3 PROPORTIONAL REASONING-2

0889CH03

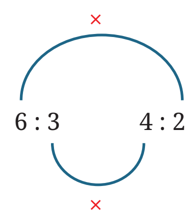
3.1 Proportionality—A Quick Recap

In an earlier chapter, we explored proportional relationships between quantities and we used the ratio notation to represent such relationships. When two or more related quantities change by the same factor, we call that relationship a proportional relationship. For example, *idli* batter is made by mixing rice and *urad dal.* The proportion of these two can have regional variations. One of the proportions used is: for 2 cups of rice, we add 1 cup of *urad dal*. We represent this relationship using the ratio notation 2 : 1.

? Viswanath made *idlis* by mixing 6 cups of rice with 3 cups of *urad dal*, while Puneet made *idlis* by mixing 4 cups of rice with 2 cups of *urad dal*. If cooked in the same way, would their *idlis* taste the same?

Viswanath's mixture can be represented as 6 : 3 and Puneet's mixture as 4 : 2.

Recall that, to verify that these two ratios are proportional, we can use the cross-multiplication method:



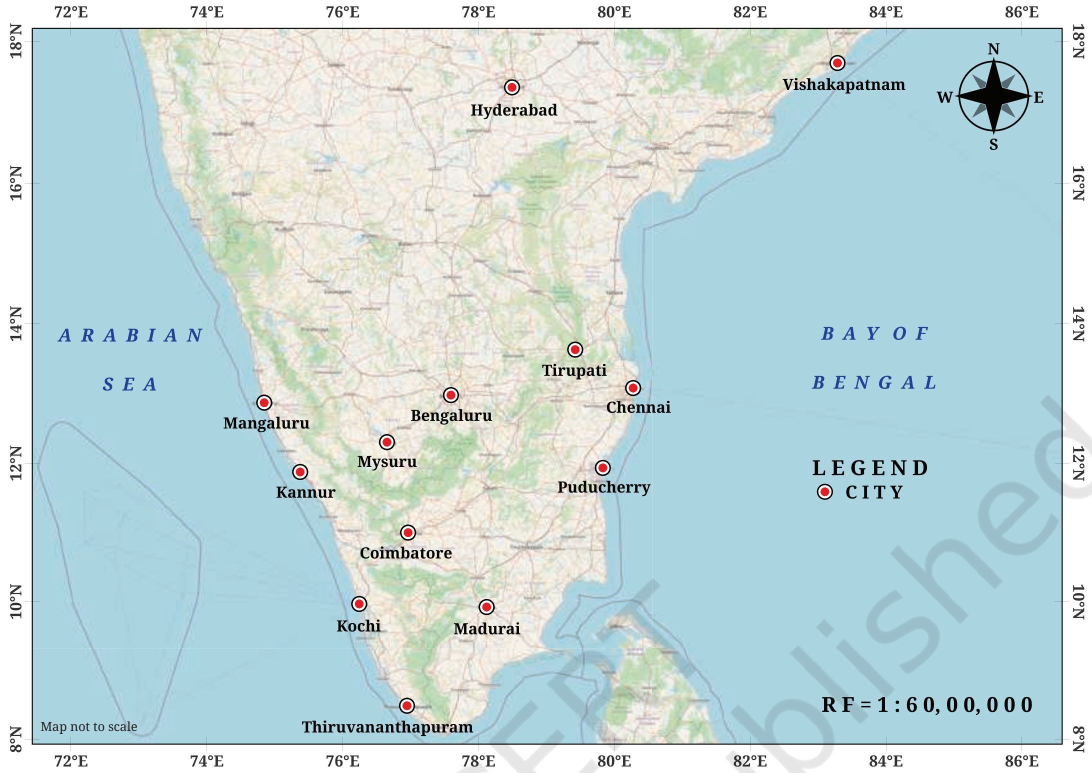
The two products are the same (12). So, the two ratios are proportional. It is likely that the *idlis* would taste the same, if all the other ingredients are proportional too!

In general, we can say that two ratios $a : b$ and $c : d$ are proportional if

$$a \times d = b \times c, \text{ or}$$

$$\frac{a}{c} = \frac{b}{d}.$$

3.2 Ratios in Maps



Have you noticed that in many maps there is a ratio given, usually in the lower right corner of the map? It usually contains 1 and a very large number, such as 1 : 60,00,000. What does RF 1 : 60,00,000 mean? What does it indicate? Can you guess?

