Introduction to Linear Polynomials Class 9 Solutions Maths Ganita Manjari Chapter 2

Think and Reflect (NCERT Textbook Page No. 17)

Question 1.
Can you identify the terms, variables and coefficients of this algebraic expression?

Solution:
The given algebraic expression = 200l + 160w + 50lw Terms: 200l, 160w and 50lw Variables: l and w Coefficients: 200 is of variable l, 160 is of variable w and 50 is of variable lw.

Question 2.
How is it different from the algebraic expression in Example 1 ?

Solution:
The algebraic expression given in example 1 is 4x + 5y + 3, which is a linear polynomial in variables x and y. While, the algebraic expression given in example 2 is 200l + 160w + 50lw, which has two variables, l and w, and the expression is not a linear polynomial because it contains a product term of two variables, lw.

Think and Reflect (NCERT Textbook Page No. 17)

Question 1.
Can you identify the terms, variables and coefficients of this algebraic expression?

Solution:
The given algebraic expression = $10x – x^2$
Terms: 10x and $-x^2$
Variables: x
Coefficients: 10 of x and -1 of $-x^2$

Question 2.
Can you point out any similarity or difference between the algebraic expressions obtained in Examples 1 and 3?

Solution:
Similarity: Both expressions use the same variable (x). Difference: The algebraic expression given in example 1 is 4x + 5y + 3, which is a linear

polynomial with variables x and y. While, the algebraic expression given in example 3 is $10x - x^2$, is a quadratic expression with variable x, because it contains the term x × x.

Think and Reflect (NCERT Textbook Page No. 19)

Find the perimeter of squares with sides 1 cm, 1.5 cm, 2 cm, 2.5 cm and 3 cm. What will happen to the perimeters if the sides increase by 0.5 cm?

Solution:
Since, perimeter of a square of side x cm = 4x cm Perimeter of a square of side 1 cm = 4 × 1=4 cm Perimeter of a square of side 1.5 cm = 4 × 1.5 = 6 cm Perimeter of a square of side 2 cm =4 × 2 = 8 cm Perimeter of a square of side 2.5 cm = 4 × 2.5 = 10 cm Perimeter of a square of side 3cm= 4 × 3 = 12 cm
Observation: Every time the side increases by 0.5 cm, the perimeter increases by exactly 2 cm. This constant rate of change is a hallmark of linear polynomials.

Think and Reflect (NCERT Textbook Page No. 19)

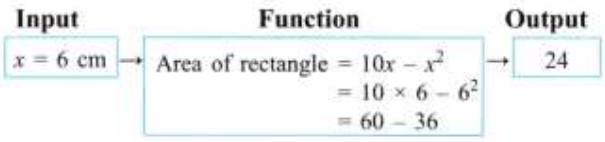
If a player paid ₹ 750, how many matches did he play?

Solution:
Here, the polynomial given is 200 + 50m, where m is the number of matches. If a player paid ₹ 750, we can find m using a simple linear equation as follows:
200 + 50m = 750 ⇒ 50 m =750 – 200 = 550 ⇒ m = 55050 = 11 matches
Hence, the player played 11 matches.

Think and Reflect (NCERT Textbook Page No. 20)

We have learnt that to evaluate the value of an algebraic expression, we substitute a value of the variable in the given expression. Consider Example 3, where the wire is bent to form a rectangle. Here, the area of the rectangle, $10x - x^2$, is a function of x. Can you interpret this as an input-output process? What value does the expression take when x = 6 cm?

Solution:
The given expression is $10x - x^2$,



Think and Reflect (NCERT Textbook Page No. 22)

Predict the number of squares in the next three stages of the pattern and write the sequence of numbers up to Stage 7 of the pattern.

Solution Do it Yourself

Think and Reflect (NCERT Textbook Page No. 22)

Using the expression 2n – 1, can you find out how many tiles will be there in the 15th stage and the 26th stage of the pattern? Also, which stage will contain 21 tiles and 47 tiles?

Solution:
For the 15th stage: Number of tiles = 2(15) – 1 = 30 – 1 =29 So, the 15th stage will have 29 tiles.

For the 26th stage: Number of tiles = 2(26) – 1 = 52-1 = 51 So, the 26th stage will have 51 tiles.

For 21 tiles: Solve the equation: In – 1 = 21 2n – 1 = 21 ⇒ 2n = 22 ⇒ n = 11
So, the stage with 21 tiles is the 11th stage.

For 47 tiles: Solve the equation: 2n – 1 = 47 2n – 1 = 47 ⇒ 2n = 48 ⇒ n = 24
So, the stage with 47 tiles is the 24th stage.

Think and Reflect (NCERT Textbook Page No. 23)

What amount will be left on the 15th day? How many days will it take for the entire amount to be spent?

Solution:
Since the amount left on the nth day will be ₹(100 – 5n). Therefore, on the 15th day the amount left will be ₹(100 – 5 x 15) = ₹ 25. Since the amount spent each day is ₹5. ∴ Number of days required to spend entire amount = 1005 = 20 days.

Think and Reflect (NCERT Textbook Page No. 23)

For how many km will the fare be ₹ 130?

Solution:
The expression describing the fare = 15n – 5, when n is the number of kilometres and n > 2. ATQ, 15n – 5 = 130 ⇒ 15n = 135 ⇒ n = 9 Hence, for 9 km, the fare is ₹ 130.

Think and Reflect (NCERT Textbook Page No. 24)

What is the cost for travelling 15 km? For how many kilometres will the cost of the journey be ₹ 700?

Solution:
The cost of a journey is given by the linear function C(d) = 100 + 60d, where C indicates total cost in rupees and d the distance travelled in km .'. Cost of travelling 15 km = 100 + 60 × 15 = 100 + 900 = ₹ 1000 Further, cost of a journey = ₹ 700 (Given) ⇒ 700= 100 + 60(7 ⇒ 600 = 60d ⇒ d = 10 Hence, for 10 km, the cost of the journey is ₹ 700.

Think and Reflect (NCERT Textbook Page No. 25)

What will be the height of the water at the end of 5 months?

Solution:
The height of water, h m, at the end of t months is given by the linear function h(t) = 3 – 0.5; The height of water at the end of 5 months = 3 – 0.5 × 5 = 0.5 m.

