value** (4, 5) | Rs 22000 (8, 0) | Rs 24000 (9, 0) | Rs 27000	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Concludes that the consultancy firm should book 4 large buses and 5 medium buses.	0.5
13	Assumes the number of pairs of shoes transported from Noida to Dwarka as x, from Noida to Saket as y and from Noida to Greater Kailash as $(450 - x - y)$.	1
	Finds the number of pairs of shoes transported from Gurgaon to Dwarka as $(250 - x)$, from Gurgaon to Saket as $(350 - y)$ and from Gurgaon to Greater Kailash as $500 - (250 - x + 350 - y)$ or $(x + y - 100)$.	1
	Writes the objective function to minimise the cost of transportation as: Minimise $Z = 10x + 15y + 12(450 - x - y) + 12(250 - x) + 15(350 - y) + 10(x + y - 100)$ or Minimise $Z = 2(6325 - 2x - y)$ (Award full marks if the objective function is framed by taking x and y as number of pairs of shoes transported from Gurgaon to Dwarka and Saket respectively.)	1
14	Assumes the quantity of figs and cashews to be f and c quintals respectively. Frames the objective function of the given problem as: Maximise $Z = 2{,}000f + 3{,}000c$	0.5
	Writes the constraints of the given problem as: $f + c \leq 50$ $40{,}000f + 60{,}000c \leq 24{,}00{,}000$ or $2f + 3c \leq 120$ $f \geq 0, c \geq 0$	1

<table>
<tr><th>Q.No</th><th>Teacher should award marks if students have done the following:</th><th>Marks</th></tr>
<tr><td rowspan="3"></td><td>

Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows:



(Award 0.5 marks each for drawing the correct line and 0.5 mark for shading the feasible region correctly.)

</td><td>1.5</td></tr>
<tr><td>

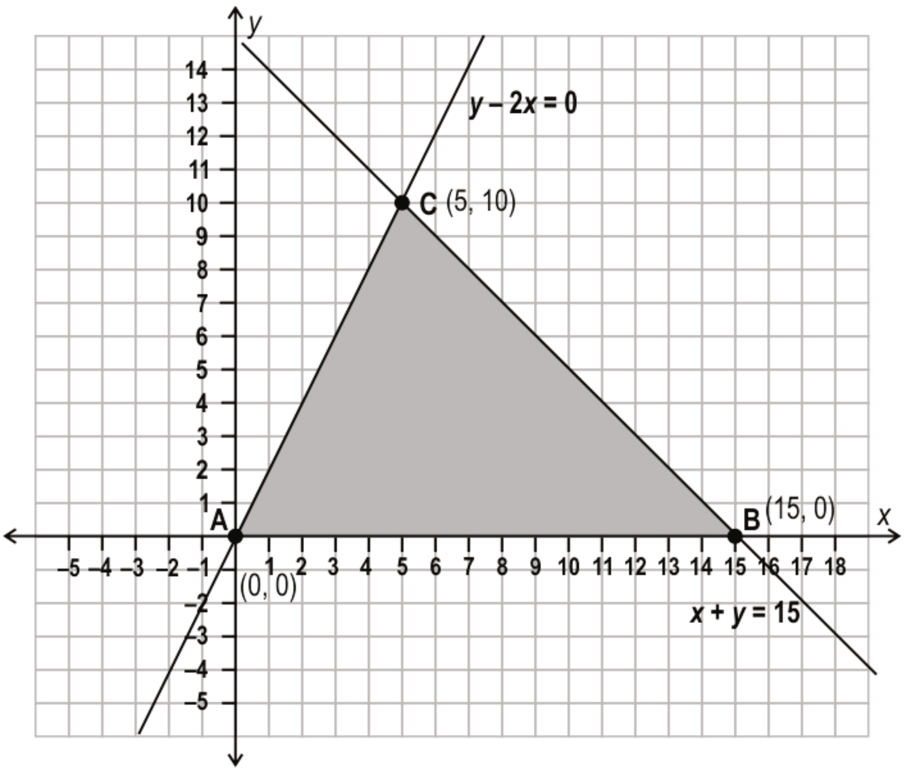
Evaluates the objective function at the corner points to find the optimal solution as follows:

Corner points	Z = 2000 f + 3000 c
O (0, 0)	0
A (0, 40)	1,20,000
B (30, 20)	1,20,000
C (50, 0)	1,00,000

</td><td>1</td></tr>
<tr><td>

Writes that the maximum value of Z is obtained at both the corner points A and B. Hence, concludes that every point on the line segment joining A and B is an optimal solution and Shaanta can buy any combination of weights corresponding to a point on the line segment joining A and B.