A Representative Fraction (RF) is an expression that shows the ratio between a distance on the map and the corresponding actual distance on the ground.

For example, if the ratio on a map is 1 : 60,00,000, that means a distance of 1 cm on the map is equivalent to a geographical distance of 60,00,000 cm. Remember, this is geographical distance and not road distance!

Convert 60,00,000 cm to kilometres.

It is 60 km. Verify this.

Using the map above, can you find the geographical distance between Bengaluru and Chennai? Also, find the geographical distance between Mangaluru and Chennai.

[**Hint:** Use a ruler to find the distance between the cities on the map. Then, use the ratio given on the map to find the actual geographical distance.]

? Try to find the distances between the same two pairs of cities with different maps that have different scales (ratios). Do they all give the same geographical distance, approximately?

Note to the Teacher: Bring maps and atlases to the classroom and encourage students to observe the scale given as a ratio (usually in the lower right corner) on the map. Use the maps and atlases in the library and ask students to find the geographical distances between two locations on the map that are of local interest. Ask them to verify with each other if they get similar distances and to find the reasons if the distances are very different.

Map Making Activity: Guide students to make a sketch of their classroom with an accurate scale (ratio of 1: 50). They should mark the location of various objects in the classroom like the teacher's desk, blackboard, fans and lights, according to scale. Students can use appropriate symbols to represent different objects like fans, lights, tables, chairs, and so on.

3.3 Ratios with More than 2 Terms

Viswanath is experimenting with a spice mix powder. He makes the powder by grinding 8 spoons of coriander seeds, 4 red chillies, 2 spoons of *toor dal*, and 1 spoon of fenugreek (*methi*) seeds. For his spice mix powder, the ratio of coriander seeds to red chillies to *toor dal* to fenugreek seeds is

8 : 4 : 2 : 1.

Notice that the ratio has 4 terms. Ratios can have many terms if each of the quantities change by the same factor to maintain the proportional relationship.

? Puneet has only 2 red chillies in his kitchen. But he wants to make spice mix powder that tastes the same as Viswanath's spice mix powder. How much of the other ingredients should Puneet use to make his spice mix powder?

For Puneet's spice mix powder to be similar to Viswanath's, the ratio of all the ingredients should be the same as Viswanath's spice mix powder. Puneet has only 2 red chillies. He has half the number of chillies that Viswanath used in his mixture. So, the quantity of the other ingredients should also be reduced to half.

Thus, Puneet should add 4 spoons of coriander seeds, 2 red chillies, 1 spoon of *toor dal*, and half a spoon of fenugreek seeds. The ratio is

4 : 2 : 1 : 0.5.

Both the ratios are proportional to each other. We denote this by

$$8 : 4 : 2 : 1 :: 4 : 2 : 1 : 0.5.$$

In general, when two ratios with multiple terms are proportional

$$a : b : c : d :: p : q : r : s$$

then,

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s}.$$

Example 1: To make a special shade of purple, paint must be mixed in the ratio, Red : Blue : White :: 2 : 3 : 5. If Yasmin has 10 litres of white paint, how many litres of red and blue paint should she add to get the same shade of purple?

In the ratio 2 : 3 : 5, the white paint corresponds to 5 parts.

If 5 parts is 10 litres, 1 part is 10 ÷ 5 = 2 litres.

Red = 2 parts = 2 × 2 = 4 litres.

Blue = 3 parts = 3 × 2 = 6 litres.

So, the purple paint will have 4 litres of red, 6 litres of blue, and 10 litres of white paint.

What is the total volume of this purple paint?

The total volume of purple paint is 4 + 6 + 10 = 20 litres.

Example 2: Cement concrete is a mixture of cement, sand, and gravel, and is widely used in construction. The ratio of the components in the mixture varies depending on how strong the structure needs to be. For structures that need greater strength like pillars, beams, and roofs, the ratio is 1 : 1.5 : 3, and the construction is also reinforced with steel rods. Using this ratio, if we have 3 bags of cement, how many bags of concrete mixture can we make?

