The World of Numbers Class 9 Solutions Maths Ganita Manjari Chapter 3

Think and Reflect (NCERT Textbook Page No. 46)

Why does a negative times a negative equal a positive? Think of it in terms of action and debt. If a negative number represents a debt, then multiplying by a negative number represents the removal of that debt. (Hint: If someone takes away (-) four of your debts that are each worth ₹ 3 (that is, -3), you are effectively ₹ 12 richer! Therefore, (-3) × (-4) = +12.)

Solution:
Think of a negative number as a debt and multiplication as an action being repeated. Suppose you have a debt of ₹ 3. This is written as -3. Now, if someone removes this debt, what happens? You are no longer required to pay ₹ 3, so your position improves. Mathematically, you go from -3 to 0, which is an increase of ₹ 3. That is why removing a debt is the same as gaining money.

Now apply this idea to: (-3) × (-4) Here, -3 represents a debt of ₹ 3, and -4 represents the action of removing this debt 4 times. So the expression means: “Remove four debts of ₹ 3 each.” Each time you remove a debt of ₹ 3, you gain ₹ 3. Repeating this action four times:

- After removing one debt → gain ₹ 3
- After removing two debts → gain ₹ 6
- After removing three debts → gain ₹ 9
- After removing four debts → gain ₹ 12

So, after all debts are removed, you are ₹ 12 better off. Therefore, (—3) × (—4) = +12 In this way, a negative times a negative becomes positive.

Think and Reflect (NCERT Textbook Page No. 47)

Can you explain why we need q ≠ 0 in the definition of a rational number?

Solution:
A rational number is written as pq, where p and q are integers and q ≠ 0, because the denominator represents division, and division by zero is not defined. If q = 0, then pq would mean dividing a number into zero equal parts, which has no meaning.

Think and Reflect (NCERT Textbook Page No. 49)

Question 1.
While adding or subtracting two rational numbers having different denominators, how will you make the denominators equal?

Solution:
While adding or subtracting two rational numbers with different denominators, we make the denominators equal by finding a common denominator, usually the LCM (Least Common Multiple) of the two denominators. Then, we convert each fraction into an equivalent fraction with this common denominator by multiplying the numerator and denominator by the required number. Once the denominators are the same, we can easily add or subtract the numerators while keeping the denominator unchanged.

Question 2.
Verify the distributive law for rational numbers.

Solution:
To verify the distributive law for rational numbers, we check whether a × (b + c) = a × b + a × c Let us take three rational numbers: a = 23, b = 14, c = 38

First, find b + c: 14+38=28+38=58

Now, a × (b + c) = 23×58=1024=512

Next, find a × b and a × c

23×14=212=16
23×38=624=14

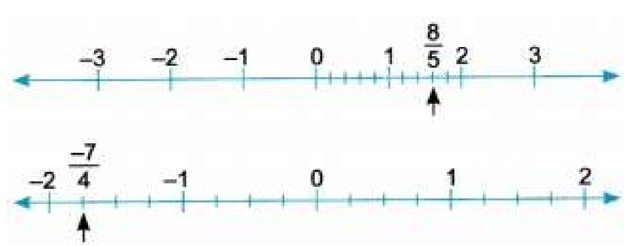
Add them 16+14=212+312=512

Since both sides are equal, a × (b + c) = a × b + a × c Hence, the distributive law is verified for rational numbers.

Think and Reflect (NCERT Textbook Page No. 51)

Try and represent 85 and –74 on a number line.

Solution:



Think and Reflect (NCERT Textbook Page No. 53)

Can √2 be written as a rational number pq ?

Solution: Do it yourself.

Think and Reflect (NCERT Textbook Page No. 55)

Question 1.
Try to prove the irrationality of √3 using the approach of proof by contradiction. Will the same approach work for √5, √7 or √10?

Solution:
To prove that √3 is irrational using contradiction, assume the opposite of what we want to prove. Suppose √3 is rational. Then it can be written as a fraction in lowest terms: √3 = pq
where p and q are integers with no common factor other than 1, that means p and q are co-prime and q ≠ 0.
Now square both sides:
3 = p2q2 ⇒ $p^2 = 3q^2$

This shows that p^2 is divisible by 3, so p must also be divisible by 3 (since 3 is prime). Let p = 3k.
Substitute back:
$(3k)^2 = 3q^2$
⇒ $9k^2 = 3q^2$
⇒ $q^2 = 3k^2$
So, q^2 is also divisible by 3, which means q is divisible by 3.

But now both p and q are divisible by 3, which contradicts our assumption that — is in lowest terms. Hence, our original assumption is false, and thus √3 is irrational.

Next, for √5 or √7, the exact same method works. Both are prime numbers, and the argument depends on the fact that if a prime divides p2, it must divide p. So 45 and 4I can be proved irrational in the same way.

