Finding Common Ground Class 7 Solutions Maths Ganita Prakash Part 2 Chapter 3

3.1 The Greatest of All

Figure It Out (Page 51)

Question 1.
List all the factors of the following numbers: (a) 90 (b) 105 (c) 132 (d) 360 (this number has 24 factors) (e) 840 (this number has 32 factors)

Solution:
(a) 90 Here

2	90
3	45
3	15
5	5
	1

∴ 90 = 2 × 3 × 3 × 5 Hence, factors of 90 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, and 90. Total factors = 12

(b) 105 Here

3	105
5	35
7	7
	1

∴ 105 = 3 × 5 × 7 Hence, factors of 105 are 1, 3, 5, 7, 15, 21, 35, and 105. Total factors = 8

(c) 132 Here

2	132
2	66
3	33
11	11
	1

∴ 132 = 2 × 2 × 3 × 11 Hence, factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132. Total factors = 12

(d) 360 Here

2	360
2	180
2	90
3	45
3	15
5	5
	1

∴ 360 = 2 × 2 × 2 × 3 × 3 × 5 Hence, factors of 360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360. Total factors = 24

(e) 840 Here

2	840
2	420
2	210
3	105
5	35
7	7
	1

∴ 840 = 2 × 2 × 2 × 3 × 5 × 7 Hence, factors of 840 are 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 20, 21, 24, 28, 30, 35, 40, 42, 56, 60, 70, 84, 105, 120, 140, 168, 210, 280, 420, 840. Total factors = 32

Figure It Out (Page 53)

Question 1.
Find the common factors and the HCF of the following numbers: (a) 50, 60 (b) 140, 275 (c) 77, 725 (d) 370, 592 (e) 81, 243 How do we directly find the HCF without listing all the factors?

Solution:
(a) Here, 50

2	50
5	25
	5

$\therefore 50 = \boxed{2} \times 5 \times \boxed{5}$

and 60

2	60
2	30
3	15
5	5
	1

$\therefore 60 = \boxed{2} \times 2 \times 3 \times \boxed{5}$

∴ Common factors of 50 and 60 are 2, 5, and HCF (50, 60) = 2 × 5 = 10.

(b) Here 140 and 275

2	140
2	70
5	35
7	7
	1

5	275
5	55
11	11
	1

∴ 140 = 2 × 2 × 5 × 7 and 275 = 5 × 5 × 11 ∴ Common factor of 140 and 275 = 5, and HCF of 140 and 275 = 5.

(c) Here, 77 and 725

7	77
11	11
	1

and

5	725
5	145
29	29
	1

∴ 77 = 7 × 11 and 725 = 5 × 5 × 29 ∴ Common factor = 1 and HCF (77, 725) = 1 (as there are no common prime factors)

(d) Here, 370 and 592

2	370
5	185
37	37
	1

and

2	592
2	296
2	148
2	74
37	37
	1

∴ 370 = 2 × 5 × 37 and 592 = 2 × 2 × 2 × 2 × 37 ∴ Common factors = 1, 2, 37, and HCF (370, 592) = 2 × 37 = 74

(e) Here 81 and 243

$$\begin{array}{c|c} 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \quad \text{and} \quad \begin{array}{c|c} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

∴ 81 = 3 × 3 × 3 × 3 and 243 = 3 × 3 × 3 × 3 × 3 ∴ Common factors = 3 × 3 × 3 × 3 and HCF (81, 243) = 3 × 3 × 3 × 3 = 81.

Figure It Out (Page 54)

Question 1.
Find the HCF of the following numbers: (a) 24, 180 (b) 42, 75, 24 (c) 240, 378 (d) 400, 2500 (e) 300, 800

Solution:
(a) Given 24, 180

$$\begin{aligned} \text{Now } 24 &= \boxed{2 \times 2} \times 2 \times \boxed{3} \\ \text{and } 180 &= \boxed{2 \times 2} \times 3 \times \boxed{3} \times 5 \end{aligned}$$

Clearly, the common prime factors are 2 and 3. ∴ HCF (24, 180) = 2 × 2 × 3 = 4 × 3 = 12

(b) Given 42, 75, 24 Now

$$\begin{aligned} 42 &= 2 \times 7 \times \boxed{3} \\ 75 &= 5 \times 5 \times \boxed{3} \\ 24 &= 2 \times 2 \times \boxed{3} \times 2 \end{aligned}$$

Clearly only common prime factor is 3. ∴ HCF (42, 75, 24) = 3.