The concrete mixture is in the ratio

Bags of cement : bags of sand : bags of gravel :: 1 : 1.5 : 3.

If we have 3 bags of cement, we have to multiply the other terms by 3. So, the ratio is

cement : sand : gravel :: 3 : 4.5 : 9.

In total, we have 3 + 4.5 + 9 = 16.5 bags of concrete.

3.4 Dividing a Whole in a Given Ratio

In an earlier chapter, we learnt how to divide a whole in a ratio, e.g., 12 in the ratio 2 : 1. To do this, we add the terms (2 + 1 = 3), and divide the whole by this sum (12 ÷ 3 = 4). We multiply each term by this quotient: 2 × 4 = 8 and 1 × 4 = 4 . So, 12 divided in the ratio 2 : 1 is 8 : 4.

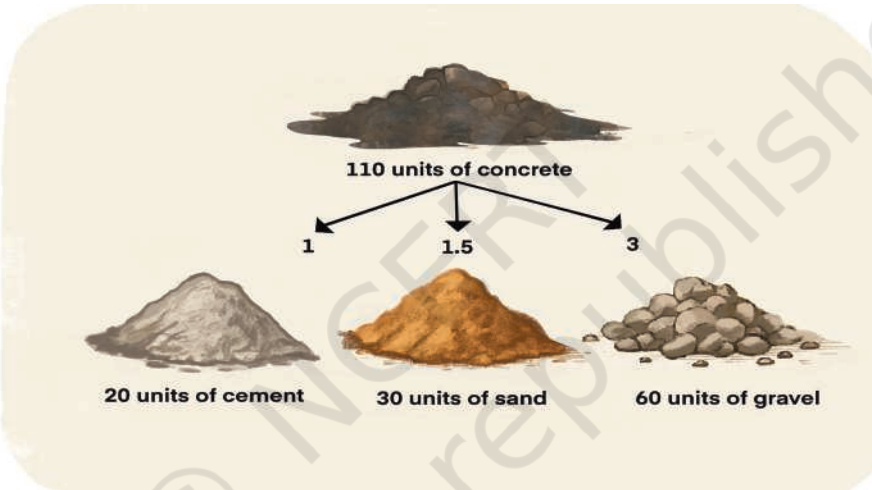
We can extend this to ratios with multiple terms.

Let us look at the earlier example of making a concrete mixture. We need to mix cement, sand, and gravel in the ratio of 1 : 1.5 : 3 to get the concrete.

Example 3: For some construction, 110 units of concrete are needed. How many units of cement, sand, and gravel are needed if these are to be mixed in the ratio 1 : 1.5 : 3?

For 1 unit of cement, we need to add 1.5 units of sand and 3 units of gravel. Together, they add up to 5.5 units of concrete. We need to do this 20 times (110 ÷ 5.5 = 20) to get 110 units of concrete. So each term has to be multiplied by 20.

$1 \times 20 = 20$ units of cement,
$1.5 \times 20 = 30$ units of sand, and
$3 \times 20 = 60$ units of gravel.



So, we need 20 units of cement, 30 units of sand, and 60 units of gravel to make the concrete.

When we divide a quantity x in the ratio $a : b : c : \cdots$, the terms in the ratio are —

$$x \times \frac{a}{(a+b+c+\cdots)},\ x \times \frac{b}{(a+b+c+\cdots)},\ x \times \frac{c}{(a+b+c+\cdots)}, \text{ and so on.}$$

Example 4: You get a particular shade of purple paint by mixing red, blue, and white paint in the ratio 2 : 3 : 5. If you need 50 ml of purple paint, how many ml of red, blue, and white paint will you mix together?

$$\text{Red paint} = 50 \times \frac{2}{(2+3+5)} = 50 \times \frac{2}{10} = 10 \text{ ml.}$$

$$\text{Blue paint} = 50 \times \frac{3}{(2+3+5)} = 50 \times \frac{3}{10} = 15 \text{ ml.}$$

$$\text{White paint} = 50 \times \frac{5}{(2+3+5)} = 50 \times \frac{5}{10} = 25 \text{ ml.}$$

Example 5: Construct a triangle with angles in the ratio 1 : 3 : 5.