For √10, the situation is slightly different because 10 is not prime (it is 2 ≠ 5). Still, the method can be adapted. If you assume √10 = pq, you get $p^2 = 10q^2$, which implies both 2 and 5 divide p^2, hence they divide p. This leads again to p and q sharing common factors, giving a contradiction. So, the same contradiction approach works for all these numbers—in fact, it works for the square root of any number that is not a perfect square.

Question 2.
We have seen how to obtain a line whose length is a rational number. How do we obtain lines whose lengths are irrational?

Solution: Do it yourself.

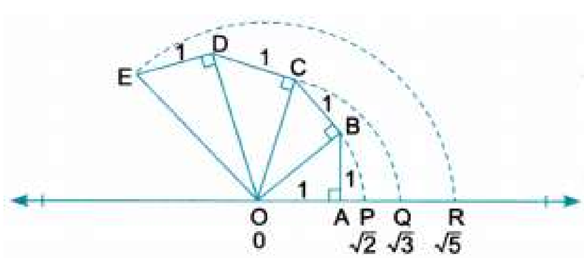
Think and Reflect (NCERT Textbook Page No. 56)

Try to extend this method for constructing line segments of lengths 43 and 45 using a ruler and a compass. Generalise this method to construct a line segment of any length of the form 4n , where n is a positive integer.

Solution:
To extend the same idea, we keep using right triangles and the Baudhayana-Pythagoras Theorem step by step. For √3 : Start exactly like the construction of √2. We already have a right triangle with legs 1 and 1, so the hypotenuse is √2. Now, from the endpoint of this √2, draw a perpendicular and mark a length of 1 unit again. Join this new point to the origin. By the Baudhayana- Pythagoras theorem: $(\sqrt{2})^2 + 1^2 = 2 + 1 = 3$

So, the new hypotenuse is √3. Then, with centre at O and this length as radius, cut the number line to locate √3 . For √5 : We can repeat the process, but there’s a simpler way. First construct √4 = 2 on the number line. At the point 2, draw a perpendicular of length 1 unit. Join this point to O. Then: $2^2 + 1^2 = 4 + 1 = 5$

So, the hypotenuse gives √5. Transfer it onto the number line using a compass.



Generalisation for √n : This method works by building a chain of right triangles. Each time:

- Take the previously obtained length n−1−−−−−√
- Draw a perpendicular of length 1 unit

- Join the new point to the origin

Then, (n−1−−−−−√)2 + 1^2 = (n – 1) + 1 = n So, by stacking right triangles, you can construct √2 , √3, √4, √5, ……… and hence any √n where n is a positive integer.

Think and Reflect (NCERT Textbook Page No. 57)

Try to find the decimal expansions of 103 and 1112. What do you observe about the repetition of the digits after the decimal point?

Solution: Do it yourself.

Exercise Set 3.1 Solutions

Question 1.
A merchant in the port city of Lothal is exchanging bags of spices for copper ingots. He receives 15 ingots for every 2 bags of spices. If he brings 12 bags of spices to the market, how many copper ingots will he leave with?

Solution: Given:

- 2 bags of spices → 15 ingots
- 12 bags of spices → ? ingots

First, find how many times 2 bags fit into 12 bags: 12 ÷ 2 = 6 So, the trader makes 6 such exchanges. Total ingots received: 6 × 15 = 90 The trader will receive 90 copper ingots.

Question 2.
Look at the sequence of numbers on one column of the Ishango bone: 11, 13, 17, 19. What do these numbers have in common? List the next three numbers that fit this pattern.

Solution:
Given sequence: 11, 13, 17, 19 All these numbers are prime numbers (numbers with only 2 factors: 1 and itself). Next three prime numbers after 19: 23, 29, 31 Therefore, the next three numbers are 23, 29, 31.

Question 3.
We know that Natural Numbers are closed under addition (the sum of any two natural numbers is always a natural number). Are they closed under subtraction? Provide a couple of examples to justify your answer.

Solution:
Natural numbers are 1, 2, 3, 4,… Check subtraction: 5 – 3 = 2 (natural number) 3 – 5 = -2 (not a natural number) 4 – 5 = -1 (not a natural number) Therefore, natural numbers are not closed under subtraction.

Question 4.
Ancient Indians used the joints of their fingers to count, a practice still seen today. Each finger has 3 joints, and the thumb is used to count them. How many can you count on one hand? How does this relate to the ancient base-12 counting systems?

Solution:
On one hand, we use 4 fingers (excluding the thumb) for counting. Each finger has 3 joints.

So, total joints: 4 × 3 = 12

Using the thumb to touch each joint, we can count from 1 to 12 on one hand. Since one hand allows counting up to 12, ancient people likely used this method to count in groups of 12. This led to the development of the base-12 (duodecimal) system, where numbers are grouped in 12s instead of 10s.

Exercise Set 3.2 Solutions

Question 1.
The temperature in the high-altitude desert of Ladakh is recorded as 4 °C at noon. By midnight, it drops by 15 °C. What is the midnight temperature?