(c) Here 240 and 378

2	240
2	120
2	60
2	30
3	15
5	5
	1

2	378
3	189
3	63
3	21
7	7
	1

$\therefore \quad 240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

and $\quad 378 = 2 \times 3 \times 3 \times 3 \times 7$

Hence, HCF(240, 378) = 2 × 3 = 6

(d) Here 400 and 2500

2	400
2	200
2	100
2	50
5	25
5	5
	1

2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$\therefore \quad 400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

and $\quad 2500 = 2 \times 2 \times 5 \times 5 \times 5 \times 5$

∴ HCF(400, 2500) = 2 × 2 × 5 × 5 = 100.

(e) Here 300 and 800

2	300
2	150
3	75
5	25
5	5
	1

and

2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

$\therefore \quad 300 = 2 \times 2 \times 3 \times 5 \times 5$

$\quad 800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

∴ HCF(300, 800) = 2 × 2 × 5 × 5 = 100.

Question 2.
Consider the numbers 72 and 144. Suppose they are factorised into composite numbers as: 72 = 6 × 12 and 144 = 8 × 18. Seeing this, can one say that these two numbers have no common factor other than 1? Why not?

Solution:
No, one cannot say that 72 and 144 have no common factor other than 1 because their factorisations have composite numbers. Here 72 = 6 × 12 144 = 8 × 18 These are not prime factorisations. Both 6 and 12 are composite numbers, as are 8 and 18. Prime factorisation of 72 = 2 × 2 × 2 × 3 × 3 Prime factorisation of 144 = 2 × 2 × 2 × 2 × 3 × 3 ∴ HCF(72, 144) = 2 × 2 × 2 × 3 × 3 = 8 × 9 = 72 Since the HCF is 72, which is greater than 1, the numbers have common factors other than 1.

3.2 Least, but not Last!

Figure It Out (Page 58)

Question 1.
Find the LCM of the following numbers: (a) 30, 72 (b) 36, 54 (c) 105, 195, 65 (d) 222, 370

Solution:
(a) 30, 72 Here 30 = 2 × 3 × 5 (One occurrence of 2, one occurrence of 3, and one occurrence of 5) (Three occurrences of 2s and two occurrences of 3s) ∴ LCM (30, 72) = 2 × 2 × 2 × 3 × 3 × 5 (Three occurrences of 2s and two occurrences of 3s, and one occurrence of 5) = 8 × 9 × 5 = 360

(b) 36, 54 Here 36 = 2 × 2 × 3 × 3 (Two occurrences of 2s and two occurrences of 3s) (One occurrence of 2, and three occurrences of 3s) ∴ LCM(36, 54) = 2 × 2 × 3 × 3 × 3 = 4 × 27 = 108 (One occurrence of 2, and three occurrences of 3s) ∴ LCM(36, 54) = 2 × 2 × 3 × 3 × 3 = 4 × 27 = 108

(c) 105, 195, 65 Here 105, 195, and 65 Now 105 = 3 × 5 × 7 195 = 3 × 5 × 13 65 = 5 × 13 ∴ LCM(105, 195, 65) = 3 × 5 × 7 × 13 = 1365

(d) 222, 370 Here 222, 370

2	222
3	111
37	37
	1

2	370
5	185
37	37
	1

222 = 2 × 3 × 37 and 370 = 2 × 5 × 37 ∴ LCM(222, 370) = 2 × 3 × 5 × 37 = 1110.

