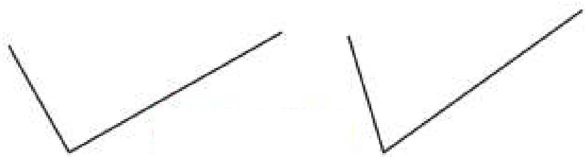
Geometric Twins Class 7 Solutions Maths Ganita Prakash Part 2 Chapter 1

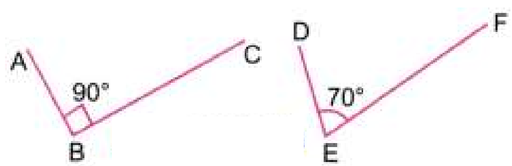
1.1 Geometric Twins

Figure It Out (Pages 3-4)

Question 1.
Check if the two figures are congruent.

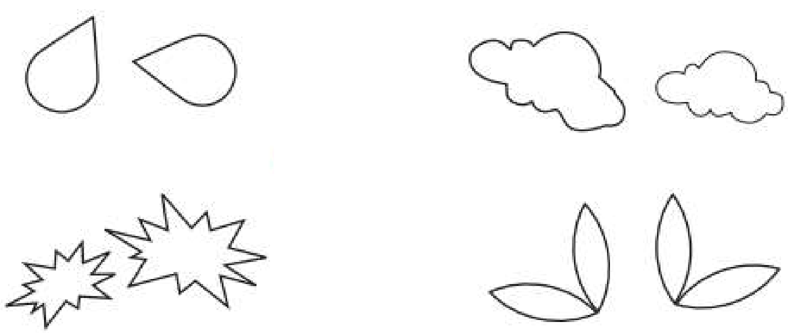


Solution:
Let's measure the angles above with a protractor.
We found as follows:



Here, ∠ABC does not coincide with ∠DEF. Hence, the given figures are not congruent.

Question 2.
Circle the pairs that appear congruent.



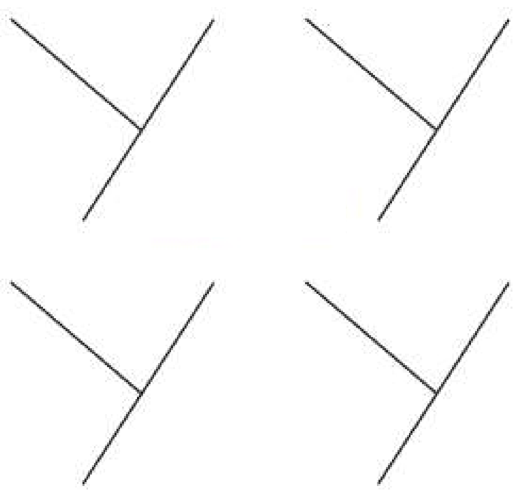
Solution: (a) and (d) are congruent. As they can be superimposed exactly.

Question 3.
What measurements would you take to create a figure congruent to a given: (a) Circle (b) Rectangle Using this, state how you would check if two (a) Are circles congruent? (b) Rectangles are congruent?

Solution:
(a) I will measure the radius or diameter of the given circle. (b) I will measure the length and breadth of the given rectangle. (a) I will place one circle over another circle. If they exactly superimpose, they are congruent. In this case, both will have the same radius. (b) I will place one rectangle over another rectangle. If they exactly superimpose, they are congruent. In such a case, both will have the same length and breadth.

Question 4.
How would we check if two figures like the one below are congruent?

Use this to identify whether each of the following pairs is congruent.



Solution:
To check if the two figures are congruent, one would need to measure the lengths of the corresponding line segments and the angle between them. Yes, each of the given figures is congruent. Length of line segments in each figure is 3.3 cm (Horizontal line) and 2.3 cm (Vertical line), and the angle between them is 82°.