</td><td>1</td></tr>
</table>

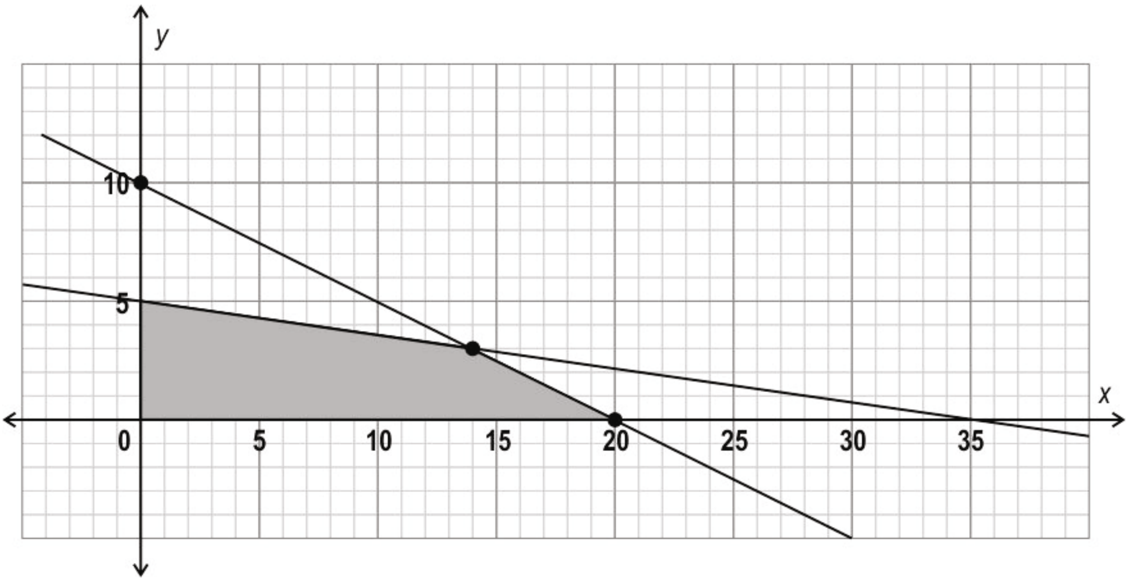
Q.No	Teacher should award marks if students have done the following:	Marks
15	**Writes the mathematical formulation of the given problem as follows:** **Maximise Z = 55 x + 70 y** **subject to the constraints:** $y - 2x \leq 0$ $x + y \leq 15$ $x \geq 0$ $y \geq 0$ **where x and y are the weights of Ajwa dates and Omani dates (in kgs) in each packet respectively.**	**2**
	Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows: 	**1.5**

<table>
<tr><th>Q.No</th><th>Teacher should award marks if students have done the following:</th><th>Marks</th></tr>
<tr><td></td><td>Evaluates the objective function at the corner points to find the optimal solution as follows:

<table><tr><th>Corner points</th><th>Z = 55 x + 70 y</th></tr><tr><td>A (0, 0)</td><td>0</td></tr><tr><td>B (15, 0)</td><td>825</td></tr><tr><td>C (5, 10)</td><td>975</td></tr></table></td><td>1</td></tr>
<tr><td></td><td>Uses the above step and writes that the maximum profit that can be attained on one packet is Rs 975 when 5 kgs of Ajwa dates and 10 kg of Omani dates are packed in each packet.</td><td>0.5</td></tr>
<tr><td>16</td><td>Assumes the number of units of Roller Bush and Hydraulic Valve to be manufactured as x and y respectively and writes the mathematical formulation of the above scenario as:

Maximise Z = 20 x + 50 y

Subject to constraints:
$2x + 14y \le 70$
$x + 2y \le 20$
$x \ge 0$
$y \ge 0$</td><td>1</td></tr>
</table>

<table>
<tr><th>Q.No</th><th>Teacher should award marks if students have done the following:</th><th>Marks</th></tr>
<tr><td>17</td><td>Plots the graph of the constraints and shades the feasible region as follows:


(Award only 1 mark if the constraints are plotted correctly but feasible region is not shaded/shaded incorrectly.)</td><td>2</td></tr>
<tr><td rowspan="2">18</td><td>Identifies the corner points and evaluates the value of Z at those points as follows:
<table><tr><th>Corner point</th><th>Z = 20 x + 50 y</th></tr><tr><td>(0, 0)</td><td>0</td></tr><tr><td>(0, 5)</td><td>250</td></tr><tr><td>(20, 0)</td><td>400</td></tr><tr><td>(14, 3)</td><td>430</td></tr></table>
Writes that 430 is the largest value of Z and is obtained at (14, 3).</td><td>1.5</td></tr>
<tr><td>Concludes that 14 units of Roller Bush and 3 units of Hydraulic Valve must be manufactured by SAPL in order to maximize its profit.</td><td>0.5</td></tr>
</table>

Chapter - 13
Probability

Q: 1 **Which of the following represents a Bernoulli trial?**

1 Tossing a coin until a tail is obtained.
2 Recording eye colours found in a group of 500 people.
3 Asking a random sample of 50 people if they have ever been to Germany.
4 A bag contains 8 red marbles and 5 blue marbles. A marble is picked from the bag 5 times without replacement.

Q: 2 **Study the following experiments.**

Experiment 1:
A fair coin is flipped 7 times with the objective of getting heads each time. The outcome is heads 5 times out of 7.

Experiment 2:
A pen is selected at random from a box 4 times with the objective of getting a red pen. The box contains 2 blue, 2 red and 2 black pens and the selected pen is replaced after each trial.

In which experiment(s) is/are the trials Bernoulli trials?

1 Experiment 1
2 Experiment 2
3 both Experiment 1 and Experiment 2
4 Neither Experiment 1 nor Experiment 2

Q: 3 **During a job interview, the probability of a candidate having a B.Tech degree was $\frac{5}{12}$ while the probability of a candidate having an MBA degree was $\frac{7}{16}$. The probability of a candidate having a B.Tech degree or an MBA degree or both is $\frac{11}{24}$.**

What is the probability of a candidate having a B.Tech degree given that the candidate has an MBA degree?