We know that the sum of the angles in a triangle is 180°. So the angles are

$\angle A = 180° \times \frac{1}{(1+3+5)} = 180° \times \frac{1}{9} = 20°.$

$\angle B = 180° \times \frac{3}{(1+3+5)} = 180° \times \frac{3}{9} = 60°.$

$\angle C = 180° \times \frac{5}{(1+3+5)} = 180° \times \frac{5}{9} = 100°.$

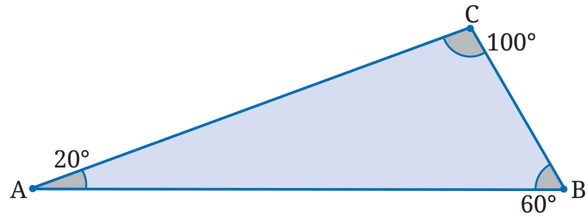


Figure it Out

1. A cricket coach schedules practice sessions that include different activities in a specific ratio — time for warm-up/cool-down : time for batting : time for bowling : time for fielding :: 3 : 4 : 3 : 5. If each session is 150 minutes long, how much time is spent on each activity?

2. A school library has books in different languages in the following ratio — no. of Odiya books : no. of Hindi books : no. of English books :: 3 : 2 : 1. If the library has 288 Odiya books, how many Hindi and English books does it have?

3. I have 100 coins in the ratio — no. of ₹10 coins : no. of ₹5 coins : no. of ₹2 coins : no. of ₹1 coins :: 4 : 3 : 2 : 1. How much money do I have in coins?

4. Construct a triangle with sidelengths in the ratio 3 : 4 : 5. Will all the triangles drawn with this ratio of sidelengths be congruent to each other? Why or why not?

Math Talk

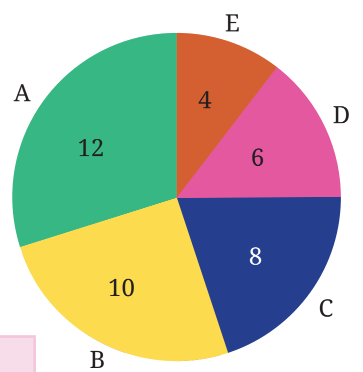
5. Can you construct a triangle with sidelengths in the ratio 1 : 3 : 5? Why or why not?

3.5 A Slice of the Pie

Have you seen pie charts like the one shown in the figure? Pie charts show different proportions of a whole. This one shows the proportions of students that have scored each grade in an assessment. These kinds of visualisations help us quickly interpret data.

Let us try to create this pie chart. Here is a table showing the grades scored by students:

Grade Distribution (40 students)



Grade	A	B	C	D	E
Students	12	10	8	6	4

How do we mark the different slices of the pie chart?

To mark a slice in the pie chart, the angle corresponding to a grade should be proportional to the number of students who have scored that grade. The total angle in a circle is 360°. So, we need to divide 360 in the ratio of 12 : 10 : 8 : 6 : 4.

Can we reduce this ratio to its simplest form?

We can use the same procedure we used to reduce a ratio with two terms to its simplest form. We divide all the terms by their HCF to get the simplest form.

In this example, 2 is the HCF of the terms. We get the simplest form by dividing all the terms by 2. The ratio becomes 6 : 5 : 4 : 3 : 2.

So, the angles are:

$$\text{Grade A} = \frac{6}{(6+5+4+3+2)} \times 360° = \frac{6}{20} \times 360° = 6 \times 18 = 108°.$$

$$\text{Grade B} = \frac{5}{(6+5+4+3+2)} \times 360° = 5 \times 18 = 90°.$$

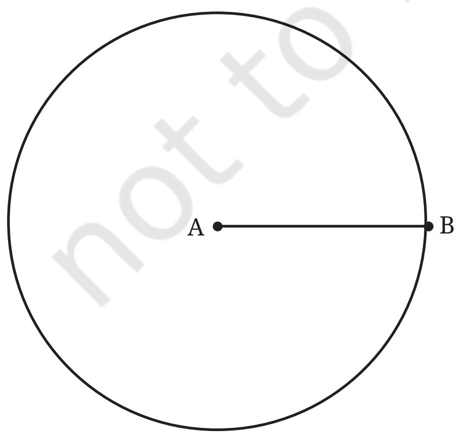
Grade C = 4 × 18 = 72°.