Think and Reflect (NCERT Textbook Page No. 27)

Can you guess what the numbers 20 and 150 in the equation y = 20x + 150 represent?

Solution:
The equation y = ax + b, where y represents the monthly bill in ₹ for data x used in GB. So, in the equation y = 20x + 150, 20 represents the additional cost per GB of the internet data used and 150 represents fixed monthly fee.

Think and Reflect (NCERT Textbook Page No. 28)

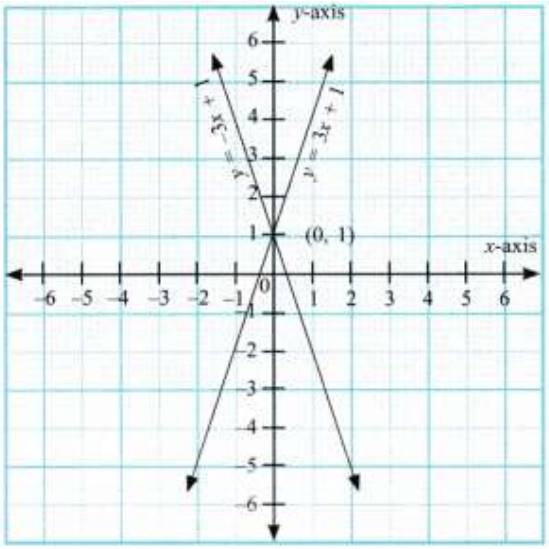
Identify other points on the line by completing the following table.

Solution:
Equation of the straight line y = 2x + 1
so, x = 2 ⇒ y = 2 × 2 + 1 = 5
x = 5 ⇒ y = 2 × 5 + 1 = 11
x = 9 ⇒ y = 2 × 9 + 1 = 19
x= 12 ⇒ y = 2 × 12 + 1 =25
x = 20 ⇒ y = 2 × 20 + 1 = 41

Think and Reflect (NCERT Textbook Page No. 33)

Differentiate between the graphs of the equations y = 3x + 1, and y = –3x + 1

Solution:



The graphs of the equations y = 3x + 1 and y = -3x + 1 are straight lines, but they have different slopes and orientations. Below are the distinctions: 1. Slope: In y = 3x + 1, the slope is +3. This means the line rises by 3 units on the y-axis for every 1 unit it moves to the right along the x-axis. In y = -3x + 1, the slope is-3. This means the line falls by 3 units on the y-axis for every 1 unit it moves to the right along the x-axis.

2. Orientation: The graph of y = 3x + 1 is an ascending line (increasing from left to right), sloping upwards. The graph of y = -3x + 1 is a descending line (decreasing from left to right), sloping downwards.

Think and Reflect (NCERT Textbook Page No. 35)

Does this help you to conclude anything about the linear equation y = ax + b when a is fixed but b varies?

Solution:
When a is fixed and b changes, the lines formed are parallel to each other, but its vertical location shifts. This means all the lines have the same slope.

Exercise Set 2.1 Solutions

Question 1.
Find the degrees of the following polynomials. (i) $2x^2 – 5x + 3$

Solution:
$2x^2 – 5x + 3$; Here the highest power of variable x is 2, so the degree of the given polynomial is 2.

(ii) $y^3 + 2y – 1$

Solution:
$y^3 + 2y – 1$; Here the highest power of variable y is 3, so the degree of the given polynomial is 3.

(iii) – 9

Solution:
-9; since we can write -9 as -9x°, here the power of variable x is 0 so the degree of constant polynomial -9 is 0.

(iv) 4z – 3

Solution:
4z – 3; Here the highest power of variable z is 1, so the degree of the given polynomial is 1.

Question 2.
Write the polynomials of degree 1, 2 and 3.

Solution:
Degree 1 (Linear): 4x + 2 Degree 2 (Quadratic): $3x^2$ -4x +7
Degree 3 (Cubic): $x^3 + 2x^2 – x + 10$

Question 3.
What are the coefficients of x2 and x3 in the polynomial $x^4 – 3x^3 + 6x^2 – 2x + 7$?

Solution:
The term containing x^2 is $6x^2$, so, the coefficient of x^2 is 6. The term containing x^3 is $-3x^3$, so the coefficient of x^3 is -3.

Question 4.
What is the coefficient of z in the polynomial $4z^3 + 5z^2 – 11$?

Solution:
Since we can rewrite $4z^3 + 5z^2 – 11$ as $4z^3 + 5z^2 + 0z – 11$, ∴ Coefficient of z in the given polynomial is 0.

Question 5.
What is the constant term of the polynomial $9x^3 + 5x^2 – 8x – 10$?

Solution:
The constant term of the given polynomial is -10.

Exercise Set 2.2 Solutions

Question 1.
Find the value of the linear polynomial 5x – 3 if: (i) x = 0 Solution: When x = 0; the linear polynomial 5x – 3 = 5 × 0 – 3 = -3

(ii) x = -1

Solution:
When x = -1; the linear polynomial 5x – 3 = 5 × (-1) – 3 = -5 – 3 = -8

(iii) x = 2

Solution:
When x = 2; the linear polynomial 5x – 3 = 5 × 2 – 3 = 10 – 3 = 7

Question 2. Find the value of the quadratic polynomial ls2 -4s+ 6 if: (i) s = 0 (ii) s = -3 (iii) s = 4

Solution:
(i) s = 0; the quadratic polynomial $7s^2$ – 4s + 6 = $7(0)^2$ – 4(0) + 6 = 0 – 0 + 6 = 6

(ii) s = -3; the quadratic polynomial $7s^2$ – 4s + 6 = $7(-3)^2$ – 4(-3) + 6 = 7(9) + 12 + 6 = 63 + 12 + 6 = 81

(iii) s = 4; the quadratic polynomial $7s^2$ – 4s + 6
= $7(4)^2$ – 4(4) + 6 = 7(16) – 16 + 6 = 112 – 16 + 6 = 102

Question 3.
The present age of Salihs mother is three times Salil's present age. After 5 years, their ages will add up to 70 years. Find their present ages.