1 $\frac{5}{12}$
2 $\frac{19}{21}$
3 $\frac{20}{21}$
4 $\frac{19}{20}$

Q: 4 **A dog has six puppies. Assume that each puppy is equally likely to be a male or a female puppy.**

What is the probability that the dog has exactly 4 female puppies?

1 $\frac{1}{16}$
2 $\frac{15}{64}$
3 $\frac{15}{16}$
4 $\frac{2}{3}$

Q: 5 **M and N are two independent events.**

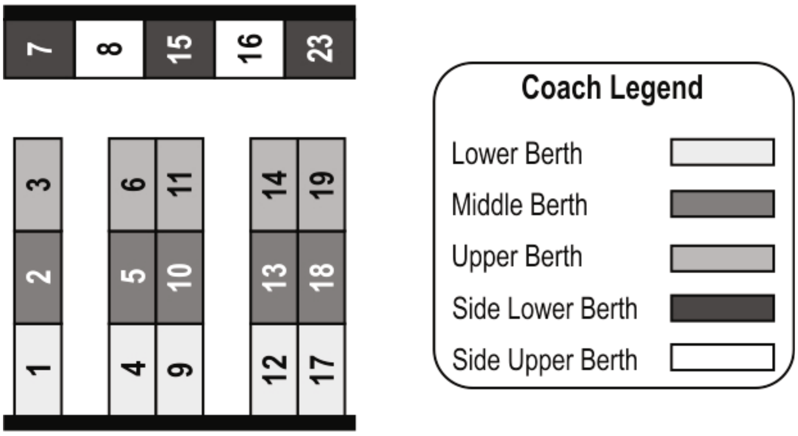
Which of the following statement(s) is/are DEFINITELY true?

i) M and N are mutually exclusive.
ii) The sum of the probabilities of M and N is 1.

1 only (i)
2 only (ii)
3 both (i) and (ii)
4 neither (i) nor (ii)

Use the information given below to solve the problems that follow.

A sleeper train has 5 types of seats. They are lower berth, middle berth, upper berth and side lower berth and side upper berth.



(Note: This picture is only to give you an idea of what a railway coach looks like. It DOES NOT represent the actual number of seats.)

Raj was wait-listed for a train from Dehradun to Delhi on a railway ticket booking application. In case his ticket got confirmed in this train, the probability that he got:

i) side berth (side lower and side upper berth) was, $P(S) = \frac{1}{3}$.
ii) upper berth was, $P(U) = \frac{2}{9}$.
iii) lower berth given that it is the side berth was, $P(L|S) = \frac{1}{4}$.
iv) upper berth or the side berth (side lower and side upper berth) was, $P(U \cup S) = \frac{17}{36}$.
v) middle berth was not 0.

(Note: P(U) does not include the side upper berths.)

Q: 6 Which of the following statements is true?

1 The probability of getting a lower berth is $\frac{7}{9}$.

2 The probability of getting a middle berth or side berth is $\frac{7}{9}$.

3 The probability of getting a middle berth or lower berth or side berth is $\frac{7}{9}$.

4 (none of the given statements are correct.)

Q: 7 Which of the following is the probability of the seat being a side berth as well as a lower berth?

1 $\frac{1}{12}$ **2** $\frac{1}{3}$ **3** $\frac{1}{4}$ **4** $\frac{7}{12}$

Q: 8 Which of the following is the probability of Raj getting an upper berth as well as a side berth?

1 $\frac{1}{12}$ **2** $\frac{2}{9}$ **3** $\frac{17}{36}$ **4** $\frac{5}{9}$

Q: 9 **Which of the following is the probability of Raj getting a side berth given that he has got an upper berth?**

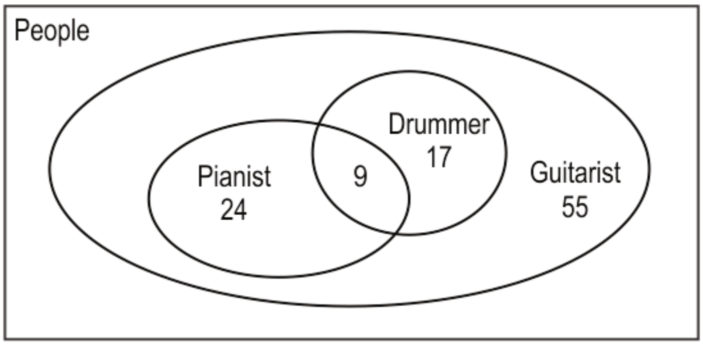
1 $\frac{1}{3}$ **2** $\frac{3}{8}$ **3** $\frac{1}{4}$ **4** $\frac{5}{9}$

Q: 10 **If the probability of Raj buying food on the train given that he got an upper berth is, P(F|U) = $\frac{1}{15}$, what can be said about the probability of Raj buying food given that he got a middle berth, P(F|M)?**

(Note: Getting food on the train and getting a lower, middle or upper berth are two independent events.)