Grade D = 3 × 18 = 54°.

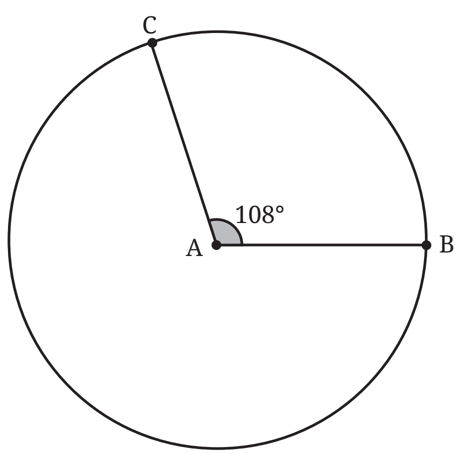
Grade E = 2 × 18 = 36°.

Now let us construct a pie chart using these angles.

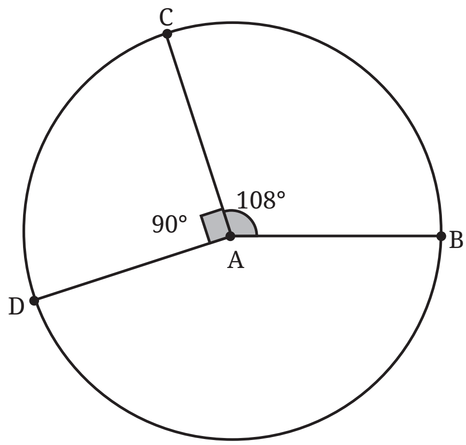
Step 1: Draw a circle and mark the radius AB as shown below:



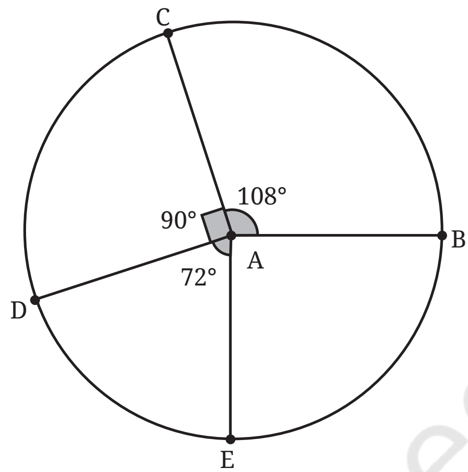
Step 2: To draw the slice to represent the proportion of students who got an A grade, measure 108° from the segment AB on A (anti-clockwise), and mark the new radius AC.



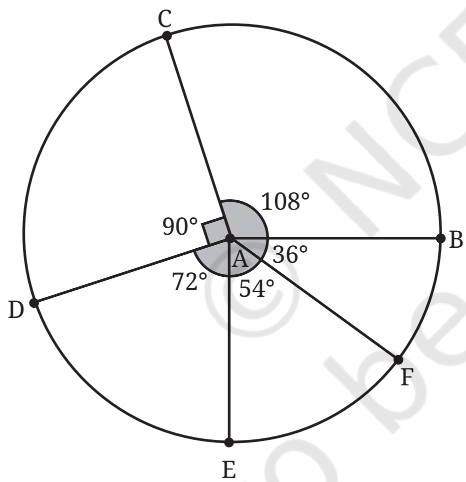
Step 3: Measure 90° from AC and draw AD.



Step 4: Measure 72° from AD and draw AE.



Steps 5,6: Similarly, we can complete the rest of the pie chart.



Step 7: You can colour and label the different slices of the pie chart appropriately.

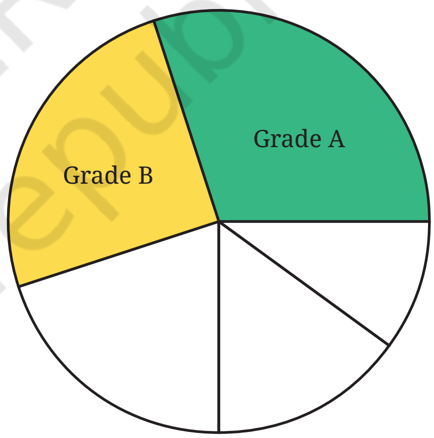


Figure it Out

1. A group of 360 people were asked to vote for their favourite season from the three seasons — rainy, winter and summer. 90 liked the summer season, 120 liked the rainy season, and the rest liked the winter. Draw a pie chart to show this information.

