Exploring Algebraic Identities Class 9 Solutions Maths Ganita Manjari Chapter 4

Think and Reflect (NCERT Textbook Page No. 71)

Question 1.
What can you say about a and b if $(a + b)^2 < a^2 + b^2$?

Solution:
Given that $(a + b)^2 < a^2 + b^2$
We have, $(a + b)^2 = a^2 + 2ab + b^2$
So, $a^2 + 2ab + b^2 < a^2 + b^2$
Subtracting $a^2 + b^2$ from both sides, we get $2ab < 0$, which is only possible when a and ft have opposite signs.
So, a and ft must have opposite signs for $(a + b)^2 < a^2 + b^2$.

Question 2.
What can you say about a and b if $(a + b)^2 > a^2 + b^2$?

Solution:
Given that $(a + b)^2 > a^2 + b^2$
We have, $(a + b)^2 = a^2 + 2ab + b^2$
So, $a^2 + 2ab + b^2 > a^2 + b^2$
Subtracting $a^2 + b^2$ from both sides, we get $2ab > 0$, which is only possible when both a and b have same sign either positive or negative.
So, a and b must have the same sign for $(a + b)^2 > a^2 + b^2$.

Question 3.
When will (a + b)2 be equal to $a^2 + b^2$?

Solution:
Given that $(a + b)^2 = a^2 + b^2$
We have, $(a + b)^2 = a^2 + b^2 + 2ab$
So, $a^2 + 2ab + b^2 = a^2 + b^2$
Subtracting $a^2 + b^2$ from both sides, we get $2ab = 0$, which is only possible when either $a = 0$ or $b = 0$.
So, either $a = 0$ or $b = 0$ for $(a + b)^2 = a^2 + b^2$.

Did you observe that $(a + b)^2$ and $a^2 + b^2$ are both positive? What term will decide which is larger? Use the expansion of $(a + b)^2$ to decide.

Solution:
Yes, both $(a + b)^2$ and $a^2 + b^2$ are indeed positive, as they are squares of numbers.
The term 2ab will determine which of the two expressions is larger. If a and b have the same sign, then $2ab > 0$, so $(a + b)^2 > a^2 + b^2$. If they have opposite signs, then $2 ab < 0$, so $(a + b)^2 < a^2 + b^2$.

Think and Reflect (NCERT Textbook Page No. 73)

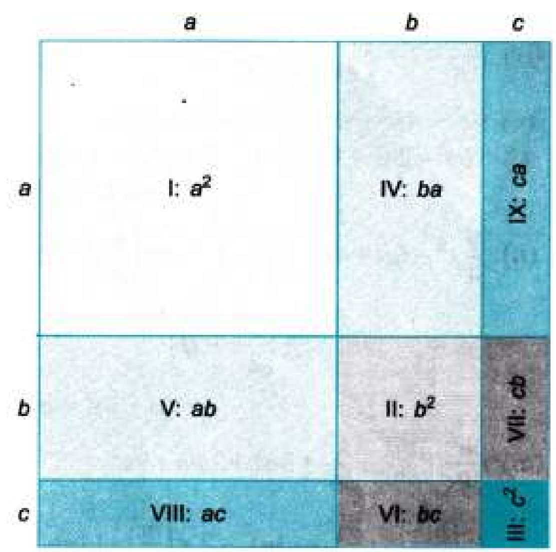
What if we replace b by -b in $(a + b)^2 = a^2 + 2ab + b^2$

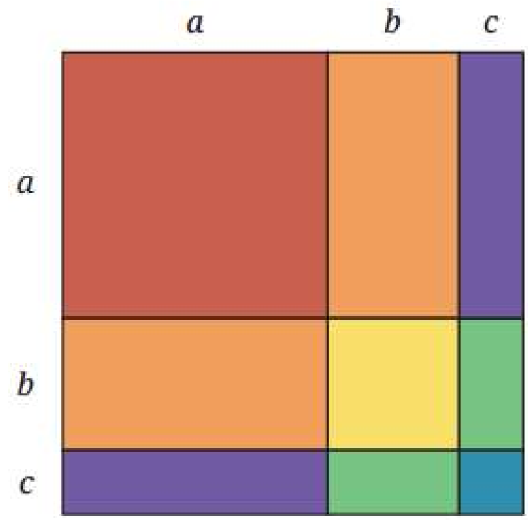
Solution:
We get $(a – b)^2 = a^2 – 2ab + b^2$, which is also an identity and can be used in ways similar to $(a + b)^2 = a^2 + 2ab + b^2$

Think and Reflect (NCERT Textbook Page No. 76)

Label the squares and rectangles in Fig. 4.4 so that it represents the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

Solution:





Think and Reflect (NCERT Textbook Page No. 78)

Question 1.
Try to evaluate the following using a suitable identity:
(i) 35^2
(ii) 65^2
(iii) 85^2
(iv) 105^2
Do you observe any interesting pattern?