3.3 Patterns, Properties, and a Pretty Procedure!

Figure It Out (Page 59)

Question 1.
Make a general statement about the HCF for the following pairs of numbers. You could consider examples before coming up with general statements. Look for possible explanations of why they hold. (a) Two consecutive even numbers (b) Two consecutive odd numbers (c) Two even numbers (d) Two consecutive numbers (e) Two co-prime numbers Share your observations with the class.

Solution:
(a) Two Consecutive Even Numbers Examples: (i) (2, 4) HCF (2, 4) = 2 (ii) (6, 8) HCF (6, 8) = 2 (iii) (10, 12) HCF (10, 12) = 2 General Statement: The HCF of any two consecutive even numbers is 2. Reason: All even numbers are divisible by 2, and consecutive even numbers differ by 2. They will not have any other common factor except 2.

(b) Two Consecutive Odd Numbers Examples: (i) (3, 5) HCF (3, 5) = 1 (ii) (7, 9) HCF (7, 9) = 1 (iii) (11, 13) HCF (11, 13) = 1 General Statement: The HCF of any two consecutive odd numbers is 1. Reason: Consecutive odd numbers are not divisible by any common even or odd factor other than 1, so they are always co-prime.

(c) Two Even Numbers Examples: (i) (4, 10) HCF (4, 10) = 2 (ii) (8, 12) HCF (8, 12) = 4 (iii) (14, 20) HCF (14, 20) = 2 General Statement: The HCF of two even numbers is always an even number. Reason: Since all even numbers are divisible by 2, their HCF will include 2 as a factor. If both have more factors in common, the HCF will be a multiple of 2.

(d) Two Consecutive Numbers Examples: (i) (7, 8) HCF (7, 8) = 1 (ii) (14, 15) HCF (14, 15) = 1 (iii) (20, 21) HCF (20, 21) = 1 General Statement: The HCF of any two consecutive numbers is 1. Reason: Consecutive numbers can never share any common factor other than 1, because every next number is exactly 1 more than the previous number.

(e) Two Co-prime Numbers Examples: (i) (4, 9) HCF (4, 9) = 1 (ii) (5, 8) HCF (5, 8) = 1 (iii) (7, 10) HCF (7, 10) = 1 General Statement: The HCF of two co-prime numbers is always 1. Reason: Co-prime numbers are defined as numbers that have no common factor other than 1.

Question 2.
The LCM of 3 and 24 is 24 (it is one of the two given numbers). (a) Find more such number pairs where the LCM is one of the two numbers. (b) Make a general statement about such numbers. Describe such number pairs using algebra.

Solution:
(a) Here are more number pairs where the LCM of the two numbers is one of the given numbers: (i) LCM of 2 and 4: The LCM is 4. Prime factorization of 2: 2 Prime factorization of 4: 2 × 2 LCM (2, 4) = 2 × 2 = 4

(ii) LCM of 5 and 10: The LCM is 10. Prime factorization of 5: 5 Prime factorization of 10: 2 × 5 LCM (5, 10) = 2 × 5 = 10

(iii) LCM of 6 and 12: The LCM is 12. Prime factorization of 6: 2 × 3 Prime factorization of 12: 2 × 2 × 3 LCM (6, 12) = 2 × 2 × 3 = 12

(iv) LCM of 7 and 49: The LCM is 49. Prime factorization of 7: 7 Prime factorization of 49: 7 × 7 LCM (7, 49) = 49

(v) LCM of 10 and 100: The LCM is 100. Prime factorization of 10: 2 × 5 Prime factorization of 100: 2 × 2 × 5 × 5 LCM (10, 100) = 2 × 2 × 5 × 5 = 100

(b) General Statement: For any two positive integers, let's call them a and b, the Least Common Multiple (LCM) will be equal to one of the numbers (specifically, the larger number) if and only if the smaller number is a factor (or a divisor) of the larger number. In the original example, the LCM of 3 and 24 is 24 because 3 is a factor of 24 (24 ÷ 3 = 8) Algebraic Description: Let the two positive integers be a and b, where $a < b$. The LCM of a and b will be b if and only if b is a multiple of a. This can be expressed using algebra as: LCM(a, b) = b if and only if $b = k \times a$, where k is a positive integer.