Solution:
The temperature at noon is 4 °C. By midnight, it drops by 15° C, which means we subtract 15 from 4: 4 – 15 = -11 So, the midnight temperature is -11 °C.

Question 2.
A spice trader takes a loan (debt) of ₹ 850. The next day, he makes a profit (fortune) of ₹ 1,200. The following week, he incurs a loss of ₹ 450. Write this sequence as an equation using integers and calculate his final financial standing.

Solution:
The trader first takes a loan of ₹ 850, which is represented as -850. The next day, he earns a profit of ₹ 1200, represented as +1200. Then he incurs a loss of ₹ 450, represented as -450.

So, the situation can be written as: -850 + 1200 – 450

Now calculate step by step: -850 + 1200 =350 350 – 450 = -100 So, his final financial standing is -₹100, which means he is still in debt of ?100.

Question 3.
Calculate the following using Brahmagupta's laws:
(i) (-12) × 5

Solution:
(-12) × 5 The product of a debt and a fortune is a debt(A negative multiplied by a positive is negative): -12 × 5 = -60

(ii) (-8) × (-7)

Solution:
(-8) × (-7) The product of two debts is a fortune (A negative multiplied by a negative is positive): -8 × -7 = 56

(iii) 0 – (-14)

Solution:
0 – (-14) Zero minus debt is a fortune (Subtracting a negative becomes addition): 0 + 14 = 14

(iv) (-20)+ 4

Solution:
(-20) + 4 A debt divided by fortune is debt (A negative divided by a positive is negative): -20 + 4 = -5

Question 4.
Explain, using a real-world example of debt, why subtracting a negative number is the same as adding a positive number (e.g., 10 – (-5) = 15).

Solution:
Suppose you have ₹ 10, and a debt of ₹ 5 is represented as -₹ 5. Now, if this debt is removed, it means you no longer owe that amount. Removing a debt increases what you effectively have.

So, starting with ₹ 10 and subtracting a debt of ₹ 5: 10 – (-5) means you are removing a debt of ₹ 5, which is the same as gaining ₹ 5. Therefore, 10 – (-5) = 10 + 5 = 15 Thus, subtracting a negative number is the same as adding a positive number because removing a debt increases your total.

Exercise Set 3.1 Solutions

Question 1.
Prove that the following rational numbers are equal:
(i) 23 and 46

Solution:
Two rational numbers ab and cd are said to be equal if ad = bc.
23 and 46 a = 2, b = 3 and c = 4, d = 6 Here, 2 × 6 = 3 × 4 ⇒ 12 = 12 [a × d = b × c] So, both are equal.

(ii) 54 and 108

Solution:
54 and 108; a = 5 , b = 4 and c = 10, d = 8 5 × 8 = 4 × 10 ⇒ 40 = 40 [a × d = b × c] So, both are equal.

(iii) −35 and −610

Solution:
−35 and −610; a = -3, b = 5 and c = -6, d = 10 ⇒ -30 = -30 [a × d = b × c] So, both are equal.

(iv) 93 and 3

Solution:
93 and 3; a = 9, b = 3 and c = 3 d = 1 9 × 1 = 3 × 3 ⇒ 9 = 9 [a × d = b × c] So, both are equal.

Question 2. Find the sum:
(i) 25+310

Solution:
To find the sum of rational numbers, first make the denominators the same (LCM of denominators), then add the numerators.
25+310
LCM of 5 and 10 = 10
25=410
410+310=710

(ii) 712+58

Solution:
712+58
LCM of 12 and 8 = 24

712=1424,58=1524
1424+1524=2924

(iii) −47+314

Solution:
LCM of 7 and 14 = 14
−47=−814
−814+314=−514

Question 3. Find the difference:
(i) 56−14

Solution:
56−14
LCM of 6 and 4 = 12
56=1012,14=312
1012−312=712

(ii) 118−34

Solution:
LCM of 8 and 4 = 8
34=68
118−68=58

(iii) −79−(−23)

Solution:
LCM of 9 and 3 = 9
23=69
−79+69=−19

Question 4. Find the product:
(i) 23×310

Solution:
To find the product of rational numbers, multiply the numerators and multiply the denominators, then simplify if possible.
23×310
2×33×10=630=15

(ii) 711×58

Solution:
711×58
7×511×8=3588

(iii) −47×514

Solution:
−47×514
−4×57×14=−2098=−1049

Question 5. Find the quotient:
(i) 25÷310

Solution:
25÷310=25×103=2015=43

(ii) 711÷58

Solution:
711÷58=711×85=5655

(iii) −47÷514

Solution:
−47÷514=−47×145=−85

Question 6.
Show that: (12+34)×83=12×83+34×83

Solution:
To show:

$$\left(\frac{1}{2}+\frac{3}{4}\right)\times\frac{8}{3} = \frac{1}{2}\times\frac{8}{3}+\frac{3}{4}\times\frac{8}{3}$$