1 P(F|M) > P(F|U) **2** P(F|M) < P(F|U)
3 P(F|M) = P(F|U) **4** (cannot be said without knowing P(M))

Q: 11 **Shown below is a Venn diagram representing the number of people in a music class who play different instruments.** **[1]**



Find the probability that a guitarist selected at random is also a pianist.

Q: 12 **There are three types of seats in an airplane: aisle, middle and window. While booking a ticket, one can pay extra money to reserve a specific seat type.** **[1]**
While booking a ticket on a website for a specific flight one particular day, the following observations were recorded:

i) the probability of a person not paying for a specific seat type was $\frac{1}{6}$.
ii) the probability of getting a window seat given that the person did not pay for a specific seat type was $\frac{1}{12}$.

Find the probability of a person getting a window seat without paying for a specific seat type. Show your work and give a valid reason.

Q: 13 **X and Y are the two events such that P(X|Y) = 0.2 and P(Y) = 0.5.** **[2]**

Find the value of P(X' ∩ Y). Show your steps.

Q: 14 **On a particular day, Vidit is going to play a game of carrom against one of three opponents. All opponents are equally likely to be paired with him. The table below shows the chances of Vidit winning when paired against each opponent.** **[2]**

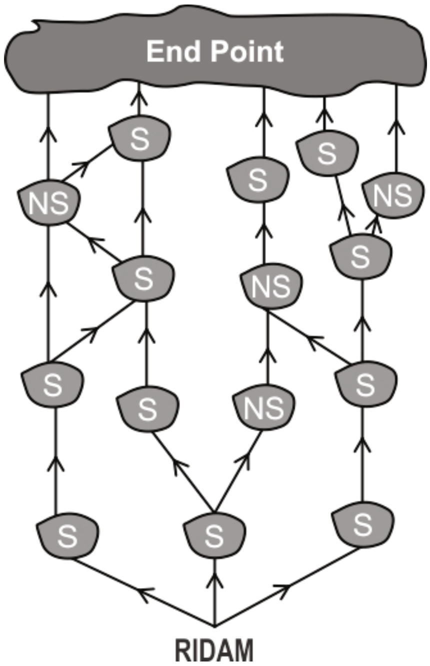
Opponent	Opponent 1	Opponent 2	Opponent 3
Vidit's chances of winning	80%	40%	x

If the probability that Vidit wins the game that day is $\frac{7}{15}$, find the chances of Vidit winning the game when paired with Opponent 3. Show your steps.

Q: 15 **Sukriti is playing a game with 16 identical white balls such that she can paint them as many times as she wants. She has the same set of balls in both the rounds. In the first round, she randomly spray paints 4 balls green. In the second round, she randomly spray paints 8 balls green.** **[2]**

At the end of the second round, what is the probability that a ball chosen at random is white? Show your steps.

Q: 16 Ridam went to a game show where she was asked to cross a puddle using stones. She cannot skip a stone. Shown below is a representation of the game. [2]



Stones marked S are stationary while those marked NS will sink as soon as she steps on them. She can only reach the other side of the puddle if she steps on all stationary stones.

If she does not know which stones are stationary, draw a tree diagram and find the probability that she reaches the other side of the puddle. Show your work.

Q: 17 A survey was conducted for all grade 10 students to test their inclination towards opting for science or commerce in grade 11. 75% of class 10 students are fluent in mathematics while the rest are fluent in economics. Based on past data, it is known that: [2]

♦ 80% of the students are fluent in business studies if they are fluent in economics.
♦ The probability of a student being fluent in business studies if they are fluent in mathematics is 15%.

If one of the students surveyed was found to be fluent in business studies, use Bayes' theorem to find the probability that the applicant is also fluent in economics? Show your work.

Q: 18 **In a particular week, the probability that it will rain on Monday, Tuesday and Wednesday are $\frac{1}{6}$, $\frac{2}{5}$ and $\frac{4}{5}$ respectively.** **[3]**

What is the probability that it will rain on at least one of the three days? Show your steps.

(Note: Whether it rains or does not rain on one of the days does not affect the probabilities of it raining on the following days.)

Q: 19 **In a particular week, a flight from Jaipur to Shillong is found to have a late arrival on:** **[3]**

♦ $\frac{1}{5}$ of the occasions if it is on time the previous day and

♦ $\frac{3}{10}$ of the occasions if it is late the previous day.

If it was on time on Wednesday, draw a tree diagram to find the probability that it will be on time on Friday. Show your work.

Q: 20 **A class 6 student, Himansh, has a 60% chance of having a misconception in decimals. His math teacher asks him take a remedial test to check if he a misconception. The test outcome is red if Himansh has the misconception and green if he does not have the misconception. Based on past data for class 6 students, the following points are known:** **[3]**

♦ If the student has a misconception, the test is 80% accurate.
♦ If the student doesn't have a misconception, the test is 90% accurate.

Find the probability that Himansh has a misconception given his test outcome was green. Show your steps.

Q: 21 **At a magic show, a magician has a sealed box that has a ball. The ball is equally likely to be either black or white. He opens it and without looking, puts a black ball in the box. He then randomly picks one of the two balls from the box.** **[5]**

If he picked a black ball, use Bayes' Theorem to find the probability that the original ball in the box is also black? Show your steps.