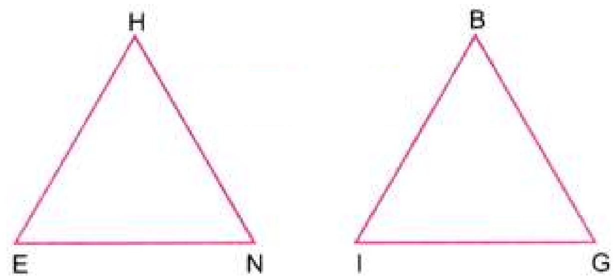
1.2 Congruence of Triangles

Figure It Out (Pages 8-9)

Question 1.

Suppose ∆HEN is congruent to ∆BIG. List all the other correct ways of expressing this congruence.

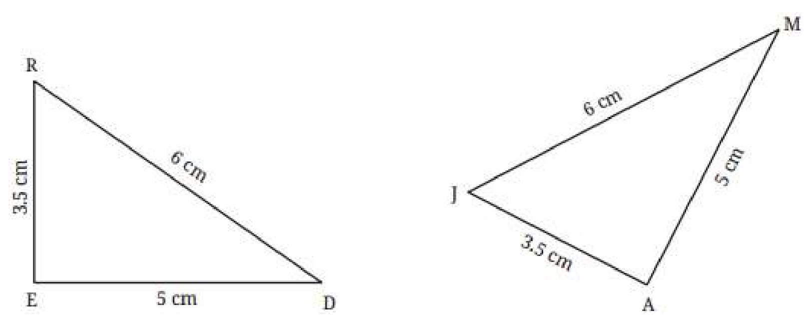
Solution:
Given



∆HEN = ∆BIG means that the vertices H, E, and N correspond to B, I, and G, respectively. There are six ways to write a congruence statement for two congruent triangles. The other five ways are (i) ∆HNE ≅ ∆BGI (ii) ∆EHN ≅ ∆IBG (iii) ∆ENH ≅ ∆IGB (iv) ∆NHE ≅ ∆GBI (v) ∆NEH ≅ ∆GIB

Question 2.

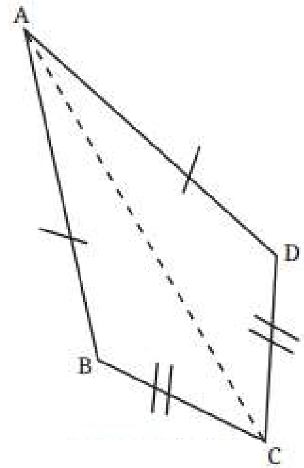
Determine whether the triangles are congruent. If yes, express the congruence.



Solution:
Given the side lengths of the two triangles RE = 3.5 cm, ED = 5 cm, RD = 6 cm and JA = 3.5 cm, AM = 5 cm, JM = 6 cm Clearly RE = JA = 3.5 cm ED = AM = 5 cm RD = JM = 6 cm Hence ∆RED ≅ ∆JAM.

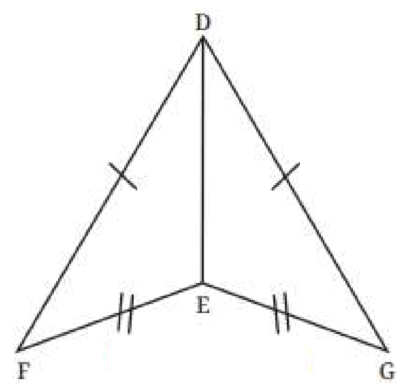
Question 3.

In the figure below, AB = AD, CB = CD. Can you identify any pair of congruent triangles? If yes, explain why they are congruent. Does AC divide ∠BAD and ∠BCD into two equal parts? Give reasons.



Solution:
CB = CD AC = AC (Common side) Since all three sides of ∆ABC are equal to the corresponding three sides ∆ADC, the triangles are congruent by the side-side-side (SSS) congruence criterion. Hence ∆ABC ≅ ∆ADC Yes, AC divides ∠BAD and ∠BCD into equal parts. Since ∆ABC ≅ ∆ADC Then, ∠BAC = ∠DAC and ∠BCA = ∠DCA This means that AC bisects both ∠BAD and ∠BCD.