LHS:

$$\left(\frac{1}{2}+\frac{3}{4}\right)\times\frac{8}{3} = \left(\frac{2}{4}+\frac{3}{4}\right)\times\frac{8}{3} = \frac{5}{4}\times\frac{8}{3}$$

$$\frac{5}{4}\times\frac{8}{3} = \frac{40}{12} = \frac{10}{3}$$

RHS:

$$\frac{1}{2}\times\frac{8}{3}+\frac{3}{4}\times\frac{8}{3} = \frac{8}{6}+\frac{24}{12} = \frac{4}{3}+2$$

$$\frac{4}{3}+2 = \frac{4}{3}+\frac{6}{3} = \frac{10}{3}$$

Since LHS = RHS = $\frac{10}{3}$,

$$\left(\frac{1}{2}+\frac{3}{4}\right)\times\frac{8}{3} = \frac{1}{2}\times\frac{8}{3}+\frac{3}{4}\times\frac{8}{3}$$

Hence verified.

Question 7.
Simplify the following using the distributive property: 79(67−34)

Solution:
Apply distributive property:
79(67−34)=79×67−79×34

Simplify each term: 79×67=23,79×34=712

Now subtract: 23−712=812−712=112

Therefore, 79(67−34)=112

Question 8.
Find the rational number x such that: 56(x+35)=56x+12.

Solution:
Apply the distributive property on the LHS:
56(x+35)=56x+56×35

Simplify the product: 56×35=36=12

So, LHS becomes: 56(x+35)=56x+12

So, the equation becomes: 56x+12=56x+12

Subtract 56 x from both sides: 12=12 This is always true, so the equation holds true for all rational numbers. So, x can be any rational number.

Exercise Set 3.4 Solutions

Question 1.
Represent the rational numbers 23, –54 and 112 on a single number line.

Solution:
To represent the rational numbers 23, –54 and 1 12on a single number line, first identify their positions.

Identify the values 23 lies between 0 and 1.
–54 = -114, which lies between -2 and -1.
112 = 32, which lies between 1 and 2.

Use suitable divisions Mark integers -2, -1,0, 1 and 2 on the number line.

- For –54 : Divide the interval from -2 to -1 into 4 equal parts and mark one part to the left of -1 (-114)
- For 23: Divide the interval from 0 to 1 into 3 equal parts and mark the second part.
- For 112: Divide the interval from 1 to 2 into 2 equal parts and mark the midpoint.

Thus, the number line:

Question 2.
Find three distinct rational numbers that lie strictly between –12 and 14

Solution:
First, take the average of –12 and 14:
(−12+14)2=−24+142=−142=−18

So, –18 lies between –12 and 14.

Now find the average of –12 and –18:
(−12−18)2=−48−182=−582=−516

Next, find the average of –18 and 14:
(−18+14)2=−18+282=182=116

Thus, three distinct rational numbers strictly between –12 and 14 are:
−516,−18,116 (Answer may vary)

Question 3.
Simplify the expression

Solution: Find the LCM of 4 and 12, which is 12. Convert – 14 to denominator 12:
−14=−312

Now add: −312+512=212

Simplify 212=16

Therefore, −14+512=16

Question 4.
A tailor has 1534 metres of fine silk. If making one kurta requires 2 14 metres of silk, exactly how many kurtas can 4 he make?

Solution:
Convert mixed fractions to improper fractions:
1534=634,214=94

Now divide: 634÷94=634×49=639 = 7 Therefore, he can make 7 kurtas.

Question 5.
Find three rational numbers between 3.1415 and 3.1416.

Solution:
To find rational numbers between 3.1415 and 3.1416, write them with more decimal places: 3.1415 = 3.14150, 3.1416 = 3.14160

Now choose any three numbers between these: 3.14151, 3.14152, 3.14153

These are all rational numbers and lie strictly between the given numbers. Therefore, the required rational numbers are: 3.14151, 3.14152, 3.14153 (Answers may vary).

Question 6.
Can you think of other way(s) to find a rational number between any two rational numbers?

Solution:
One simple way is to make the denominators the same. Suppose the two rational numbers are ab and cd with ab<cd.

Convert them to equivalent fractions with a common denominator, then choose any fraction whose numerator lies between the two numerators. Another way is to multiply both numbers by the same positive number to create more space, then choose a number in between and divide back.

Example: To find a rational number between 13 and 12, write them with denominator 6:

13=26,12=36

Now a number between them is:

512

since

26<512<36 We can also express the rational numbers in decimal form and then find the rational numbers between them. So, besides the average method, we can use common denominators, scaling, by expressing in decimal form or simply pick a fraction with a suitable numerator after rewriting.

Exercise Set 3.5 Solutions

Question 1.

Without performing long division, determine which of the following rational numbers will have terminating decimals and which will be repeating: 720,415 and 13250

Then check your answers by explicitly performing the long divisions and expressing these rational numbers as decimals.

Solution:

For 720 :

$20 = 2^2 \times 5$ Only 2 and 5 appear in the denominator, so the decimal terminates.