Solution:
Let Salil's present age be x. Then, Salil's mother's present age = 3x. After 5 years, Salil's age = x + 5 and his mother's age = 3x + 5. Their total age after 5 years = (x + 5) + (3x + 5) = 70. 4x + 10 = 70 ⇒ 4x = 60 ⇒ x = 15 Thus, Salil's present age is 15 years and his mother's present age is 3 × 15 = 45 years.

Question 4.
The difference between two positive integers is 63. The ratio of the two integers is 2 : 5. Find the two integers.

Solution:
Let the two integers be 2x and 5x. Their difference: 5x – 2x = 63 ⇒ 3x = 63 ⇒ x = 21 ∴ The first integer = 2 × 21 = 42 And the second integer = 5 × 21 = 105

Thus, the two integers are 42 and 105.

Question 5.
Ruby has 3 times as many two-rupee coins as she has five- rupee coins. If she has a total of ?88, how many coins does she have of each type?

Solution:
Let the number of five-rupee coins be x. Then, the number of two-rupee coins = 3x. Total value of coins = 2(3x) + 5(x) = 88 ⇒ 6x + 5x = 88 ⇒ 11x = 88 ⇒ x = 8 Number of five-rupee coins = 8 And number of two-rupee coins = 8 × 3 = 24 Thus, Ruby has 8 five-rupee coins and 24 two-rupee coins.

Question 6.
A farmer cuts a 300 feet fence into two pieces of different sizes. The longer piece is four times as long as the shorter piece. How long are the two pieces?

Solution:
Let the length of the shorter piece be x. Then, the length of the longer piece is 4x. Total length of the fence = x + 4x = 300 ⇒ 5x = 300 ⇒ x = 60 Thus, the shorter piece is 60 feet long, and the longer piece is 60 × 4 = 240 feet long.

Question 7.
If the length of a rectangle is three more than twice its width and its perimeter is 24 cm, what are the dimensions of the rectangle?

Solution:
Let the width of the rectangle be x. Then, the length of the rectangle = 2x + 3. Perimeter: 2[(2x + 3) + x] = 24 ⇒ 2(3x + 3) = 24 ⇒ 6x + 6 = 24 ⇒ 6x = 18 ⇒ x = 3 Thus, the width is 3 cm, and the length is 2 × 3 + 3 = 9 cm.

Exercise Set 2.3 Solutions

Question 1. A student has ₹ 500 in her savings bank account. She gets ₹ 150 every month as pocket money. How much money will she have at the end of every month from the second month onwards? Find a linear expression to represent the amount she will have in the nth month.

Solution:
Initial amount = ₹ 500 Pocket money per month = ₹ 150 Money she will have at the end of every month:

Month	2	3	4	5	...
Money (₹)	500 + 150 × 2 = 800	500 + 150 × 3 = 950	500 + 150 × 4 = 1100	500 + 150 × 5 = 1250	...

The total amount, A(n), at the end of the 1th month can be expressed as: A(n) = ₹ (500 + 150n), where n is the number of months.

Question 2.
A rally starts with 120 members. Each hour, 9 members drop out of the group. How many members will remain after 1,2,3,… hours? Find a linear expression to represent the number of members at the end of the nth hour.

Solution:
Number of initial members = 120 Members dropping per hour = 9 Number of members after 1, 2, 3,… hours:

Hours	1	2	3	4	...
Number of members	120 – 9 × 1 = 111	120 – 9 × 2 = 102	120 – 9 × 3 = 93	120 – 9 × 4 = 84	...

The total number of members remaining, M(n), at the end of the nth hour can be expressed as: M(n) = 120 – 9n, where n is the number of hours.

Question 3.
Suppose the length of a rectangle is 13 cm. Find the area if the breadth is (i) 12 cm, (ii) 10 cm, (iii) 8 cm. Find the linear pattern representing the area of the rectangle.

Solution:
Length = 13 cm Area = Length × Breadth (i) When the breadth is 12 cm: Area = 13 × 12 = 156 cm^2
(ii) When the breadth is 10 cm:
Area = 13 × 10 = 130 cm^2
(iii) When the breadth is 8 cm:
Area = 13 × 8 = 104 cm^2 The linear expression representing the area A(b) in terms of the breadth b is: A (b) = 13 b, where b is the breadth of the rectangle.

Question 4.
Suppose the length of a rectangular box is 7 cm and breadth is 11 cm. Find the volume if the height is (i) 5 cm, (ii) 9 cm, (iii) 13 cm. Find the linear pattern representing the volume of the rectangular box.

Solution:
Since, length = 7 cm, breadth = 11 cm And Volume (V) = Length × Breadth × Height (i) When the height is 5 cm: V = 7 × 11 × 5 = 385 cm^3
(ii) When the height is 9 cm: V = 7 × 11 × 9 = 693 cm^3
(iii) When the height is 13 cm: V = 7 × 11 × 13 = 1001 cm^3 The linear expression representing the volume V(h) in terms of the height h is: V(h) = 77h, where h is the height of the rectangular box.

Question 5.
Sarita is reading a book of 500 pages. She reads 20 pages every day. How many pages will be left after 15 days? Express this as a linear pattern.

Solution:
Total pages in the book = 500 The number of pages read per day = 20 The number of pages left after n days, L(n), can be expressed as: L(n) = 500 – 20n, For n = 15 days: L(15) = 500 – 20(15) = 500 – 300 = 200 So, after 15 days, 200 pages of the book will be left to read.

Exercise Set 2.4 Solutions

Question 1. Suppose a plant has height 1.75 feet and it grows by 0.5 feet each month. (i) Find the height after 7 months.

Solution:
Initial height = 1.75 feet Growth per month = 0.5 feet Height after 7 months = Initial height + (Growth per month × 7) ∴ h= 1.75+ (0.5 × 7) = 1.75 + 3.5 = 5.25 feet

(ii) Make a table of values for t varying from 0 to 10 months and show how the height, h, increases every month. Solution:

Time (months)	Height (h feet)
0	1.75
1	2.25
2	2.75
3	3.25
4	3.75
5	4.25

6	4.75
7	5.25
8	5.75
9	6.25
10	6.75

(iii) Find an expression that relates h and t, and explain why it represents linear growth.

Solution:
The height h after t months can be expressed as: h(t)= 1.75 + 0.5; This expression represents linear growth because the height increases by a constant amount (0.5 feet) for each unit increase in time (months).