Question 3.
Make a general statement about the LCM for the following pairs of numbers. You could consider examples before coming up with these general statements. Look for possible explanations of why they hold. (a) Two multiples of 3 (b) Two consecutive even numbers (c) Two consecutive numbers (d) Two co-prime numbers

Solution:
(a) Two Multiples of 3 Examples: (i) (6, 9) LCM (6, 9) = 18 (ii) (9, 12) LCM (9, 12) = 36 (iii) (12, 18) LCM (12, 18) = 36 Observation: The LCM of two multiples of 3 is also a multiple of 3. Reason: Since both numbers are divisible by 3, their

common multiples will also be divisible by 3. Hence, the LCM must include 3 as a factor. General Statement: The LCM of two multiples of 3 is always a multiple of 3.

(b) Two Consecutive Even Numbers Examples: (i) (2, 4) LCM (2, 4) = 4 (ii) (6, 8) LCM (6, 8) = 24 (iii) (10, 12) LCM (10, 12) = 60 Observation: The LCM of two consecutive even numbers is half of their product. Reason: Consecutive even numbers always share a common factor of 2, but not more. Therefore, when finding the LCM, one factor of 2 overlaps, so the LCM becomes smaller than their product. General Statement: The LCM of two consecutive even numbers 2n and 2n + 2 is always equal to half of their product. or LCM (2n, 2n + 2) = 2n×(2n+2)2
= n(2n + 2)
= $2n^2 + 2n$

(c) Two Consecutive Numbers Examples: (i) (7, 8) LCM (7, 8) = 56 (ii) (9, 10) LCM (9, 10) = 90 (iii) (10, 11) LCM (10, 11) = 110 Observation: The LCM of two consecutive numbers is equal to their product. Reason: Consecutive numbers have no common factors other than 1, so their product is the smallest number divisible by both. General Statement: The LCM of two consecutive numbers is their product.

(d) Two Co-prime Numbers Examples: (i) (4, 9) LCM (4, 9) = 36 (ii) (5, 8) LCM (5, 8) = 40 (iii) (7, 10) LCM (7, 10) = 70 Observation: The LCM of two co-prime numbers is equal to their product. Reason: Co-prime numbers do not share any common factors except 1, so the smallest number that contains both is simply their product. General Statement: The LCM of two co-prime numbers is equal to their product. Note: Co-prime numbers are any two natural numbers that have no common factor other than 1.

Figure It Out (Pages 63-64)

Question 1.
In the two rows below, colours repeat as shown. When will the black stars meet next?

Solution:
The positions of the black star in the first row are 4 and 10. The difference in position = 10 – 4 = 6. So, the next positions of the black star in the first row are:

4, 10, 16, 22, 28, … The positions of the black star in the second row are 4 and 8. The difference in position = 8 – 4 = 4. So, the next positions of the black star in the second row are: 4, 8, 12, 16, 20, … The 16th position is common in both rows. Hence, the black stars meet again at the 16th position.

Question 2.
(a) Is 5 × 7 × 11 × 11 a multiple of 5 × 7 × 7 × 11 × 2? (b) Is 5 × 7 × 11 × 11 a factor of 5 × 7 × 7 × 11 × 2?

Solution:
(a) Here let a = 5 × 7 × 11 × 11 and b = 5 × 7 × 7 × 11 × 2 For a number a to be a multiple of number b, all prime factors of b must be present in a with at least the same power. Number a does not have the prime factor 2. Number a has 71, while number b has 7. Hence, a is not a multiple of b.

(b) Let a = 5 × 7 × 11 × 11 6 = 5 × 7 × 7 × 11 × 2 For a to be a factor of b, b must contain all the prime factors of a, in equal or higher powers. But here b has only one 11 (a has two 11s), so b does not include all factors of a. Hence, 5 × 7 × 11 × 11 is not a factor of 5 × 7 × 7 × 11 × 2.