Q: 22 **Danish had one hundred cards. Each card had a natural number from 1 to 12 on its face.** **[5]**

Number on the card	1	2	3	4	5	6	7	8	9	10	11	12
Number of cards	5	10	12	8	6	7	6	10	12	15	4	5

He said, "If a card is drawn at random, the probability that the number on the card is even provided that it is a multiple of 3 is the same as the probability that the number on the card is a multiple of 3 provided that it is an even number."

Is Danish right in his conclusion? Justify your answer.

Q: 23 **Sundar has a basket of fruits that contains only 6 guavas and 4 apples. He picks three fruits, one after the other, at random without replacing any of the fruits.** **[5]**

Find the probability that:

i) all three fruits picked are apples.
ii) the first fruit picked is guava and the next two are apples.
iii) at least one of the fruits picked is a guava.

Show your steps.

Read the information and answer the questions that follow.

In a clinic, out of 40 patients who are waiting to see the doctor:

♦ 24 have cold
♦ 18 have fever
♦ 9 have migraine
♦ 4 have both cold and fever
♦ 6 have both fever and migraine
♦ 3 have both migraine and cold
♦ 2 have all the three

(Note: Assume that each of the 40 patients present is suffering from one or more of the 3 ailments - cold, fever or migraine only.)

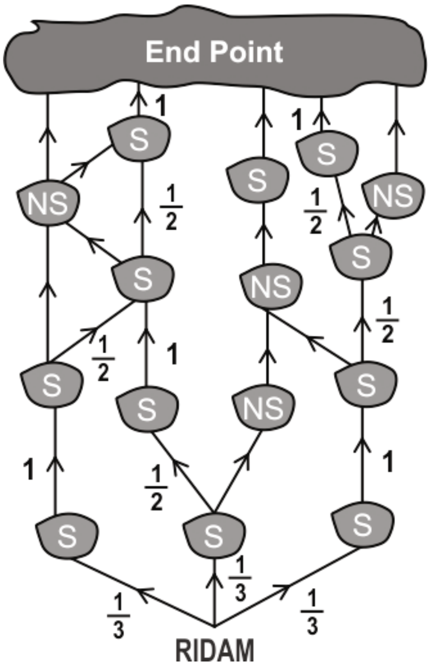
Q: 24 **If a patient is selected at random, find the probability that he/she DOES NOT have migraine given that he/she has fever. Show your work.** **[2]**

Q: 25 **If a patient is selected at random, what is the probability that he/she has cold or fever or both, given that he/she has migraine? Show your work.** **[3]**

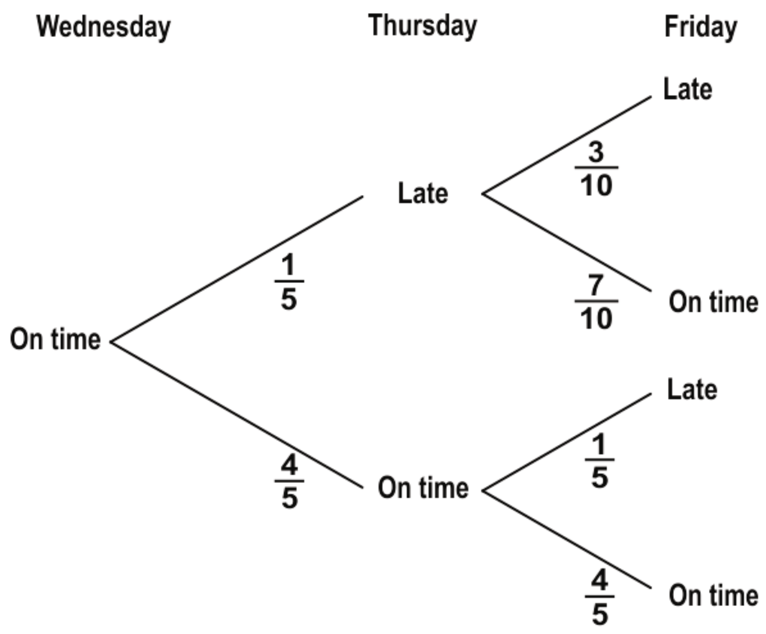
The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	3
3	2
4	2
5	4
6	3
7	1
8	1
9	2
10	3