Question 2.
A mobile phone is bought for ₹ 10,000. Its value decreases by ₹ 800 every year. (i) Find the value of the phone after 3 years.

Solution:
Initial value = ₹ 10,000 Decrease per year = ₹ 800 Value after 3 years = Initial value – (Decrease per year × 3) ∴ Value (v) = ₹ 10000 – (₹ 800 × 3) = ₹ 10000 – ₹ 2400 = ₹ 7600

(ii) Make a table of values for; varying from 0 to 8 years and show how the value of the phone, v, depreciates with time.

Solution:

Time (t years)	Value (v in ₹)
0	10,000
1	9,200
2	8,400
3	7,600
4	6,800
5	6,000

6	5,200
7	4,400
8	3,600

(iii) Find an expression that relates v and;, and explain why it represents linear decay.

Solution:
The value v after t years can be expressed as: v(t)= 10000 – 800; This expression represents linear decay because the value v decreases by a constant amount (₹ 800) for each unit increase in time (years).

Question 3.
The initial population of a village is 750. Every year, 50 people move from a nearby city to the village. (i) Find the population of the village after 6 years.

Solution:
Initial population of village = 750 Population increase per year = 50 ∴ Population after 6 years = Initial population + (Increase per year × 6) ⇒ Population (P) = 750 + (50 × 6) = 750 + 300 = 1050

(ii) Make a table of values for t varying from 0 to 10 years and show how the population, P, increases every year.

Solution:

Time (years)	Population (P)
0	750
1	800
2	850
3	900
4	950
5	1,000
6	1,050

7	1,100
8	1,150
9	1.200
10	1,250

(iii) Find an expression that relates P and t, and explain why it represents linear growth.

Solution:
The population P after t years can be expressed as: P(t) = 750 + 50t This expression represents linear growth because the population increases by a constant number (50 people) every year.

Question 4.
A telecom company charges ₹ 600 for a certain recharge scheme. This prepaid balance is reduced by ? 15 each day after the recharge. (i) Write an equation that models the remaining balance b(x) after using the scheme forx days. Explain why it represents linear decay.

Solution:
Initial balance = ₹ 600 Reduction per day = ₹ 15 ∴ Remaining balance after x days can be expressed as: b(x) = 600 – 15x This expression represents linear decay because the balance decreases by a constant amount (₹ 15) each day.

(ii) After how many days will the balance run out? Solution: To find the number of days after which the balance will runs out, set b(x) = 0. ⇒ 600 – 15x = 0 ⇒ 15x = 600 ⇒ x = 40 Therefore, the balance will ran out after 40 days.

(iii) Make a table of values for x varying from 1 to 10 days and show how the balance b(x), reduces with time. Solution:

Days (x)	Remaining Balance b(x) (₹)
1	585
2	570
3	555
4	540
5	525

6	510
7	495
8	480
9	465
10	450

Exercise Set 2.5 Solutions

Question 1. A learning platform charges a fixed monthly fee and an additional cost per digital learning module accessed. A student observes that when she accessed 10 modules, her bill was ?400. When she accessed 14 modules, her bill was ?500. If the monthly bill y depends on the number of modules accessed, x, according to the relation y = ax + b, find the values of a and b.

Solution:
Given the linear relation y = ax + b, where y is the monthly bill and x is the number of modules accessed: When x = 10, y = 400, and when x = 14, y = 500 We now have two equations: 400= 10a + b …(i) 500 = 14a + b ……. (ii) To solve for a and b, subtract the first equation from the second one: 500 – 400 = (14a + b) – (10a + b) ⇒ 100 = 4a ⇒ a = 25 Substitute a = 25 equation (i): 400= 10(25) + b ⇒ 400 = 250 + b ⇒ b = 150 Thus, the equation for the monthly bill is y = 25x + 150, and a = 25 and 6 = 150.

Question 2.
A gym charges a fixed monthly fee and an additional cost per hour for using the badminton court. A student using the gym observed that when she used the badminton court for 10 hours, her bill was ₹ 800. When she used it for 15 hours, her bill was ₹ 1100. If the monthly billy depends on the hours of the use of the badminton court, x, according to the relation y = ax + b, find the values of a and b.

Solution:
Given the linear relation y = ax + b, wherey is the monthly bill and x is the number of hours: When x = 10 , y = 800, and when x = 15 ,y = 1100 We now have two equations: 800= 10a + b … (i) 1100= 15a + b …(ii) Subtract the first equation from the second one: 1100-800= (15a+ 6)-(10a+ 6) ⇒ 300 = 5a ⇒ a = 60 Substitute a = 60 in first equation: 800= 10(60) + b ⇒ 800 = 600 + 6 ⇒ b = 200 Thus, the equation for the gym's monthly bill is y = 60x + 200, and a = 60 and b = 200.

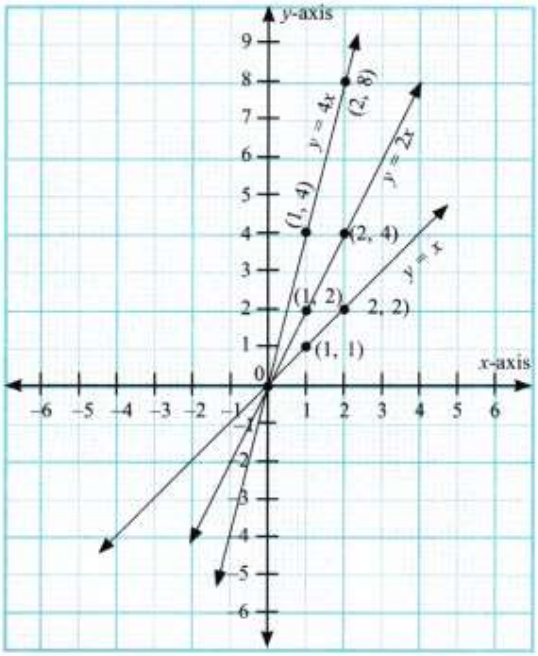
Question 3.
Consider the relationship between temperature measured in degrees Celsius (°C) and degrees Fahrenheit (°F), which is given by °C = a °F + b. Find a and b, given that ice melts at 0 degrees Celsius and 32 degrees Fahrenheit, and water boils at 100 degrees Celsius and 212 degrees Fahrenheit. (Hint: When °C = 0, °F = 32 and when °C = 100, °F = 212. Use this information to find a and b, and thus, the linear relationship between °C and °F.)