Question 3.
Find the HCF and LCM of the following (state your answers in the form of prime factorisations): (a) 3 × 3 × 5 × 7 × 7 and 12 × 7 × 11 (b) 45 and 36

Solution:
(a) Here 3 × 3 × 5 × 7 × 7 and 12 × 7 × 11 = 2 × 2 × 3 × 7 × 11 ∴ HCF = 3 × 7 = 21 ∴ LCM = 2 × 2 × 3 × 3 × 5 × 7 × 7 × 11.

(b) Here 45 = 3 × 3 × 5 36 = 2 × 2 × 3 × 3 HCF (45, 36) = 3 × 3 LCM (45, 36) = 2 × 2 × 3 × 3 × 5

Question 4.
Find two numbers whose HCF is 1, and LCM is 66.

Solution:
For two numbers a and b, the product of the numbers is equal to the product of their LCM and HCF. ∴ a × b = HCF(a, b) × LCM(a, b) Here HCF = 1, LCM = 66 ⇒ a × b = 1 × 66 = 66 ∴ Pairs of factors of 66 are (1, 66), (2, 33), (3, 22), (6, 11) Any of the following pairs of numbers will satisfy the conditions: (1, 66), (2, 33), (3, 22), (6, 11).

Question 5.
A cowherd took all his cows to graze in the fields. The cows can go to a crossing with 3 gates. An equal number of cows passed through each gate. Later, at another crossing with 5 gates again an equal number of

cows passed through each gate. The same happened at the third crossing with 7 gates. If the cowherd had fewer than 200 cows, how many cows did he have? (Based on the folklore mathematics from Karnataka).

Solution:
No. of cows is divisible by 3, 5, 7. This means the number of cows is a multiple of the LCM(3, 5, 7). The total number of cows is also less than 200. LCM (3, 5, 7) = 3 × 5 × 7 = 105 No. of cows must be a multiple of 105. Multiples of 105 are 105, 210, 315. The problem states that the cowherd had fewer than 200 cows. Only multiples of 105 that are less than 200 are 105. Hence cowherd had 105 cows.

Question 6.
The length, width, and height of a box are 12 cm, 18 cm, and 36 cm, respectively. Which of the following-sized cubes can be packed in this box without leaving gaps? (a) 9 cm (b) 6 cm (c) 4 cm (d) 3 cm (e) 2 cm

Solution:
Here, dimensions of box: Length = 12 cm, Width = 18 cm, Height = 36 cm The size of the largest cube that can exactly fit (without gaps) That means the side of the cube must exactly divide all three dimensions of the box. So, we need to find the HCF (Highest Common Factor) of 12, 18, and 36. Prime factorisation 12 = 2 × 2 × 3, 18 = 2 × 3 × 3, and 36 = 2 × 2 × 3 × 3 Common factors = 2 × 3 = 6 HCF = 6 cm The cube must have a side length equal to a factor of the HCF. From the options: (a) 9 cm ✗ (9 doesn't divide 12 evenly) (b) 6 cm ✓ (divides 12, 18, and 36 exactly) (c) 4 cm ✗ (doesn't divide 18 evenly) (d) 3 cm ✓ (also divides all) (e) 2 cm ✓ (also divides all) Hence, the largest possible cube that fits without gaps is (b) 6 cm.

Question 7.
Among the numbers below, which is the largest number that perfectly divides both 306 and 36? (a) 36 (b) 612 (c) 18 (d) 3 (e) 2 (f) 360

Solution:
Longest number that perfectly divides (306, 36) = HCF (306, 36)

Now

2	306
3	153
3	51
17	17
	1

$$\therefore 306 = \boxed{2} \times \boxed{3 \times 3} \times 17$$
$$36 = \boxed{2} \times 2 \times \boxed{3 \times 3}$$

HCF (306, 36) = 2 × 3 × 3 = 18. ∴ (c) is the correct option.

Question 8.
Find the smallest number that is divisible by 3, 4, 5, and 7, but leaves a remainder of 10 when divided by 11.