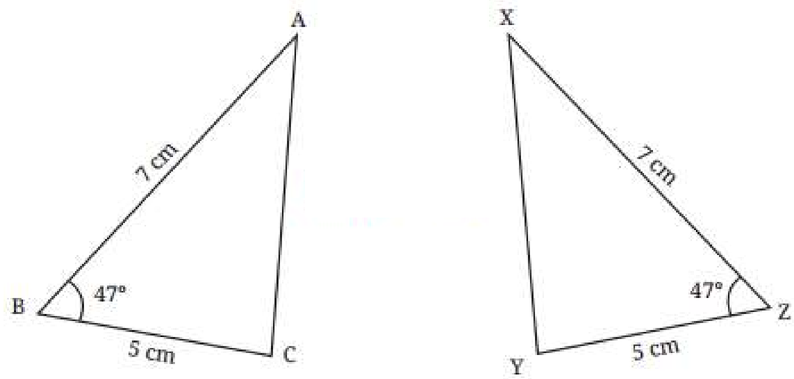
Question 4.
In the figure below, are ∆DFE and ∆GED congruent to each other? It is given that DF = DG and FE = GE.



Solution:
Given DF = DG and FE = GE The side DE is common to both triangles ∆DFE and ∆DGE Hence, by the SSS congruence criterion ∆DFE ≅ ∆DGE The order of the vertices matters in congruence statements. The vertices must correspond correctly. In ∆DFE and ∆DGE DF corresponds to DG FE corresponds to EG ED is common. Given statements DF = DG and FE = GE do not support the congruence of ∆DFE and ∆GED because the corresponding sides are not equal.

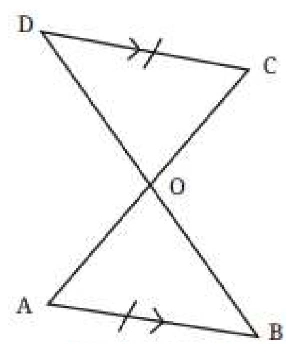
Figure It Out (Pages 13-14)

Question 1.
Identify whether the triangles below are congruent. What conditions did you use to establish their congruence? Express the congruence.



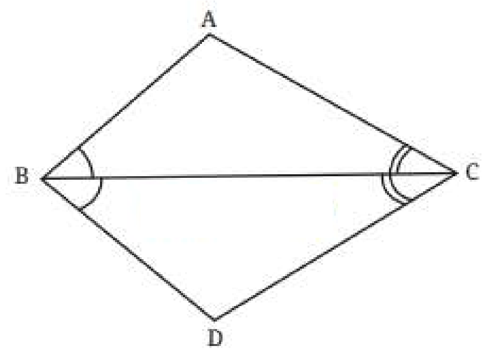
Solution:
Here, BC = ZY = 5 cm BA = ZX = 7 cm ∠ABC = ∠XZY = 47° ∆ABC ≅ ∆XZY
Since the two sides and the included angle of triangle ABC are equal to the two sides and the included angle of triangle XZY. The triangles are congruent by the side-angle-side condition. It can be expressed as ∆ABC ≅ ∆XZY.

Question 2.
Given that CD and AB are parallel, and AB = CD, what are the other equal parts in this figure? (Hint: When the lines are parallel, the alternate angles are equal. Are the two resulting triangles congruent? If so, express the congruence.)