Now check by long division:

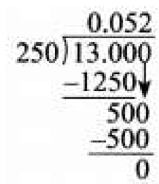
So 720 = 0.35

For 415 : 15 = 3 × 5 Since 3 appears in the denominator, the decimal will be repeating.

Long division:

4 ÷ 15= $0.2\overline{6}$
So, 13250
$250 = 2 \times 5^3$ Only 2 and 5 appear in the denominator, so the decimal terminates.

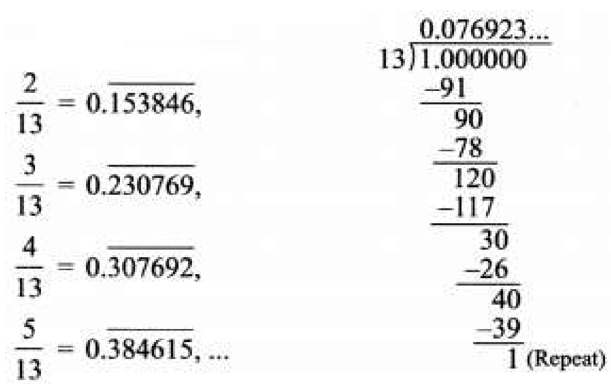
Long division:



13 ÷ 250 = 0.052
So, 13250 = 0.052

Question 2.
Perform the long division for 113. Identify the repeating block of digits. Does it show cyclic properties if you evaluate 213? Now compute 313,413etc. What do you notice?

Solution:
Long division for 113:
So, the repeating block is: 076923
Now,



repeating digits are the product of 2, 3, 4, 5, … to the repeating digit of 113.

Question 3.
Classify the following numbers as rational or irrational:
(i) 81−−√
(ii) 12−−√
(iii) 0.33333 …

(iv) 0.123451234512345…
(v) 1.01001000100001 … (Notice the pattern: Is it repeating a single block?)
(vi) 23.560185612239874790120
Find the explicit fractions in case they are rational.

Solution:
(i) 81−−√
81−−√ = 9
So it is rational.
Explicit fraction: 9 = 91

(ii) 12−−√
12−−√ = 4×3−−−−√=23–√
Since √3 is irrational, 2√3 is also irrational.
So 12−−√ is irrational.

(iii) 0.33333… This is a repeating decimal. Let x= 0.33333… Then 10x = 3.33333… Sub tract: 10x – x = 3.33333…- 0.33333… 9x = 3 x = 39=13
So it is rational. Explicit fraction:
0.33333…= 13

(iv) 0.123451234512345… The block 12345 repeats again and again. So, this is a repeating decimal, hence rational. Let x = 0.1234512345… Then 100000 x = 12345.1234512345… Subtract: 100000x – x = 12345 99999 x = 12345 x = 1234599999
This can be simplified as 3:
1234599999=411533333
So, the number is rational.
Explicit fraction:
0.123451234512345… = 411533333

(v) 1.01001000100001… Here, the digits do not repeat in one fixed block. The pattern keeps changing because the zeros keep increasing. So it is non-terminating, non-repeating. Therefore, it is irrational.

(vi) 23.560185612239874790120 This is a terminating decimal because it ends after a finite number of digits. So, it is rational. As a fraction: =
235601856122398747901201021
This may also be simplified to:
589004640305996869753250000000000000000

Question 4.
The number 0.9 (which means 0.99999…) is a rational number. Using algebra (let x = 0.9, multiply by 10, and subtract), explain why 0.9 is exactly equal to 1.

Solution:
Let x = 0.9999… Multiply both sides by 10: 10x = 9.9999…

Now subtract the original equation from this: 10x – x = 9.9999…-0.9999… 9x = 9

Divide both sides by 9: x = 1 But we assumed x = 0.9999…, So 0.9999… = 1 Hence, 0.9 (i.e., 0.9999…) is exactly equal to 1.

Question 5.
We have seen that the repeating block of 17 is a cyclic number. Try to find more numbers (n) whose reciprocals (1n) produce decimals with repeating blocks that are cyclic.

Solution:
A cyclic number is one where its repeating digits rotate when multiplied by numbers.
We already know:
17 = 0.142857 The block 142857 is cyclic because multiplying it by 2,3, 4, 5, or 6 gives rotations of the same digits.

Now, to find more such numbers, we look for values of n where:

- The decimal expansion of 1n is repeating.
- The repeating block has special rotation properties.

These usually occur when:

- n is a prime number.
- The length of the repeating cycle is n -1 (called a full reptend prime).

Examples: 1. 117 =0.0588235294117647 17 The repeating block has 16 digits and shows cyclic-like properties.

2. 119 = 0.052631578947368421 19
A number n (more precisely, a prime p) has this cyclic property when the decimal expansion of 1p repeats with the maximum possible length, which is P – 1. Such primes are called full reptend primes. These primes generate repeating decimals where the digit block cycles under multiplication, forming cyclic numbers.