Q.No	Teacher should award marks if students have done the following:	Marks
11	**Finds the probability that a guitarist selected at random is also a pianist as:** $P(pianist \mid guitarist)$ $= \frac{P(pianist \cap guitarist)}{P(guitarist)}$ $= \frac{33}{55} = \frac{3}{5}$ **(Award 0.5 marks if only the correct formula for conditional probability is written.)**	**1**
12	**Writes that, by using the multiplication theorem of probability, the probability of getting a window seat without paying for a specific seat type can be found as** $\frac{1}{6} \times \frac{1}{12} = \frac{1}{72}$. **(Award full marks for any other valid reason.)**	**1**
13	**Writes that P(X' ∩ Y) = P(X'|Y) × P(Y).**	**0.5**
	Uses the property of conditional probability and simplifies the above equation as: **P(X' ∩ Y) = [1 - P(X|Y)] × P(Y)**	**1**
	Substitutes the given values in the above equation to find P(X' ∩ Y) as: **P(X' ∩ Y) = 0.8 × 0.5 = 0.4**	**0.5**
14	**Uses the given information and writes the equation for Vidit winning the game that day as:** $\frac{7}{15} = \left(\frac{1}{3} \times \frac{80}{100}\right) + \left(\frac{1}{3} \times \frac{40}{100}\right) + \left(\frac{1}{3} \times \frac{x}{100}\right)$	**1**
	Simplifies the above equation as: $\frac{x}{100} = \frac{7}{5} - \frac{120}{100}$	**0.5**
	Solves the above equation for *x* and finds the chances of Vidit winning the game when paired with Opponent 3 as 20%.	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
15	Finds the probability of a ball being white in the first round as $1 - \frac{4}{16} = \frac{3}{4}$.	0.5
	Finds the probability of a ball being white in the second round as $1 - \frac{8}{16} = \frac{1}{2}$.	0.5
	Finds the probability of a ball being white at the end of the second round as $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.	1
16	Identifies that there are three paths which Ridam can take to successfully reach the other side of the puddle and draws a tree diagram. The diagram may look as follows: 	1
	Finds the probability of Ridam successfully reaching the other side of the puddle as: $3 \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	1
17	Finds the probability of a student being fluent in economics as: P(E) = 1 - P(M) = 1 - 0.75 = 0.25	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Represents mathematics as M and business studies as BS and uses Bayes' theorem to write: $$P(E\|BS) = \frac{P(E)P(BS\|E)}{P(E)P(BS\|E) + P(M)P(BS\|M)}$$ $$= \frac{0.25 \times 0.80}{(0.25 \times 0.80) + (0.75 \times 0.15)}$$ (Award 0.5 marks if only the formula for Bayes' theorem is written correctly.)	1
	Finds the probability of a student being fluent in economics given that he is fluent in business studies as $\frac{16}{25}$ or 64%.	0.5
18	Finds the P(it will not rain on Monday) as $1 - \frac{1}{6} = \frac{5}{6}$.	0.5
	Finds the P(it will not rain on Tuesday) as $1 - \frac{2}{5} = \frac{3}{5}$.	0.5
	Finds the P(it will not rain on Wednesday) as $1 - \frac{4}{5} = \frac{1}{5}$.	0.5
	Finds the P(it will not rain on all the three days) as $\frac{5}{6} \times \frac{3}{5} \times \frac{1}{5} = \frac{1}{10}$.	0.5
	Finds the P(it will rain on at least one of the three days) as: 1 - P(it will not rain on all the three days) $= 1 - \frac{1}{10} = \frac{9}{10}$ (Award 0.5 marks if only the first part of the step is correctly written.)	1

Q.No	Teacher should award marks if students have done the following:	Marks
19	Draws a tree diagram to illustrate the given situation. The diagram may look as follows: 	2
	Uses the above diagram and finds the probability of the flight being on time on Friday given that it was on time on Wednesday as $\left(\frac{1}{5} \times \frac{7}{10}\right) + \left(\frac{4}{5} \times \frac{4}{5}\right) = \frac{39}{50}$.	1
20	Writes the probability of Himansh having a misconception as P(M) = 0.6 and of not having a misconception as P(M') = 1 - 0.6 = 0.4.	0.5
	Writes the probability of the test outcome being green given he has a misconception as P(G|M) = 0.2.	0.5
	Writes the probability of the test outcome being green given he doesn't have a misconception as P(G|M') = 0.9.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses Bayes' theorem and finds the probability that Himansh has a misconception given his test outcome was green as: $$P(M\|G) = \frac{P(M)P(G\|M)}{P(M)P(G\|M) + P(M')P(G\|M')}$$ $$= \frac{0.6 \times 0.2}{(0.6 \times 0.2) + (0.4 \times 0.9)}$$ **(Award 0.5 marks if only the formula for Bayes' theorem is written correctly.)**	**1**
	Evaluates the above expression to find the required probability as $\frac{1}{4}$ or 25%.	**0.5**
21	**Finds the probability of a black ball being in the sealed box as $P(B_1) = \frac{1}{2}$ and that of a white ball being in the sealed box as $P(B_1') = \frac{1}{2}$.**	**1**
	Finds the probability of picking a black ball, B_2 given that B_1 is black as: $P(B_2\|B_1) = 1$	**1**
	Finds the probability of picking a black ball given that B_1 is white as: $P(B_2\|B_1') = \frac{1}{2}$	**1**
	Finds the total probability of picking a black ball as: $P(B_2) = P(B_2\|B_1)P(B_1) + P(B_2\|B_1')P(B_1')$ $= \frac{1}{2} + (\frac{1}{2} \times \frac{1}{2})$ $= \frac{3}{4}$	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the probability that the original ball in the box is black given that he picked a black ball as: $$P(B_1 \mid B_2) = \frac{P(B_1)P(B_2 \mid B_1)}{P(B_2)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$	1
22	Takes A to be an event of drawing a card with an even number on it and finds P(A) as $\frac{10+8+7+10+15+5}{100} = \frac{55}{100}$.	1
	Takes B to be an event of drawing a card with a multiple of 3 on it and finds P(B) as $\frac{12+7+12+5}{100} = \frac{36}{100}$.	1
	Writes that $A \cap B = \{6, 12\}$ and finds $P(A \cap B)$ as $\frac{7+5}{100} = \frac{12}{100}$.	0.5
	Finds the probability that the number on the card is even provided that it is a multiple of 3 as: $P(A\|B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{12}{100} \div \frac{36}{100} = \frac{1}{3}$	1
	Finds the probability that the number on the card is a multiple of 3 provided that it is an even number as: $P(B\|A) = \frac{P(A \cap B)}{P(A)}$ $= \frac{12}{100} \div \frac{55}{100} = \frac{12}{55}$	1
	Uses step 4 and 5 to write $P(A\|B) \neq P(B\|A)$ and hence concludes that what Danish said was wrong.	0.5
23	i) Takes A to be the event of picking an apple and finds $P(A) = \frac{4}{10} = \frac{2}{5}$.	0.5
	Finds the probability of getting an apple in the second pick with the condition that one apple has already been picked as $P(A\|A) = \frac{3}{9} = \frac{1}{3}$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the probability of getting an apple in the third pick with the condition that two apples have already been picked as P(A|AA) = $\frac{2}{8} = \frac{1}{4}$.	**0.5**
	Uses the multiplication theorem of probability and finds P(all three fruits picked are apples) as: **P(AAA) = $\frac{2}{5} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{30}$** **(Award 2 marks if the problem is solved correctly using combinatorics.)**	**0.5**
	ii) Takes G to be the event of picking a guava and finds P(G) = $\frac{6}{10} = \frac{3}{5}$.	**0.5**
	Finds the probability of getting an apple in the second pick with the condition that one guava has already been picked as P(A|G) = $\frac{4}{9}$.	**0.5**
	Finds the probability of getting an apple in the third pick with the condition that one guava and one apple have already been picked as P(A|GA) = $\frac{3}{8}$.	**0.5**
	Uses the multiplication theorem of probability and finds P(the first fruit is guava and the next two are apples) as: **P(GAA) = $\frac{3}{5} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{10}$** **(Award 2 marks if the problem is solved correctly using combinatorics.)**	**0.5**
	iii) Uses step 4 and finds P(at least one of the fruits picked is a guava) as: **1 - P(all three fruits picked are apples)** **= 1 - $\frac{1}{30}$** **= $\frac{29}{30}$** **(Award 0.5 marks if only the first part of the step is correctly written.)**	**1**
24	**Assumes the set of patients having fever and migraine to be F and M respectively.** **Finds the required probability as:** **P(M'|F) = 1 - P(M|F)** **= 1 - $\frac{P(M \cap F)}{P(F)}$** **= 1 - $\frac{6}{18}$** **= $\frac{2}{3}$** **(Award 0.5 marks if just the formula is written correctly.)**	**2**