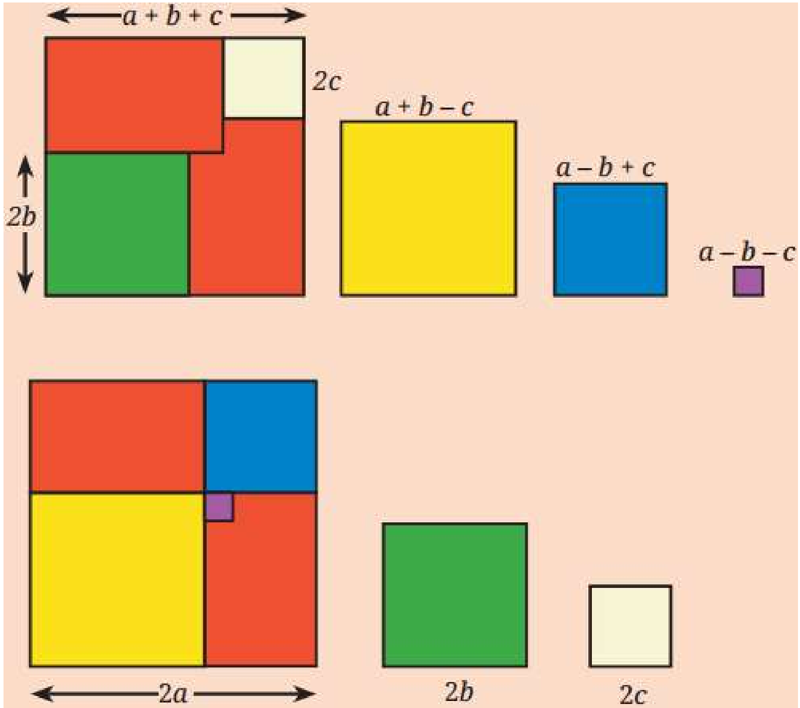
Solution:
(i) We have, $a^2 = (a + b)(a – b) + b^2$
$35^2 = (35 + 5)(35 – 5) + 5^2 = 40 × 30 + 25 = 1225$

(ii) $652 = (65 + 5 × 65 – 5) + 5^2 = 70 × 60 + 25 = 4200 + 25 = 4225$

(iii) $852 = (85 + 5)(85 – 5) + 5^2 = 90 × 80 + 25 = 7200 + 25 = 7225$

(iv) $1052= (105 + 5)(105 – 5)+ 5^2$
$= 110 × 100 + 5^2 = 11000 + 25 = 11025$ Here we observe that to square a number ending in 5, multiply the leading part (the part except 5) by its successor and write 25 at the end.

Question 2.
Observe the two rows of figures below. They represent an algebraic identity. Try to identify it.



Solution:
The top row shows squares with sides
(a + b + c), (a + b – c), (a – b + c), (a – b – c)
So the total area of squares in the top row is
$(a + b + c)^2 + (a + b – c)^2 + (a – b + c)^2 + (a – b – c)^2$.

The bottom row rearranges all those pieces into:

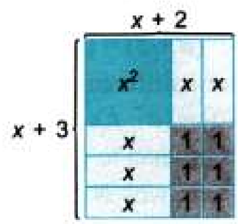
- one square of side 2a
- one square of side 2b
- one square of side 2c

So, their areas are $(2a)^2$, $(2b)^2$, $(2c)^2$
The total area of squares in the bottom row is
$= (2a)^2 + (2b)^2 + (2c)^2$
$= 4a^2 + 4b^2 + 4c^2$
$= 4 (a^2 + b^2 + c^2)$
So, the algebraic identify represented:
$(a + b + c)^2 + (a + b – c)^2 + (a-b + c)^2 + (a – b – c)^2 = 4 (a^2 + b^2 + c^2)$.

Think and Reflect (NCERT Textbook Page No. 79)

Question 1.
Figure out the product of x + 2 and x + 3 using algebra tiles.

Solution:
We want to find $(x + 2)(x + 3)$ Step 1: Represent each binomial with algebra tiles
Step 2: Form a rectangle (area model) with dimensions $(x + 2)$ by $(x + 3)$.

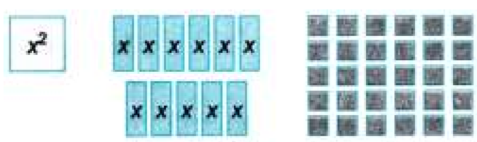


Step 3: Add the areas of all tiles
$x^2 + x + x + x + x + x + 1 + 1 + 1 + 1 + 1 + 1$
$= x^2 + 5x + 6$
Therefore, $(x + 2)(x + 3) = x^2 + 5x + 6$

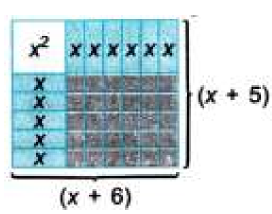
Question 2.
Lay out algebra tiles for $x^2 + 11x + 30$ in such a way that you will see its factors.

Solution:
Layout algebra tiles for $x^2 + 11x + 30$

Step 1: Use the tiles $x^2 + 11x + 30 = 1x^2 + 11x + 30$.



Step 2: Arrange to form a rectangle



We have arranged the tiles into a rectangle with dimensions $(x + 5)$ by $(x + 6)$
Step 3: Hence, $x^2 + 11x + 30 = (x + 5)(x + 6)$ So, the factors are $(x + 5)$ and $(x + 6)$.