Solution:
Given the linear relation °C = a °F + b, where the temperatures for melting ice and boiling water are: When °C = 0, °F = 32 and when °C = 100, °F = 212 We now have two equations: 0 = a(32) + b …(i) 100 = a(212) + b …(ii) From equation (i) b = -32a Substitute b = -32a into the equation (ii): 100 = 212a – 32a ⇒ 100 = 180a ⇒ a = 100180=59
Substitute a = 59 in b = -32a
b = -32 × 59=−1609
Thus, the relationship between Celsius and Fahrenheit is:
°C = 59°F – 1609 and a = 59 and b = –1609

Exercise Set 2.6 Solutions

Question 1.
Draw the graphs of the following sets of lines. In each case, reflect on the role of ‘a’ and ‘6’. (i) y = 4x,y = 2x,y = x

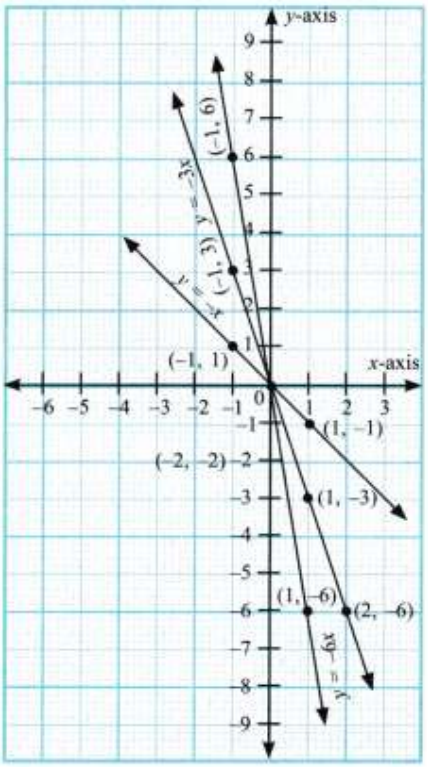
Solution:
y = 4x, y = 2x, y = x All lines are of the form y = ax (b = 0). Observation: All lines pass through the origin (0, 0). The value of ‘a’ (slope) determines steepness. Larger ‘a’ ⇒ steeper line.



(ii) y = – 6x, y = – 3x, y = – x

Solution:

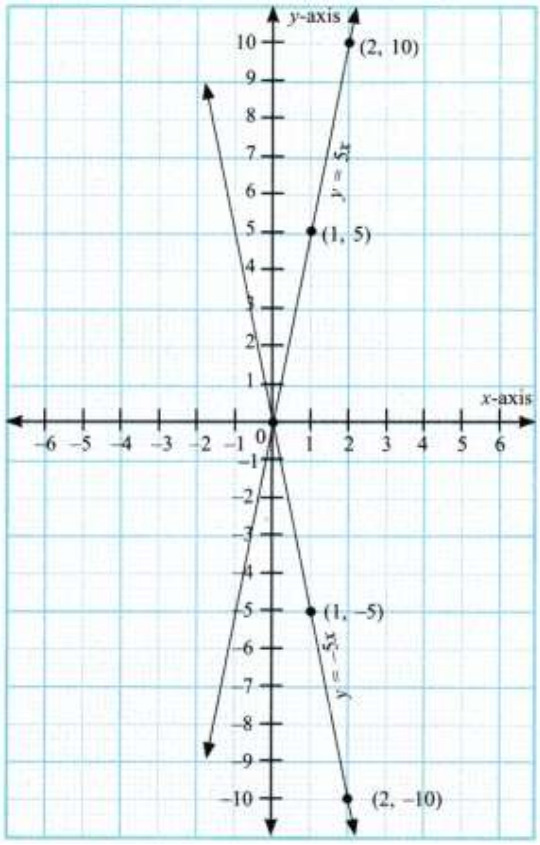
y = – 6x, y = -3x, y =-x All lines are of the form y = -ax (b = 0). Observation: All lines pass through the origin. Negative 'a' means lines slope downward. Larger magnitude of'a' ⇒ steeper downward slope.



(iii) y = 5x, y = -5x

Solution:

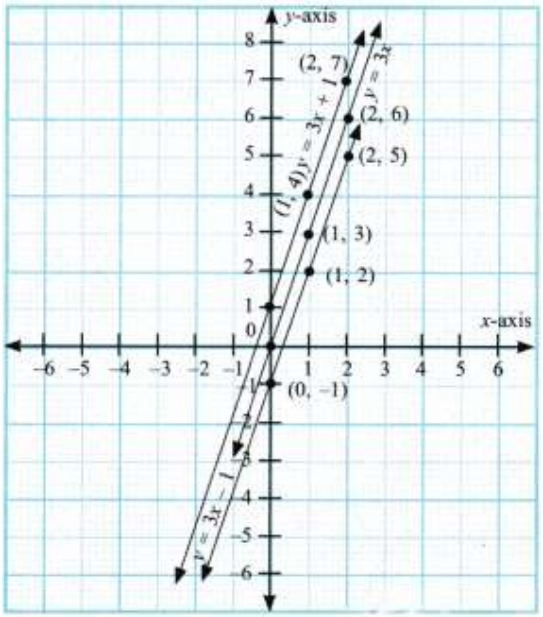
y = 5x, y = -5x All lines are of the form y = ax (b = 0). Observation: Here, both lines pass through the origin because the y-intercept is 0. y = 5x slopes upward, y = -5x slopes downward, same magnitude of a, same steepness but in opposite directions.



(iv) y = 3x – 1, y = 3x, y = 3x + 1

Solution:

y = 3x – 1, y = 3x, y = 3x + 1 All lines have same slope (a = 3). Observation: Lines are parallel as slopes are the same (3). Different values of ‘b’ shift the line. b = -1 line below origin, b = 0 passes through origin b = 1 line above origin.