End of Chapter Exercises Solutions

Question 1.
Convert the following rational numbers in the form of a terminating decimal or non-terminating and repeating decimal, whichever the case may be, by the process of long division:
(i) 350

Solution:
350 = 0.06

$$\begin{array}{r} 0.06 \\ 50\overline{)3.00} \\ \underline{-300} \\ 0 \end{array}$$

This is a terminating decimal.

(ii) 29

Solution:

$$\begin{array}{r} 0.22... \\ 9\overline{)2.000} \\ \underline{-1.8} \\ 0.20 \\ \underline{-0.18} \\ 0.02 \end{array}$$

29 = 0.2222……… = $0.\overline{2}$ This is a non-terminating repeating decimal.

Question 2. Prove that √5 is an irrational number. Solution: We will use proof by contradiction. Assume that √5 is rational.

Then it can be written in the form: √5 = pq
where p and q are integers having no common factor (i.e., in lowest form), and q 0.
Squaring both sides:
5 = p2q2
$p^2 = 5q^2$
This means p^2 is divisible by 5, so p must also be divisible by 5.

Let p = 5k for some integer k. Substitute back: $(5k)^2 = 5q^2$
$25k^2 = 5q^2$

Divide both sides by 5: $5k^2 = q^2$
This shows q^2 is divisible by 5, so q is also divisible by 5. So, both p and q are divisible by 5, which contradicts our assumption that pq is in lowest form.

Thus, our assumption is wrong. Hence, √5 is irrational.

Question 3.
Convert the following decimal numbers into the form pq.
(i) 12.6

Solution:
Let x = 12.6
x = 12610=635

(ii) 0.0120

Solution: 0.0120 = 12010000=3250

(iii) 3.052

Solution:
Let x = 3.0525252…
10x = 30.5252…
1000x = 3052.5252…
1000x – 10x = 3022
⇒ 990x = 3022
⇒ x = 3022990=1511495

(iv) 1.235

Solution:
Let x = 1.2353535…
10x = 12.3535…
1000x = 1235.3535…
1000x – 10x = 1223
⇒ 990x = 1223
⇒ x = 1223990

(v) 0.23

Solution:
Let x = 0.232323…
100x = 23.2323…
100x – x = 23
99x = 23
x = 2399

(vi) 2.05

Solution:
Let x = 20.5555…
100x = 2.0555…
100x – x = 20.5555…. – 2.0555….
9x = 18.5
⇒ x = 18.59=18590=3718

(vii) 2.125

Solution:
Let x = 2.125555…
100.x = 212.5555…
1000x = 2125.5555…
1000x – 100x = 2125.5555… – 212.5555…
900x = 1913
⇒ x = 1913900

(viii) 3.125

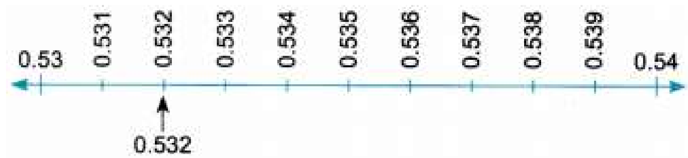
Solution:
Let x = 3.125555…
100x = 312.5555…
1000x = 3125.5555…
900x = 2813
x = 2813900

(ix) 2.1625

Solution:
Let x = 2.162516251625…
10000x= 21625.1625…
1000x – x = 21625.1625… – 2.1625…
9999x = 21623
x = 216239999

Question 4.
Locate the following rational numbers on the number line,
(i) 0.532
(ii) 1.15

Solution:
(i) 0.532 lies between 0.53 and 0.54. Divide the segment from 0.53 to 0.54 into 10 equal parts; the 2nd mark after 0.53 gives 0.532



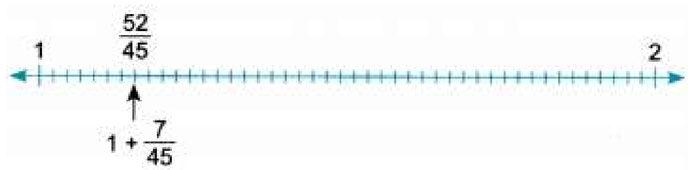
(ii) Convert the repeating decimal to a fraction first: Let x — 1.15555… Then 10x= 11.5555… 100x = 115.555… Subtract: 100x – 10x= 115.5555… — 11.555… = 10.4 90x = 104 x = 10490=5245

So,

1.15555….. = 5245

5245 = 1 + 745

It lies between 1 and 2. Divide the segment from 1 to 2 into 45 equal parts. Count 7 parts after 1.



Question 5.
Find 6 rational numbers between 3 and 4.

Solution:
Write 3 and 4 with a common denominator: 3 = 3010, 4 = 4010

Now choose any 6 rational numbers between them:
3110,3210,3310,3410,3510,3610

So, 6 rational numbers between 3 and 4 are: 3110,3210,3310,3410,3510,3610 (Answer may vary)

Question 6.
Find 5 rational numbers between 23 and 35.