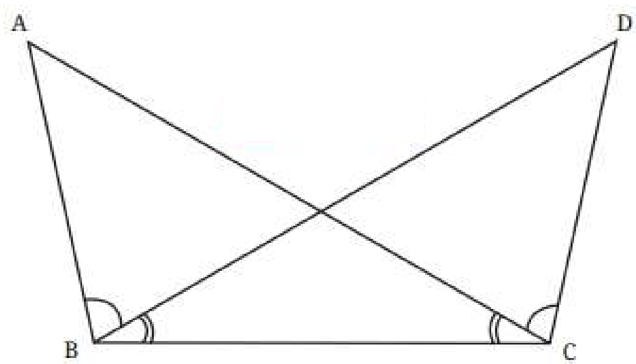
Solution:
Given that DC || AB; AB = CD ∠OCD = ∠OAB; ∠OBA = ∠ODC (∵ Alternate Angles) ∠DOC = ∠BOA OA = OC; OB = OD

Question 3.
Given that ∠ABC = ∠DBC and ∠ACB = ∠DCB, show that ∠BAC = ∠BDC. Are the two triangles congruent?



Solution:
Let ∠ABC = a = ∠DBC and let ∠ACB = b = ∠DCB BC is a common side of the two triangles, then ΔABC ≅ ΔDBC Angle Side Angle Congruence: When two triangles are congruent, their corresponding parts are equal [CPCT] Since ΔABC ≅ ΔDBC, their corresponding angles are equal. Hence ∠BAC = ∠BDC

Question 4.
Identify the equal parts in the following figure, given that ∠ABD = ∠DCA and ∠ACB = ∠DBC.



Solution:
Given ∠ABD = ∠DCA; ∠ACB = ∠DBC ∠AOB = ∠DOC (∵ They are vertically opposite angles) AO = DO; CO = BO ΔCOD ≅ ΔBOA Angle-side-angle condition

1.3 Angles of Isosceles and Equilateral Triangles

Figure It Out (Pages 20-21)

Question 1.
ΔAIR ≅ ΔFLY. Identify the corresponding vertices, sides, and angles.

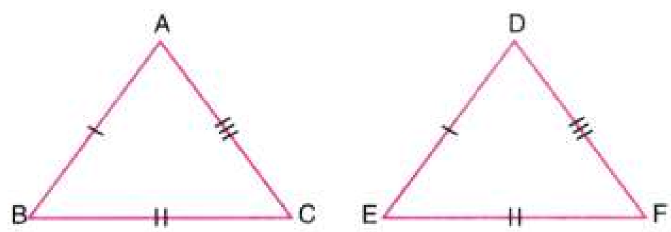
Solution:
Here ΔAIR ≅ ΔFLY. The corresponding parts are as follows: Corresponding Vertices A corresponds to F I corresponds to L R corresponds to Y

Corresponding Sides AI corresponds to FL IR corresponds to LY AR corresponds to FY

Corresponding Angles ∠A corresponds to ∠F ∠I corresponds to ∠L ∠R corresponds to ∠Y

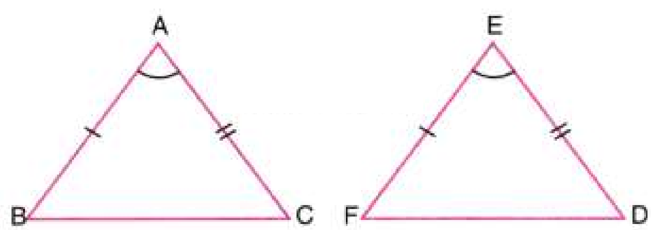
Question 2.
Each of the following cases contains certain measurements taken from two triangles. Identify the pairs in which the triangles are congruent to each other, with reason. Express the congruence whenever they are congruent. (a) AB = DE BC = EF CA = DF (b) AB = EF ∠A = ∠E AC = ED (c) AB = DF ∠B = ∠D = 90° AC = FE (d) ∠A = ∠D ∠B = ∠E AC = DF (e) AB = DF ∠B = ∠F AC = DE

Solution:
(a) Here, AB = DE BC = EF CA = FD



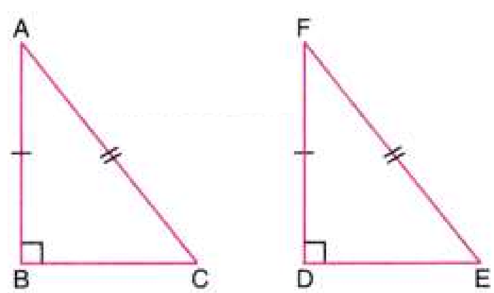
All three corresponding sides are equal. Thus, triangles are congruent by the side-side-side congruence. Hence, ∆ABC ≅ ∆DEF.