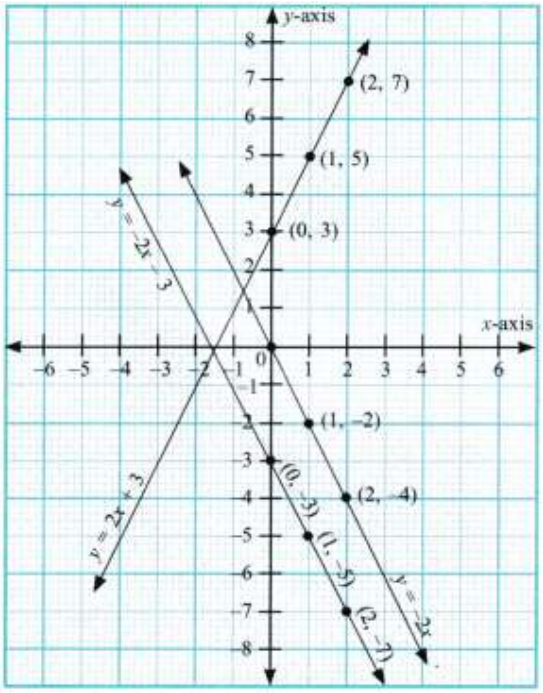


(v) y = -2x – 3, y = -2x, y = 2x + 3

Solution:

y = -2x -3 ,y = -2x, y = 2x + 3 Observation: y = -2x – 3 and y = -2x have the same slope (-2), so the lines are parallel lines. y = 2x + 3 has positive slope and represents different direction. ‘b’ changes vertical position of the line.

Conclusion: 1. a (coefficient of x) controls slope (steepness and direction). 2. b (constant term) controls vertical shift (y-intercept).



End of Chapter Exercises Solutions

Question 1.
Write a polynomial of degree 3 in the variable x, in which the coefficient of the x term is -7.

Solution:
A degree 3 polynomial is of the form $ax^3 + bx^2 + cx + d$.

So, with the coefficient of x^2 as -7, one example of a polynomial is
$P(x) = x^3 – 7x^2 + x + 1$ (Answer may vary)

Question 2.
Find the values of the following polynomials at the indicated values of the variables. (i) $5x^2 – 3x + 7$ if x = 1

Solution:
For $5x^2 – 3x + 7$ if x = 1
$\Rightarrow 5 \times 1^2 – 3 \times 1 + 7 = 5 – 3 + 7 = 9$

(ii) $4t^3 – t^2 + 6$ if t = a
Solution:
For $4t^3 – t^2 + 6$ if t = a
$\Rightarrow 4 \times a^3 – a^2 + 6$
$\Rightarrow 4a^3 – a^2 + 6$

Question 3.
If we multiply a number by 52 and add 23 to the product, we get −712. Find the number. 12

Solution:
Let the number be x,
ATQ, $x \times 52+23=−712$
$\Rightarrow 52x = −712−23$
$\Rightarrow 52x = −7−812$
$\Rightarrow 52x = −1512$
$x = −1512×25=−12$
Hence, the required number is – 12.

Question 4.
A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

Solution:
Let the two numbers be x and y, where x = 5y. ATQ, x + 21 = 2(y + 21) Substituting x = 5y: ⇒ 5y + 21 =2(y + 21) ⇒ 5y + 21 = 2_y + 42 ⇒ 3y = 21 ⇒ y = 7 So, x = 5y = 35. The numbers are 35 and 7.

Question 5.
If you have ₹ 800 and you save ₹ 250 every month, find the amount you have after (i) 6 months (ii) 2 years. Express this as a linear pattern.

Solution:
(i) After 6 months: 800 + 6 × 250 = 800 + 1500 = ₹ 2300

(ii) After 2 years (24 months): 800 + 24 × 250 = 800 + 6000 = ₹ 6800 The linear pattern is 800 + 250x, where x is the number of months.

Question 6. The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. Find both the numbers.

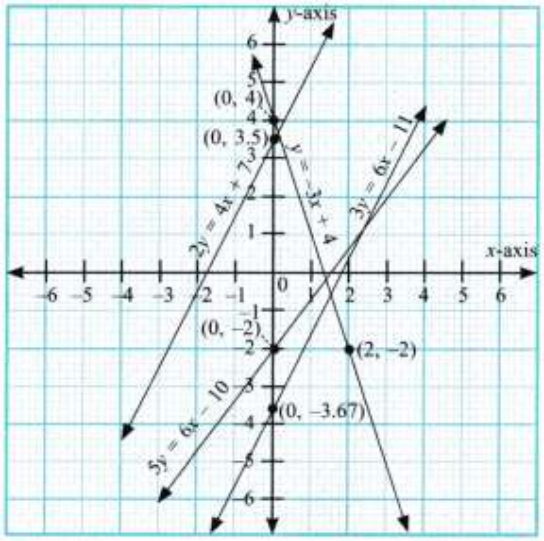
Solution:
Let the two-digit number be represented as 10a + b, where a and b are the tens and ones digits, respectively. The digits differ by 3: |a – b| = 3 When the digits are interchanged, the number becomes 10b + a. The sum of the original and the interchanged numbers is 143: ⇒ (10a+ 6)+ (106 +a) = 143 ⇒ 11a + 11 b= 143 ⇒ a + b = 13

Solving the system of equations: a – b = 3 and a + b = 13 Adding these equations: 2a = 16 ⇒ a = 8

Substituting a = 8 in a + b = 13: 8 + 6 = 13 ⇒ 6 = 5 The original number = 85 and the interchanged number = 58 Therefore, the numbers are 85 and 58.

Question 7.
Draw the graph of the following equations, and identify their slopes andy-intercepts. Also, find the coordinates of the points where these lines cut the y-axis. (i) y = -3x + 4 (ii) 2y = 4x + 7 (iii) 5y = 6x – 10 (iv) 3y = 6x – 11 Are any of the lines parallel?

Solution:



(i) y = -3x + 4 Slope: a = -3 y-intercept: b = 4 Coordinates where the line cuts the y-axis: (0, 4) Explanation: When x = 0, substitute it into the equation: y = -3(0) + 4 = 4. The line cuts the y-axis at (0, 4).

(ii) 2y = 4x + 7 Convert to slope-intercept form: y = 2x + 3.5 Slope: a = 2 y-intercept: b = 3.5 Coordinates where the line cuts the y-axis: (0, 3.5) Explanation: When x = 0, substitute it into the equation: y = 2(0) + 3.5 = 3.5. The line cuts the y-axis at (0, 3.5).