Solution:
Make denominators larger to create more:
25=2050,35=3050

Now take any 5 rational numbers between them: 2150,2250,2350,2450,2550

So the required rational numbers are: 2150,2250,2350,2450,2550 (Answer may vary)

Question 7.
Find 5 rational numbers between 16 and 25.

Solution:
Take LCM of denominators 6 and 5, which is 30:
16=530,25=1230

Now pick any 5 rational numbers between them: 630,730,830,930,1030

So the required rational numbers numbers are: 630,730,830,930,1030 (Answer may vary)

Question 8.
If 12, find the rational number x.

Solution:
Take LCM of 3 and 5, which is 15.
x3=5x15,x5=3x15

So, the equation becomes: 5x+3x15=1615
8x15=1615

Multiply both sides by 15: 8x = 16 x = 168 = 2 x = 2

Question 9.
Let a and b be two non-zero rational numbers such that a + 1b = 0
Without assigning numerical values, determine whether ab is positive or negative. Justify your answer.

Solution:
Given a + 1b = 0
Rearrange: a = –1b Now multiply both sides by b: ab = -1 Since -1 is a negative number, so, ab is negative.

Question 10.
A rational number has a terminating decimal expansion whose last non-zero digit occurs in the 4th decimal place. Show that such a number can be written in the form, p104 where p is an integer not divisible by 10. Is it necessary that the denominator of this rational number, when written in lowest form, is divisible by 24 or 54? Give reasons.

Solution:
Let the number be x. Since the decimal expansion terminates and the last non¬zero digit is at the 4th decimal place, the number must be of the form: x= a.bcde where e 0 and there are no non-zero digits beyond the 4th decimal place.

Multiply by 10^4 = 10000:
1000x = an integer = p
So, x = p104

Now, since the last non-zero digit is exactly at the 4th decimal place, the decimal does not terminate earlier. This means p cannot be divisible by 10, because if p were divisible by 10, then: p104=p10×1103 which would shift the decimal termination to an earlier place — a contradiction.

Hence, x = p104, where p is not divisible by 10.
Now consider the second part.
$10^4 = (2 \times 5)^4 = 2^4 . 5^4$
So initially, the denominator contains both 2^4 and 5^4.

When reducing p104 to lowest terms:

- Since p is not divisible by 10, it cannot be divisible by both 2 and 5 together.
- However, p may be divisible by 2 or by 5 individually.

So, some powers of 2 or 5 may cancel, but not both completely at the same time. Therefore, it is not necessary that the denominator in lowest form is divisible by 2^4 or 5^4.
After simplification, the denominator will be of the form 2^m5^n, where $m \leq 4$, $n \leq 4$, and possibly smaller depending on common factors with p.

Question 11.
Without performing division, determine whether the decimal expansion of 18125 is terminating or non-terminating. If it terminates, state the number of decimal places.

Solution:
A rational number pq (in lowest form) has a terminating decimal expansion if the prime factorisation of the denominator q contains only powers of 2 and/or 5.

First, check if the fraction is in lowest form: HCF (18, 125)= 1 So it is already in lowest term. Now factorise the denominator: $125 = 5^3$

Since the denominator contains only the prime factor 5, the decimal expansion is terminating. To find the number of decimal places, we need to convert the denominator into a power of 10. $10^3 = 2^3 \times 5^3$

Multiply numerator and denominator by $2^3 = 8$:
18125=18×8125×8=1441000 Thus, the decimal has 3 decimal places.

Question 12.
A rational number in its lowest form has denominator $2^3 \times 5$. How many decimal places will its decimal expansion have? Explain your answer.

Solution:
Given denominator:
$2^3 \times 5 = 8 \times 5 = 40$
A rational number has a terminating decimal if the denominator (in lowest form) is of the type $2^m \times 5^n$. To find the number of decimal places, we convert the denominator into a power of 10. 10 = 2 × 5

We need equal powers of 2 and 5. Here we have: $2^3 \times 5^1$

To balance, multiply by 5^2:
$2^3 \times 5^3 = 10^3 = 1000$
So, 140=251000 This shows the decimal will have 3 decimal places.

Question 13.
Let a = 712 and b = 56. Express both a and b in the form k1m and k2m where kh k2 and m are integers and $k_2 – k_1 > 6$. Using the same denominator m, write exactly five distinct rational numbers lying between a and b keeping an integer numerator. Explain why the condition $k_2 – k_1 > n + 1$ is necessary to find n such rational numbers between the two rational numbers a and b using this method.

Solution:
First, write both fractions with a common denominator.
a = 712, b = 56=1012
Here,
$k_1 = 7$, $k_2 = 10$, m = 12 => $k_2 – k_1 = 3$

But 3 < 6, so we need a larger common denominator. Multiply both by 4: a = 712=2848
b = 1012=4048

Now, $k_1 = 28$, $k_2 = 40$, m = 48
⇒ $k_2 – k_1 = 12 > 6$

Now write five rational numbers between them: 2948,3048,3148,3248,3348
These all lie strictly between ∀ and ∀.