Q.No	Teacher should award marks if students have done the following:	Marks
25	Assumes the set of patients having cold, fever and migraine to be C, F and M respectively. Writes that the required probability is given by, $P(C \cup F \mid M) = P(C \mid M) + P(F \mid M) - P(C \cap F \mid M)$	0.5
	Evaluates $P(C \mid M)$ as $\frac{P(C \cap M)}{P(M)} = \frac{3}{9}$.	0.5
	Evaluates $P(F \mid M)$ as $\frac{P(F \cap M)}{P(M)} = \frac{6}{9}$.	0.5
	Evaluates $P(C \cap F \mid M)$ as $\frac{P(C \cap F \cap M)}{P(M)} = \frac{2}{9}$.	0.5
	Finds the probability that a patient selected at random has cold or fever, provided he/she has migraine as: $\frac{3}{9} + \frac{6}{9} - \frac{2}{9} = \frac{7}{9}$ (Award 1.5 marks if the problem is solved correctly without explicitly writing the formula in step 1.)	1

Chapter - 14
Application of Multiple Concepts

Q: 1 **Find the critical point(s) of the real valued function, $f(x) = x^x$, where $x \in (0,\infty)$. Show your work.** **[2]**

Q: 2 **Prove that:** **[4]**

$$\sin^{-1}\frac{(\cos\theta - \sqrt{3}\sin\theta)(1+\tan^2\theta)}{(1+\sin^2\theta)(1+\tan^2\theta)+1} = \left(\frac{\pi}{6} - \theta\right)$$

Q.No	Teacher should award marks if students have done the following:	Marks
1	**Considers $y = x^x$ and takes logarithm to the base e on both sides to write:** **$\ln y = x \ln x$**	0.5
	Differentiates the above equation with respect to x as: $\frac{dy}{dx} = x^x(1 + \ln x)$	1
	Writes that, since x^x cannot be zero, $(1 + \ln x)$ must be zero. Solves this equation to get the critical point as: $1 + \ln x = 0$ $\Rightarrow x = e^{-1}$ or $\frac{1}{e}$ **(Award full marks if 'log' is used instead of 'ln'.)**	0.5
2	**Writes the L.H.S as:** $\sin^{-1}\left(\frac{(\cos\theta - \sqrt{3}\sin\theta)\sec^2\theta}{(1 + \sin^2\theta)\sec^2\theta + 1}\right)$	0.5
	Rewrites the L.H.S as: $\sin^{-1}\left(\frac{(\cos\theta - \sqrt{3}\sin\theta)\sec^2\theta}{\sec^2\theta + \tan^2\theta + 1}\right)$	0.5
	Simplifies the L.H.S. as: $\sin^{-1}\left(\frac{(\cos\theta - \sqrt{3}\sin\theta)\sec^2\theta}{2\sec^2\theta}\right)$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Simplifies the above step as: $\sin^{-1}\left(\frac{\cos\theta}{2} - \frac{\sqrt{3}\sin\theta}{2}\right)$	**0.5**
	Rewrites the above step as: $\sin^{-1}(\sin\frac{\pi}{6}\cos\theta - \sin\theta\ \cos\frac{\pi}{6})$	**1**
	Rewrites the above step as: $\sin^{-1}(\sin(\frac{\pi}{6} - \theta))$ **and proves that the L.H.S = R.H.S = ($\frac{\pi}{6}$ - θ).**	**1**