(b) Given AB = EF ∠A = ∠E AC = ED



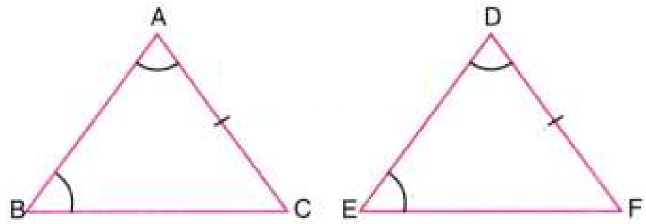
Two corresponding sides and the included angle are equal. Thus, triangles satisfy the SAS condition. Hence, ∆ABC ≅ ∆EFD

(c) Here, AB = FD ∠B = ∠D = 90° AC = FE



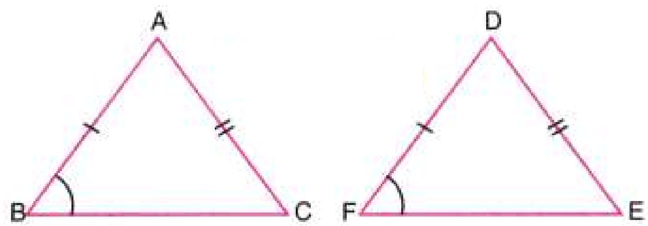
The triangles have equal right angles, equal hypotenuses, and one equal corresponding side. Thus, triangles satisfy the RHS conditions. Hence, ∆ABC ≅ ∆FDE.

(d) Here, ∠A = ∠D ∠B = ∠E AC = DF



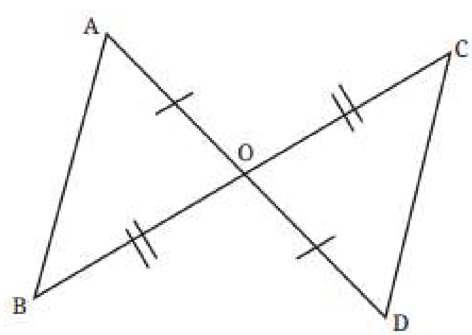
Clearly, two corresponding angles and one corresponding side are equal. Thus, triangles satisfy the AAS conditions. Hence, ∆ABC ≅ ∆DEF.

(e) Here, AB = DF ∠B = ∠F AC = DE



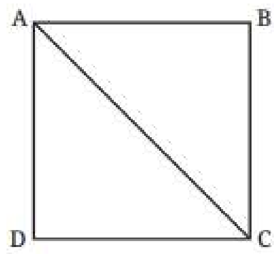
Here, two corresponding sides and a non-included angle are equal. Thus, the triangles satisfy the SSA condition, which is not a valid congruence rule. Hence, ∆ABC need not be congruent to ∆DFE.

Question 3.
It is given that OB = OC, and OA = OD. Show that AB is parallel to CD.
[Hint: AD is a transversal for these two lines. Are there any equal alternate angles?]



Solution:
Given OB = OC OA = OD Then ∠AOB = ∠COD ∆AOB ≅ ∆COD (∵ Side angle side condition) So, these two triangles can be superimposed exactly. Therefore, ∠A = ∠D ∠B = ∠C Hence, AB is parallel to CD.