Why the condition $k_2 – k_1 > n + 1$ is necessary:
Between k1m and k2m, the numbers with denominator m are:
k1+1m,k1+2m,…,k2−1m

Total numbers between them: $k_2 – k_2 – 1$
To find at least numbers, we need:
$k_2 – k_1 – 1 > n$
$k_2 – k_1 > n + 1$
Hence, the condition:
$k_2 – k_1 > n + 1$
ensures enough integers exist between k_1 and k_2 to form rational numbers.

Question 14.
Three rational numbers x, y, z satisfy x + y + z = 0 and xy + yz + zx = 0. Show that all the rational numbers x, y, z must be simultaneously zero.

Solution:
Given: $x + y + z = 0 \Rightarrow z = -(x + y)$

Substitute this in the second equation $xy + yz + zx = 0$: $xy + y(-x – y) + x(-x – y) = 0$

Simplify: $xy – xy – y^2 – x^2 – xy = 0$
$-(x^2 + y^2 + xy) = 0$
$x^2 + y^2 + xy = 0$ …(1)

Now consider: $(x + y)^2 = x^2 + y^2 + 2xy$
Using (1), substitute $x^2 + y^2 = -xy$:
$(x + y)^2 = -xy + 2xy = xy$
So, $(x + y)^2 = xy$ …(2)

But from $z = -(x + y)$, we also have: $z^2 = (x + y)^2$
So from (2):
$z^2 = xy$ …(3)

Now similarly, by symmetry: $x^2 = yz$, $y^2 = zx$
Add all three:
$x^2 + y^2 + z^2 = xy + yz + zx$

But given: $xy + yz + zx = 0$ So, $x^2 + y^2 + z^2 = 0$

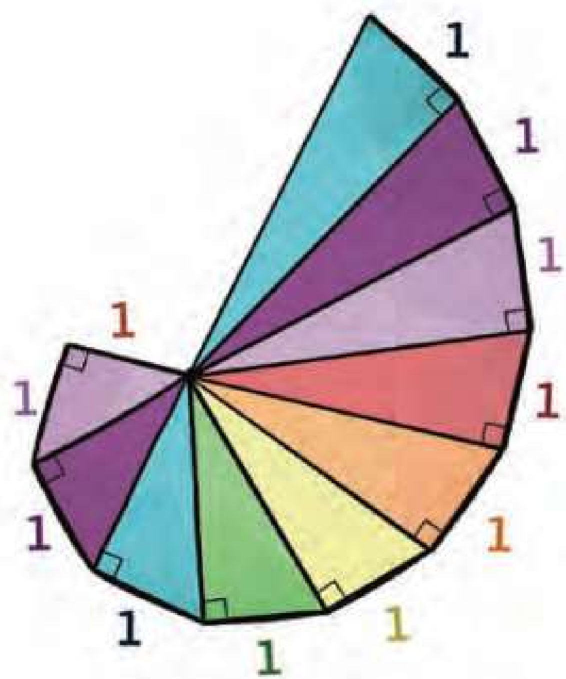
Since squares of rational numbers are non-negative, the only way their sum is zero is: $x^2 = y^2 = z^2 = 0 \Rightarrow x = y = z = 0$ $z = y = z = 0$

Question 15.
Show that the rational number lies between the rational numbers a and b.

Solution:
Assume a < b (the case b < a is similar). We need to prove: a < a+b2 < b
First part: Prove a < a+b2
Multiply both sides by 2 (positive number, so inequality stays same):
2a < a + b
a< b (which is true)
Hence,
a < a+b2

Second part: Prove a+b2 < b
Multiply both sides by 2:
a + b < 2b
a < b (which is true)
Hence,
a+b2 < b
Conclusion
a < a+b2 < b
So, a+b2 lies between a and b

Question 16.
Find the lengths of the hypotenuses of all the right triangles in Figure which isjeferred to as the square root spiral.



Square root spiral

Solution:

Each new triangle is formed by taking the previous hypotenuse as one side and 1 as the other side. We can find the hypotenuse of each triangle using the Baudhayana- Pythagoras Theorem:

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

For triangle 1: Sides: 1, 1

Hypotenuse = 12+12−−−−−−√ = √2

For triangle 2: Sides: √2, 1

Hypotenuse = (2–√)2+12−−−−−−−−−√=2+1−−−−√ = √3

Similarly: Triangle 3: Sides: √3 , 1; Hypotenuse: 2 Triangle 4: Sides: 2, 1; Hypotenuse: √5 Triangle 5: Sides: √5 , 1; Hypotenuse: √6 Triangle 6: Sides: √6, 1; Hypotenuse: √7 Triangle 7: Sides: √7, 1; Hypotenuse: √8 Triangle 8: Sides: √8,, 1; Hypotenuse: 3 Triangle 9: Sides: 3, 1; Hypotenuse: √1o Triangle 10: Sides: √10, 1; Hypotenuse: √11