Solution:
LCM(3, 4, 5,7) = 3 × 4 × 5 × 7 = 420 The number must be a multiple of 420, so the number can be written as 420k, where k is a whole number. N = 420 × 1 = 420 When divided by 11 leaves a remainder of 2 11 × 38 + 2 = 420 But we require 10 as a remainder ∴ 2k = 10 ⇒ k = 5 Hence number = 420 × 5 = 2100

Question 9.
Children are playing 'Fire in the Mountain.' When the number 6 was called out, no one got out. When the number 9 was called out, no one got out. But when the number 10 was called out, some people got out. How many children could have been playing initially? (a) 72 (b) 90 (c) 45 (d) 3 (e) 36 (f) None of these

Solution:
Interpretation is that when a number k is called, the children are grouped into rows of size k, and "no one got out" means the children formed complete rows with no remainder. So: "No one got out" when 6 was called ⇒ the total number N is divisible by 6. "No one got out" when 9 was called ⇒ N is divisible by 9. "Some people got out" when 10 was called ⇒ N is not divisible by 10. If N is divisible by both 6 and 9, then it must be divisible by their LCM: LCM (6, 9) = 18. So N is a multiple of 18, but not a multiple of 10. Now check the options: (a) 72 = 18 × 4 – divisible by 18 and not by 10 → possible. (b) 90 = 18 × 5 – divisible by 18 but is divisible by 10 → not possible. (c) 45 – not divisible by 18 → not possible. (d) 3 – not divisible by 18 → not possible. (e) 36 = 18 × 2 – divisible by 18 and not by 10 → possible. So the numbers that could have been playing are 36 and 72.

Question 10.
Tick the correct statement(s). The LCM of two different prime numbers (m, n) can be: (a) Less than both numbers (b) In between the two numbers (c) Greater than both numbers (d) Less than m × n (e) Greater than m × n

Solution:
LCM of two different prime numbers, m and n, is their product of m × n. Since both m and n are prime numbers, their only common factor is 1. Hence, LCM is always greater than both individual numbers. Hence, option (c) is correct.

Question 11.
A dog is chasing a rabbit that has a head start of 150 feet. It jumps 9 feet every time the rabbit jumps 7 feet. In how many leaps does the dog catch up with the rabbit?

Solution:
Here, distance gained per leap = Dog's leap distance – Rabbit's leap distance = 9 – 7 = 2 The dog needs to close a 150-foot head start. No. of leaps =
Head Start Distance Distance Gained Per Leap
= 1502 = 75 leaps

Question 12.
What is the smallest number that is a multiple of 1, 2, 3, 4, 5, 6, 8, 9, 10? Do you remember the answer from Grade 6, Chapter 5?

Solution:
1 = 1 2 = 2 3 = 3 4 = 2 × 2 5 = 5 6 = 2 × 3 7 = 7 8 = 2 × 2 × 2 9 = 3 × 3 10 = 2 × 5 LCM(1, 2, 3, 4, 5, 6, 7, 8, 9, 10) = 2 × 2 × 2 × 3 × 3 × 5 = 8 × 9 × 5 = 360 Yes, this is the same answer. You might remember from grade 6, Chapter 5, Prime Time.

Question 13.
Here is a problem posed by the ancient Indian Mathematician Mahaviracharya (850 C.E.). Add together 815,120,736,1163 and 121. What do you get? How can we find this sum efficiently?

Solution:
Here 815,120,736,1163,121
Now 15 = 3 × 5
20 = 2 × 2 × 5
36 = 2 × 2 × 3 × 3
63 = 3 × 3 × 7
21 = 3 × 7

LCM of denominators = 2 × 2 × 3 × 3 × 5 × 7

= 4 × 9 × 5 × 7

= 1260

$$\text{Now } \frac{8}{15}+\frac{1}{20}+\frac{7}{36}+\frac{11}{63}+\frac{1}{21}$$

$$=\frac{8\times 84}{1260}+\frac{1\times 63}{1260}+\frac{7\times 35}{1260}$$

$$=\frac{11}{1260}\times 20+\frac{1\times 60}{1260}$$

$$=\frac{1260}{1260}=1$$