2. Draw a pie chart based on the following information about viewers' favourite type of TV channel: Entertainment — 50%, Sports — 25%, News — 15%, Information — 10%.

3. Prepare a pie chart that shows the favourite subjects of the students in your class. You can collect the data of the number of students for

each subject shown in the table (each student should choose only one subject). Then write these numbers in the table and construct a pie chart:

Subject	Language	Arts Education	Vocational Education	Social Science	Physical Education	Maths	Science
Number of Students							

3.6 Inverse Proportions

Do you recall the rule of three? When two ratios are proportional, i.e., when

$a : b :: c : d$, then $d = \frac{bc}{a}$. We call such proportions **direct proportions**. We use this understanding to find the value of the fourth quantity (d), when the value of three quantities (a, b, and c) are given.

Example 1: If 5 workers can move 4500 bricks in a day, how many workers are needed to move 18000 bricks in a day?

This can be represented as a statement of proportionality — $4500 : 18000 :: 5 : x$. We can find the value of x by

$$x = \frac{18000 \times 5}{4500} = 20$$

Thus, the number of workers needed are 20.

Example 2: Puneeth's father went from Lucknow to Kanpur in 3 hours by riding his motorcycle at a speed of 30 km/h. If he takes a car instead and drives at 60 km/h, how long will it take him to reach Kanpur?

Can we represent this problem with the following statement of proportionality — $30 : 60 :: 3 : x$? Will the travel time increase or decrease as the speed of the motorcycle increases?

Math Talk

The following table shows the time taken to travel from Lucknow to Kanpur using different modes of transport:

	Walk	Bicycle	Motorcycle	Car
Speed (km/h)	5	15	30	60
Time (in hours)	18	6	3	1.5

From this table, we notice that when the speed **increases**, the time taken to travel the same distance **decreases**.

? Does it decrease by the same rate (or factor)?

Going by bicycle is 3 times faster than walking (15 ÷ 5). The speed has increased 3 times. The travel time has decreased 3 times too (18 ÷ 6). The speed has increased by the same factor by which the travel time has decreased.

? Check if this is the case for the other modes of transport.

Since both quantities, speed and time, change by the same factor, they are proportional. But they change in opposite directions, or inversely. Such proportions are called **inverse proportions**.

From the table, we can see that the product of speed and time are the same for all modes of transport, namely 90 km.

Two quantities x and y vary in inverse proportion if there exists a relation of the type $xy = k$, where k is a constant.

In the previous example, if x represents the speed and y represents the time taken, then k is the distance between Lucknow and Kanpur, which remains constant.

× 4

× 3 × 2

Speed (km/h)	5	15	30	60
Time (in hours)	18	6	3	1.5

$\times \frac{1}{3}$ $\times \frac{1}{2}$

$\times \frac{1}{4}$

Thus, if quantities x and y are inversely proportional, and x_1 and x_2 are values of x that have corresponding y values y_1 and y_2, then

$$x_1y_1 = x_2y_2 = k \text{ (some constant)}.$$

From these we can see that $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

Let us check if this is true for other modes of transport, such as walking and by car. Let us use x to represent speed and y to represent time.

$$x_1 = 5,\ x_2 = 60,\ y_1 = 18,\ y_2 = 1.5.$$

$$\frac{x_1}{x_2} = \frac{5}{60} = 0.083333...$$

What is the value of $\frac{1.5}{18}\left(\frac{y_2}{y_1}\right)$? It is 0.083333... too!

Figure it Out

1. Which of these are in inverse proportion?

(i)

x	40	80	25	16
y	20	10	32	50

(ii)

x	40	80	25	16
y	20	10	12.5	8

(iii)

x	30	90	150	10
y	15	5	3	45

2. Fill in the empty cells if x and y are in inverse proportion.

x	16	12		36
y	9		48	

Example 3: 20 workers take 4 days to complete laying a road. How many days will 10 workers take to complete laying the same length of road?

If we decrease the number of workers, then the number of days to complete the work will increase by the same factor. So these quantities are inversely proportional. So, $x_1y_1 = x_2y_2$.