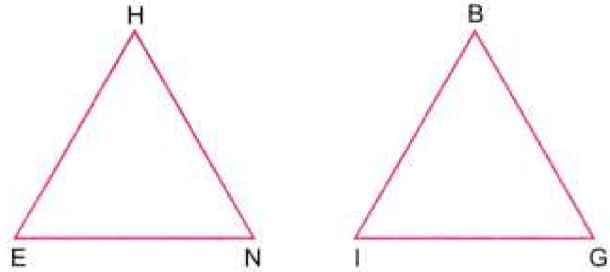
Question 4.
ABCD is a square. Show that ∆ABC ≅ ∆ADC. Is ∆ABC also congruent to ∆CDA?



Give more examples of two triangles where one triangle is congruent to the other in two different ways, as in the case above. Can you give an example of two triangles where one is congruent to the other in six different ways?

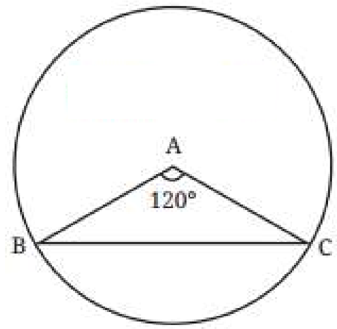
Solution:
To show ∆ABC ≅ ∆ADC (∵ ABCD is a square)

CD = CB
AC = AC (Common side)
So, ∆ABC ≅ ∆ADC (Using SSS criterion)
For ∆ABC to be congruent to ∆CDA
AB = CD
BC = DA
AC = CA
Since ABCD is a square
Hence, ∆ABC is congruent to ∆CDA by side side-side-side condition.
Now, let us take two congruent triangles ∆HEN and ∆BIG.



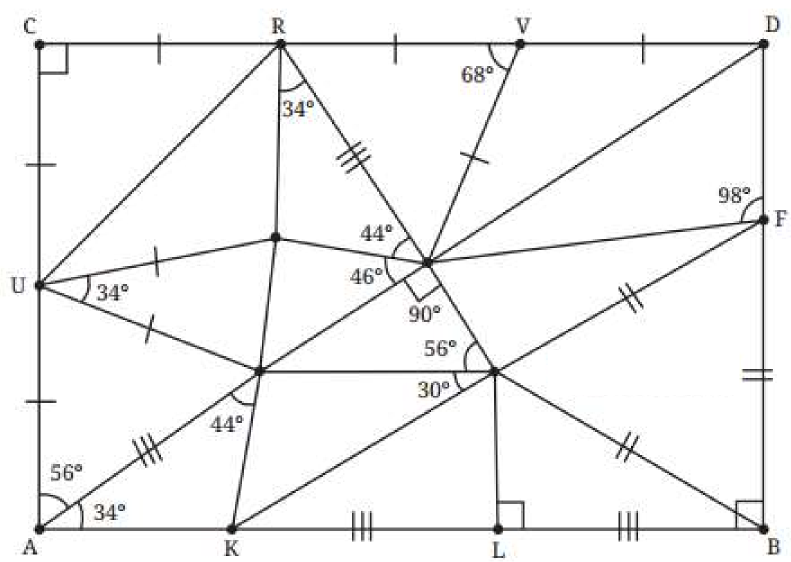
There are six ways to write a congruence statement for two congruent triangles. The other five ways are (i) ∆HNE ≅ ∆BGI (ii) ∆EHN ≅ ∆IBG (iii) ∆ENH ≅ ∆IGB (iv) ∆NHE ≅ ∆GBI (v) ∆NEH ≅ ∆GIB (vi) ∆HEN ≅ ∆BEN

Question 5.
Find ∠B and ∠C, if A is the centre of the circle.



Solution:
In ∆BAC AB = AC = Radius of the circle. Let ∠ABC = ∠ACB = x (angles opposite to equal sides are equal) Then, x + x + 120° = 180° (∵ Sum of all the angles in a triangle is 180°) ⇒ 2x = 180° – 120° ⇒ 2x = 60° ⇒ x = 30° Hence ∠B = ∠C = 30°

Question 6.
Find the missing angles. As per the convention that we have been following, all line segments marked with a single '|' are equal to each other, and those marked with a double '|' are equal to each other, etc.