(iii) 5y = 6x – 10 Convert to slope-intercept form:y = 65x – 2
Slope: a = 65
y-intercept: b = – 2
Coordinates where the line cuts the y-axis: (0, -2)
Explanation: When x = 0, substitute it into the
equation: y = (65)(0) – 2 = -2. The line cuts the y-axis at (0, -2).

(iv) 3y = 6x – 11 Convert to slope-intercept form: y = 2x – 113
Slope: a = 2
y-intercept: b = −113 = -3.67 (approximately)
Coordinates where the line cuts the y-axis: (0, -3.67)
Explanation:- When x = 0, substitute it into the
equation: y = 2(0) – (113) = −113. The line cuts the y-axis at (0, -3.67).
The lines with slopes -3, 2, and 65 are not parallel, but lines (ii) and (iv) have the same slope of 2, so they are parallel.

Question 8.
If the temperature of a liquid can be measured in Kelvin units as x K and in Fahrenheit units as y °F, the relation between the two systems of measurement of temperature is given by the linear equation y = 95 (x –

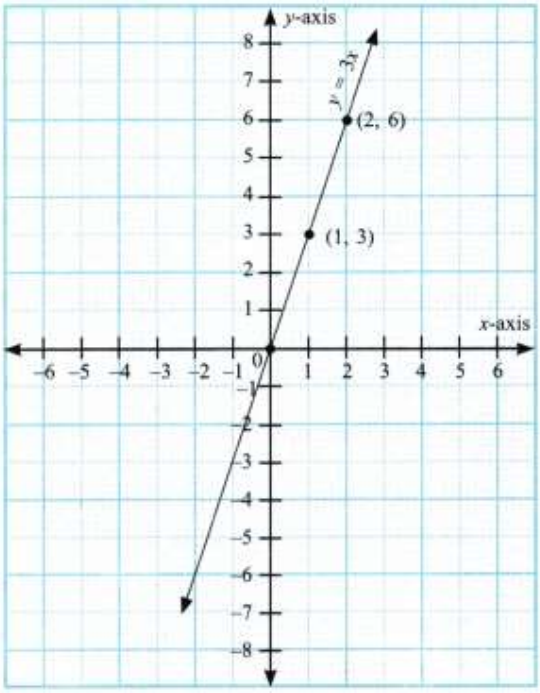
273) + 32.
(i) Find the temperature of the liquid in Fahrenheit if the temperature of the liquid is 313 K.

Solution:
When x = 313 K,
Since, y = 95(x-273) + 32
⇒ y = 95(313 – 273)+ 32
= 95 × 40 + 32 = 72 + 32 = 104°F Hence, the temperature of liquid in Fahrenheit is 104°F.

(ii) If the temperature is 158 °F, then find the temperature in Kelvin. Solution: If the temperature is 158 °F, then 158 = 95 (x – 273) + 32
⇒ 126 = 95 (x – 273) ⇒ x = 273 + 40 = 343 Hence, the temperature of liquid in Kelvin is 343 K.

Question 9.
The work done by a body on the application of a constant force is the product of the constant force and the distance travelled by the body in the direction of the force. Express this in the form of a linear equation in two variables (work w and distance d), and draw its graph by taking the constant force as 3 units. What is the work done when the distance travelled is 2 units? Verify it by plotting it on the graph.

Solution:
The work done is given by the equation. Work done = Force × Distance ATQ, w = F × d, where F = 3. ⇒ w = 3d, is the required linear equation in two variables. For d= 2: w = 3 × 2 = 6 units of work. By taking w on y-axis and d on x-axis, we can plot the graph as follows:



Question 10.

The graph of a linear polynomial p(x) passes through the points (1, 5) and (3, 11). (i) Find the polynomial p(x).

Solution:

Let p(x) = ax + b Since the graph passes through (1,5), a(1) + b = 5 ⇒ a + b = 5 …(i) Since the graph passes through (3, 11), a(3) + b = 11 ⇒ 3a + b = 11 ……(ii) Subtracting (i) from (ii), we get: 3o + b – (a + b) = 11 – 5 ⇒ 2a = 6 ⇒ a = 3 Substituting a = 3 in (i), we have: b = 5 – 3 = 2 Therefore, p(x) = 3x + 2

(ii) Find the coordinates where the graph of p(x) cuts the axes.

Solution:

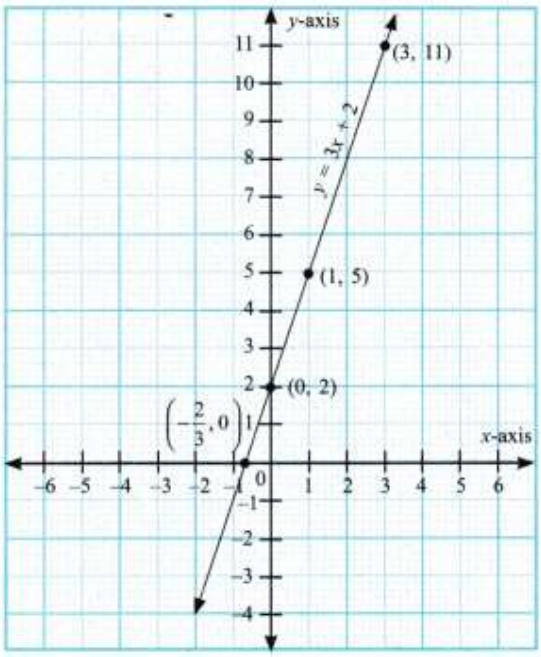
Coordinates where the graph cuts the axes: To find the y-axis intercept: Put x = 0 ⇒ y = p(0) = 3(0) + 2 = 2 So, the graph cuts the y-axis at (0, 2). To find the x-axis intercept: Put y = 0 ⇒ 3x + 2 = 0 ⇒ 3x = -2 ⇒ x = −23.

So, the graph cuts the x-axis at (-23, 0)

(iii) Draw the graph of p(x) and verify your answers.

Solution:

Graph:



Question 11.

Let p(x) = ax + b and q(x) = cx + d be two linear polynomials such that: (i) m = 5. (ii) The polynomial p(x) – q(x) cuts the x-axis at (3, 0). (iii) The sum p(x) + q(x) is equal to 6x + 4 for all real x. Find the polynomials p(x) and q(x).