15. Annexure

Correct Answer Explanation

Chapter Name	Q. No	Correct Answer	Correct Answer Explanation
Relations and Functions	2	1	Since the number of elements in the codomain (set B) is greater than that in the domain (set A), for any function, there will be at least one element in the co-domain (set B) which is not an image of any element in the domain (set A). Hence option A is the correct answer.
	4	1	The given relation is reflexive as it contains all the elements of the form (x, x).
			The given relation is symmetric since for every (x, y) in the relation, (y, x) is also in the relation.
			The given relation is transitive as it DOES NOT contain any pair of elements of the form (x, y) and (y, z).
			Hence option A is the correct answer.
	7	2	$f(1)=1$, $f(2)=0$, $f(3)=2$, $f(4)=0$, $f(5)=3$... All the odd numbers are being mapped to positive integers and all the even numbers are being mapped to 0. The function is onto as all the non-negative integers of the co-domain are the images of some elements in the domain. The function is not one-one as more than one even number is getting mapped to 0. Hence, option B is the correct answer.
Inverse Trigonometric Functions	1	1	Domain of arcsec(x) is $(-\infty, -1] \cup [1, \infty)$ and domain of arcsin(x) is $[-1, 1]$. For the given function, the domain would be intersection of the two domains. That is, $(-\infty, -1] \cup [1, \infty) \cap [-1, 1] = \{-1, 1\}$. Hence, option A is the correct answer.
	3	3	Domain of arccos(x) is $[-1, 1]$. So $1/x-3$ should belong to $[-1, 1]$. Solving the inequality $-1 \leq 1/(x - 3) \leq 1$ gives $x \leq 2$ and $x \geq 4$. Hence, option C is the correct answer.
Matrices	1	4	i) Columns 1 and 3 are identical, so upon exchanging them, the matrix is unchanged. ii) Third row is a zero row, so adding any scalar multiple of it to another row will keep the row unchanged. iii) Column 2 is a zero column, so multiplying it with (-1) keeps the matrix unaltered. Hence option D is the correct answer.

	2	4	The order of the two matrices given are 2x3. A matrix of order 2x3 is not compatible for multiplication with another matrix of order 2x3. Only matrices with orders of the form 3xk, where k is a natural number, are compatible to multiply with matrices of order 2x3. Thus, none of their answers is correct.
			Hence, option D is the correct answer.
Determinants	5	3	Taking 3 common from the second row and 2 common from the third row one can observe that the elements of first row are identical to the elements of the third row. Given that the determinant is 0, this can happen when 2 rows are the same. So $2\cos(2x)$ could come in the box which is the second element of row 3. Hence, option C is the correct answer.
Continuity and Differentiability	1	3	In $g(x)$, $(x-1)$ is differentiable everywhere. $\lvert x-1 \rvert$ is not differentiable at 1. Graph of $g(x)$ shows that it is differentiable everywhere. Thus, product of a differentiable function and a non-differentiable function could be differentiable.
			In $f(x)$, x is differentiable everywhere. $\lvert x-1 \rvert$ is not differentiable at 1. Graph of $f(x)$ is pointed at $x=1$, that means it is not differentiable at 1. Thus, product of a differentiable function and a non-differentiable function could be non-differentiable.
			Therefore, product of a differentiable function and a non-differentiable function MAY BE differentiable. Hence, option C is the correct answer.
Application of Derivatives	4	2	$f'(x)=2x^2-18$. Equating $f'(x)$ to zero gives the critical points as (+3) and (-3). $f''(x)=4x$. $f''(-3)<0$ and $f''(3)>0$, that means function attains maxima at (-3) and minima at (+3). Thus the function is decreasing in the interval [-3, 3]. Hence, option B is the correct answer.
Integrals	2	2	Statement 1 is false. For example, integral of (-x) in the interval (1, 2) is (-3/2). If integral exists in (a, b), then the function is continuous in (a, b), because, in an interval if the function is continuous only then it can be integrated. Thus, statement 2 is true.
			Hence, option B is the correct answer.
Differential Equations	5	4	The general solution of this differential equation is $a\sin x + b\cos x$, where a and b are arbitrary constants. Bulbul's particular solution is obtained when a=1 and

			b=0. Ipsita's particular solution is obtained when a=0 and b=1. Sagarik's particular solution is obtained when a= 1 and b=1. Thus all three of their answers are particular solutions. Thus, option D is the correct answer.
Vector Algebra	2	3	v x u = -w. Angle between w and (-w) is 180 degrees. Hence, option C is the correct answer.
	3	2	Cross product of two vectors is given by |a||b|sin Θ, where Θ is the angle between a and b. |a| and |b| are positive being magnitudes. sin Θ is maximum when the angle between a and b is ninety degrees. Hence, option B is the correct answer.

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