Solution:
In ∆CUR ∠CUR = ∠CRU = x (say) (∵ CU = CR) According to the angle sum property of a triangle x + x + 90° = 180° ⇒ 2x = 180° – 90° = 90° ⇒ x = 45° ∴ ∠CUR = ∠CRU = 45°

In ∆VRN ∠VRN = ∠VNR = a (say) [∵ Angles opposite to equal sides are equal] ∵ VR = VN Since the sum of the angles of a triangle is 180°. So, a + a + 68° = 180° ⇒ 2a = 180° – 68° ⇒ 2a = 112° ⇒ a = 56° ∠VRN = ∠VNR = 56°

In ΔAUP $\angle UAP = \angle UPA$ ($\because$ they are equal) $\angle UPA = 56°$ The sum of the angles of a triangle is 180°. So, $56° + 56° + \angle AUP = 180° \Rightarrow 112° + \angle AUP = 180° \Rightarrow \angle AUP = 180° – 112° \Rightarrow \angle AUP = 68°$ ΔBOF is an equilateral triangle as all sides are equal. So, $OB = OF = BF$ $\angle FOB = \angle FBO = \angle OFB = 60°$ $\angle RVN + \angle DVN = 180° \Rightarrow 68° + \angle DVN = 180° \Rightarrow \angle DVN = 180° – 68° \Rightarrow \angle DVN = 112°$ $\angle VND + \angle VDN + \angle NVD = 180°$ $\because VN = VD$ $\therefore \angle VND = \angle VDN = c$ $\therefore c + c + 112° = 180° \Rightarrow 2c = 180° – 112° \Rightarrow 2c = 68° \Rightarrow c = 34°$ $\angle VND = \angle VDN = 34°$

In ΔOLB $\angle OBL = 90° – 60° = 30°$ $\angle LOB = 60°$ [$\because$ LO || BF and BO is transversal]

In ΔOPN $\angle OPN + \angle PON + \angle PNO = 180° \Rightarrow \angle OPN + 56° + 90° = 180° \Rightarrow \angle OPN + 146° = 180° \Rightarrow \angle OPN = 180° – 146° \Rightarrow \angle OPN = 34°$ Now, $\angle APK + \angle KPO + \angle OPN = 180°$ [$\because$ Straight angle is 180°] $\Rightarrow 44° + \angle KPO + 34° = 180° \Rightarrow \angle KPO = 180° – 78° \Rightarrow \angle KPO = 102°$

In ΔKPO $\angle KPO + \angle POK + \angle PKO = 180° \Rightarrow 102° + 30° + \angle PKO = 180° \Rightarrow 132° + \angle PKO = 180° \Rightarrow \angle PKO = 180° – 132° = 48°$ $\angle KAP + \angle KPA + \angle AKP = 180°$ [$\because$ Sum of angles of a triangle is 180°] $\Rightarrow 34° + 44° + \angle AKP = 180° \Rightarrow 78° + \angle AKP = 180° \Rightarrow \angle AKP = 180° – 78° \Rightarrow \angle AKP = 102°$ and $\angle PKO = 48°$ So, $\angle AKP + \angle PKO + \angle OKL = 180° \Rightarrow 102° + 48° + \angle OKL = 180° \Rightarrow 150° + \angle OKL = 180° \Rightarrow \angle OKL = 180° – 150° \Rightarrow \angle OKL = 30°$

In ΔKOL $\angle OKL + \angle OLK + \angle KOL = 180° \Rightarrow 30° + 90° + \angle KOL = 180° \Rightarrow \angle KOL = 180° – 120° \Rightarrow \angle KOL = 60°$ Also, $\Delta OKL \cong \Delta OBL$ $KL = LB$ $\angle OLK \cong \angle OLB = 90°$ Side angle side condition.