Solution:

Let p(x) = ax + b and q(x) = cx + d Using the condition (i), we have p(0) = 5 ⇒ a(0) + b= 5 ⇒ b = 5 So, p(x) = ax + 5

Now, using condition (iii), We have p(x) + q(x) = 6x + 4 ⇒ (ax + 5) + (cx + d) = 6x + 4 ⇒ (a + c) x x + (5 + d) = 6x + 4

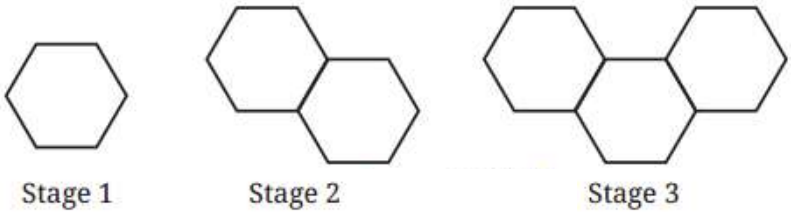
Comparing coefficients, we get a + c = 6 …(1) 5 + d = 4 ⇒ d = -1 So, q(x) = cx – 1

Using condition (ii), p(x) – q(x) cuts x-axis at (3, 0) ⇒ p(3)-q(3) = 0 Here, p(3) = 3a + 5 and g(3) = 3c – 1 So, (3a + 5) – (3c- 1) = 0 ⇒ 3a — 3c + 6 = 0 ⇒ a – c = -2 ……..(2)

Solving equations From (1): a + c = 6 From (2): a – c = -2 Adding both: 2a =4 ⇒ a = 2 Substituting a = 2 in (1), we get 2 + c= 6 ⇒ c = 4 Hence, the polynomial p(x) = 2x + 5 and q(x) = 4x – 1

Question 12.

Look at the first three stages of a growing pattern of hexagons made using matchsticks. a new hexagon gets added at every stage which shares a side with the last hexagon of the previous stage.



(i) Draw the next two stages of the pattern. How many matchsticks will be required at these stages?
(ii) Complete the following table.

Stage Number	1	2	3	4	5	...	n
Number of matchsticks							

(iii) Find a rule to determine the number of matchsticks required for the «,h stage.
(iv) How many matchsticks will be required for the 15th stage of the pattern?
(v) Can 200 matchsticks form a stage in this pattern? Justify your answer.

Solution:
A single hexagon needs 6 matchsticks.
Since each new hexagon shares one side with the previous hexagon, only 5 new matchsticks are added at every new stage.
So the pattern is:
Stage 1: 6
Stage 2: 6 + 5 = 11
Stage 3: 11 + 5= 16

(i) Next two stages:
Stage 4: Number of matchsticks = 16 + 5 = 21
Stage 5: Number of matchsticks = 21 + 5 = 26

(ii)

Stage Number	1	2	3	4	5	...	n
Number of matchsticks	6	11	16	21	26		$5n + 1$

(ii) Rule for the nth stage: The number of matchsticks forms a pattern: 6, 11, 16, 21, 26,… Where, the first term = 6, difference of two consecutive terms = 5 ∴ Number of matchsticks in the nth stage = 6 + (n – 1) × 5 = 6 + 5n – 5 = 5n + 1 Hence, the rule to determine the number of matchsticks required for the nth stage is given by $S_n = 5n + 1$

(iv) Matchsticks required for the 15th stage $S_{15} = 5(15) + 1 = 75 + 1 = 76$ Therefore, 76 matchsticks will be required for the 15th stage.

(v) Let for some stage n, 5n + 1 = 200 n = 1995 = 39.8 Since, 39.8 is not a whole number. Therefore, 200 matchsticks cannot form a stage in this pattern.

Question 13.
Let p(x) = ax + b and q(x) = cx + d be two linear polynomials such that: (i) The graph of p(x) passes through the points (2, 3) and (6, 11). (ii) The graph of q(x) passes through the point (4, -1). (iii) The graph of q(x) is parallel to the graph of p(x). Find the polynomials p(x) and q(x). Also, find the coordinates of the point where these lines meet the x-axis.

Solution:
Given: p(x) = ax + 1 q(x) = cx + d First, to find p(x), since p(x) passes through (2, 3) and (6, 11), Its slope is m = 12 = 2
So, p(x) = 2x + b
Using the point (2, 3), we have:
3 = 2(2) + b ⇒ b = – 1
∴ p(x) = 2x – 1
Now to find q(x). Since q(x) is parallel to p(x), so, it has the same slope.
So, slope of q(x) is 2.
Hence, q(x) = 2x + d
Since q(x) passes through (4,-1).
∴ -1 = 2(4) + d ⇒ d = -9

Therefore, q(x) = 2x – 9
Now we have to find where these lines meet the x-axis.
A line meets the x-axis where y = 0
For p(x) = 2x – 1
⇒ 0 = 2x – 1
⇒ x = 12

So, p(x) meets the x-axis at (12, 0)
For q(x) = 2x – 9
⇒ 0 = 2x – 9
⇒ x = 92
So, q(x) meets the x-axis a (92, 0)
Hence,p(x) = 2x – 1 and q(x) = 2x – 9
The x-axis intercepts are:
For p(x): (12, 0) and q(x): (92, o)

Question 14.
What do all linear functions of the form f(x) = ax + a, a > 0, have in common?

Solution:
Given: f(x) = ax + a, where a > 0 Common properties of all such linear functions are: 1. Slope: The slope is a, and since a > 0 all the lines have positive slope. So, all these lines rise from left to right.

2. y-intercept: Putting x = 0 f(0) = a So, the y-intercept is (0, a). Since a > 0 all the lines cut they-axis above the origin.

3. x-intercept: ax + a = 0 ⇒ a(x + 1) = 0 To find where the line cuts the x-axis, put f(x) = 0, Since a > 0, – a is not zero. So, x + 1 = 0 ⇒ x = -1 Thus, every line cuts the x-axis at the same point (-1,0). Therefore, all linear functions of the form f(x) = ax + a, a > 0 have the following common characteristic: 1. All have positive slope. 2. All cut the y-axis above the origin. 3. All pass through the fixed point (-1, 0).