Thus, $20 \times 4 = 10 \times y_2$

$$y_2 = \frac{20 \times 4}{10} = 8.$$

It will take 8 days for 10 workers to complete the work.

We notice that when the number of workers halved, the number of days to complete the work doubled. Quantities are inversely proportional if, when one quantity changes by a factor n, the other quantity changes by the inverse $\frac{1}{n}$.

Example 4: 2 pumps can fill a tank in 18 hours. How much time will it take to fill the tank if we add 2 more pumps of the same kind?

If we add 2 more pumps, we will have 4 pumps. Let us denote the time taken to fill the tank with 4 pumps by x. If we increase the number of pumps by n, then the time taken to fill the tank will decrease by the same factor n, so the quantities are inversely proportional. Since the quantities are inversely proportional, the product remains constant. So,

$$2 \times 18 = 4 \times x.$$

$$x = \frac{2 \times 18}{4} = 9.$$

It will take 9 hours to fill the tank.

Example 5: A school has food provisions to feed 80 students for 15 days. If 20 more students join the school, for how many days will the provisions last?

More students → fewer days the provisions will last. The quantities are inversely proportional. If x is the number of days,

$$80 \times 15 = 100 \times x.$$

$$x = \frac{80 \times 15}{100} = 12.$$

The provisions will last for only 12 days.

Example 6: If Ram takes 1 hour to cut a given quantity of vegetables and Shyam takes 1.5 hours to cut the same quantity of vegetables, how much time will they take to cut the vegetables if they do it together?

Consider the work done to cut the given quantity of vegetables as 1 unit of work. Let us figure out the work done by each person in 1 hour.

- Ram finishes the work in 1 hour, so in 1 hour he does 1 unit of work.
- Shyam finishes the work in 1.5 hours, so in 1 hour he does $\frac{1}{1.5} = \frac{2}{3}$ units of work.

So, the work done by both in 1 hour is $1 + \frac{2}{3} = \frac{5}{3}$ units of work.

Therefore, to complete $\frac{5}{3}$ units of work, it takes them 1 hour if they work together. How much time will it take them to complete 1 unit of work?

Math Talk

Is the quantity of work and time taken to complete it directly or inversely proportional?

It is directly proportional. So, this can be represented as

$$\frac{5}{3} : 1 :: 1 : x \text{ where } x \text{ is the time taken.}$$

Since it is a direct proportion, we know that

$$\frac{5}{3} \times x = 1 \times 1$$

$$x = \frac{1 \times 1}{\frac{5}{3}} = \frac{1}{\frac{5}{3}} = \frac{3}{5}.$$

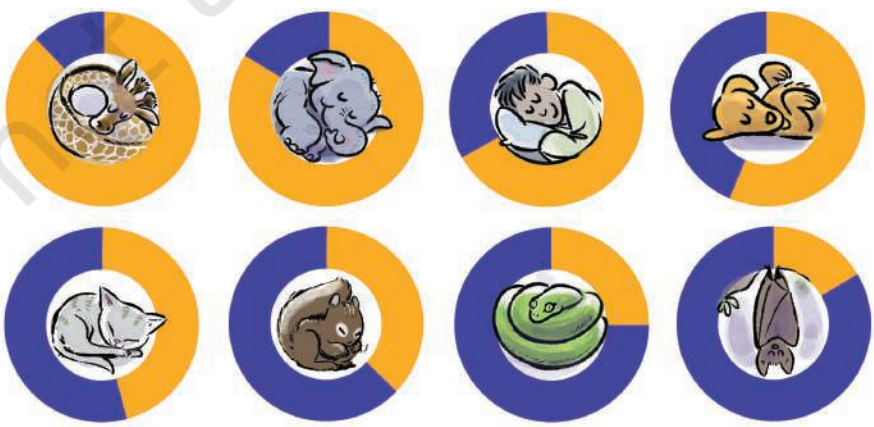
If they cut the vegetables together, they will finish in $\frac{3}{5}$ hours.

Figure it Out

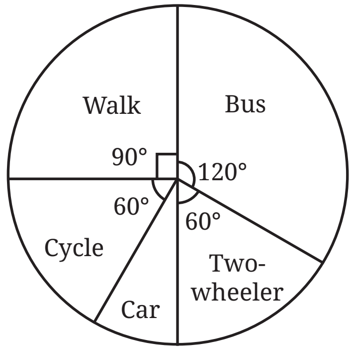
1. Which of the following pairs of quantities are in inverse proportion?
   - (i) The number of taps filling a water tank and the time taken to fill it.
   - (ii) The number of painters hired and the days needed to paint a wall of fixed size.
   - (iii) The distance a car can travel and the amount of petrol in the tank.
   - (iv) The speed of a cyclist and the time taken to cover a fixed route.
   - (v) The length of cloth bought and the price paid at a fixed rate per metre.
   - (vi) The number of pages in a book and the time required to read it at a fixed reading speed.
2. If 24 pencils cost ₹120, how much will 20 such pencils cost?
3. A tank on a building has enough water to supply 20 families living there for 6 days. If 10 more families move in there, how long will the water last? What assumptions do you need to make to work out this problem?

Math Talk

4. Fill in the average number of hours each living being sleeps in a day by looking at the charts. Select the appropriate hours from this list : 15, 2.5, 20, 8, 3.5, 13, 10.5, 18.



5. The pie chart on the right shows the result of a survey carried out to find the modes of transport used by children to go to school. Study the pie chart and answer the following questions.



   (i) What is the most common mode of transport?

   (ii) What fraction of children travel by car?

   (iii) If 18 children travel by car, how many children took part in the survey? How many children use taxis to travel to school?

   (iv) By which two modes of transport are equal numbers of children travelling?

6. Three workers can paint a fence in 4 days. If one more worker joins the team, how many days will it take them to finish the work? What are the assumptions you need to make?

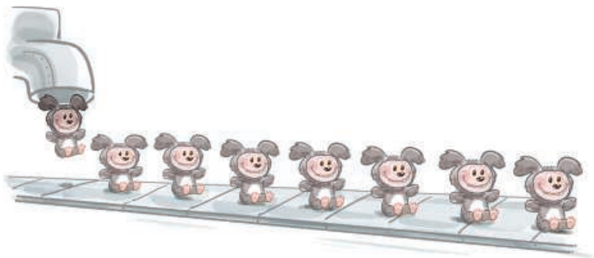
7. It takes 6 hours to fill 2 tanks of the same size with a pump. How long will it take to fill 5 such tanks with the same pump?

8. A given set of chairs are arranged in 25 rows, with 12 chairs in each row. If the chairs are rearranged with 20 chairs in each row, how many rows does this new arrangement have?

9. A school has 8 periods a day, each of 45 minutes duration. How long is each period, if the school has 9 periods a day, assuming that the number of school hours per day stays the same?

10. A small pump can fill a tank in 3 hours, while a large pump can fill the same tank in 2 hours. If both pumps are used together, how long will the tank take to fill?

11. A factory requires 42 machines to produce a given number of toys in 63 days. How many machines are required to produce the same number of toys in 54 days?



12. A car takes 2 hours to reach a destination, travelling at a speed of 60 km/h. How long will the car take if it travels at a speed of 80 km/h?

SUMMARY

- Ratios in the form $a : b : c : d : \cdots$ indicate that for every a units of the first quantity, there are b units of the second quantity, c units of the third quantity, and so on.
- If x is divided into many parts in the ratio $p : q : r : s : \cdots$, then the quantity of the first part is $x \times \frac{p}{(p + q + r + s + \cdots)}$, the quantity of the second part is $x \times \frac{q}{(p + q + r + s + \cdots)}$, and so on.
- Two quantities are directly proportional when they both change by the same factor, and their quotient remains the same. For example, if x and y are two quantities that are directly proportional, and $(x_1, x_2, x_3, \ldots)$ and $(y_1, y_2, y_3, \ldots)$ are the corresponding values of x and y, then $\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = \cdots = k$, where k is a constant.
- Quantities are inversely proportional if, when one quantity changes by a factor n, the other quantity changes by the inverse $\frac{1}{n}$. For example, if x and y are two quantities that are inversely proportional, and $(x_1, x_2, x_3, \ldots)$ and $(y_1, y_2, y_3, \ldots)$ are the corresponding values of x and y, then $x_1y_1 = x_2y_2 = x_3y_3 = \cdots = n$, where n is a constant.