Think and Reflect (NCERT Textbook Page No. 80)

We have seen that $(x + 3)(x + 4) = x^2 + 7x + 12$.
Also $(x + 6)(x + 7) = x^2 + 13x + 42$.
Generalise the pattern to get an expression for $(x + a)(x + b)$.
Solution:
$(x + 3)(x + 4) = x^2 + 3x + 4x + 3 \times 4$
$= x^2 + (3 + 4)x + 12$
$= x^2 + 7x + 12$

$(x + 6)(x +7) = x^2 + 6x + 7x + 6 \times 7$
$= x^2 + (6 + 7)x + 6 \times 7$
$= x^2 + 13x + 42$
Thus, the general pattern for $(x + a)(x + b)$ is:
$(x + a)(x + b) = x^2 + (a + b)x + ab$

Think and Reflect (NCERT Textbook Page No. 82)

James and Reshma were talking about algebraic identities they leamt in school. James: $(a – b)^2 (a + b) = (a^2 – 2ab + b^2)(a + b)$
Reshma: I have a different idea, $(a – b)^2 (a + b) = (a – b) [(a-b) (a + b)] = (a-b)(a^2 – b^2)$
I will find this product to get the answer.
According to you, who is correct and why?
Try to combine more such identities and find new results.

Solution:
Both are correct. They used different algebraic identities to reach the same result. Both identities are valid, and combining them in different ways still leads to the same final polynomial $(a^3 – a^2b – ab^2 + b^3)$.

Think and Reflect (NCERT Textbook Page No. 85)

We already know that $x^2 – y^2 = (x – y)(x + y)$
Further, we have verified that $x^3 – y^3 = (x -y)(x^2 + xy + y^2)$ Observe that $x – y$ is a common factor of $x^2 – y^2$ and $x^3 – y^3$. Do you think $x-y$ is also a factor of $x^4 – y^4$?
Note that $x^4 – y^4 = (x^2)^2 – (y^2)^2 = (x^2 – y^2) (x^2 + y^2)$.
Can you see how $x – y$ is a factor of $x^4 – y^4$?
How about $x^5 – y^5$? Does this also have $x – y$ as a factor?

Solution:
For $x^4 – y^4$
$x^4 – y^4 = (x^2)^2 – (y^2)^2 = (x^2 – y^2) (x^2 + y^2)$
We have, $x^2 – y^2 = (x – y)(x + y)$
So, $x^4 – y^4 = (x – y)(x + y)(x^2 + y^2)$

Hence, yes, $x - y$ is a factor of $x^4 - y^4$.
Since, $x^2 - y^2 = (x - y)(x + y)$
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
$x^4 - y^4 - (x - y)(x + y)(x^2 + y^2)$
$= (x - y)(x^3 + x^2y + xy^2 + y^3)$
A pattern created by above identities.
Hence, yes, $x - y$ is also a factor of $x^5 - y^5$.
As, $x^5 - y^5 = (x - y)(x^4 + x^3y + xy^3 + x^2y^2 + y^4)$

Think and Reflect (NCERT Textbook Page No. 87)

Try to simplify the following rational expression:
36s2−12st+t2t2+2ts−48s2=(6s−t)2(?+?)(?+?).
(Hint: Factor t2 + 2ts – 48s2 and simplify the rational expressions assuming that $t^2 + 2ts - 48s^2$ * 0).

Solution:
$36s^2 - 12st + t^2 = (6s)^2 - 2(6s)t + t^2$
$= (6s - t)^2$
$= t^2 + 2ts - 48s^2$
$= t^2 + [8s + (-6s)]t + (-6s)(8s)$
$= [t + 8s]\ [t + (-6s)]$
$= (t + 8s)\ (t - 6s)$
$= -(t + 8s)\ (6s - t)$
36s2−12st+t2t2+2ts−48s2=(6s−t)2(t+8s——)(−6s———+t–)=(6s−t)(6s−t)−(t+8s)(6s−t)
= −(6s−t)(t+8s)=t+6st+8s
Thus 36s2−12st+t2t2+2ts−48s2=t+6st+8s

Exercise Set 4.1 Solutions

Question 1.
Using the identity $(a + b)^2 = a^2 + 2ab + b^2$, expand the following:

(i) $(7x + 4y)^2$

Solution:
$(7x + 4y)^2 = (7x)^2 + 2 \times 7x \times 4y + (4y)^2$
$[\because (a + b)^2 = a^2 + 2ab + b^2] = 49x^2 + 56xy + 16y^2$

(ii) $(75x + 32y)^2$

Solution:

$$\left(\frac{7}{5}x+\frac{3}{2}y\right)^2 = \left(\frac{7}{5}x\right)^2 + 2\left(\frac{7}{5}x\right)\left(\frac{3}{2}y\right) + \left(\frac{3}{2}y\right)^2$$
$$= \frac{49}{25}x^2 + \frac{42}{10}xy + \frac{9}{4}y^2$$
$$= \frac{49}{25}x^2 + \frac{21}{5}xy + \frac{9}{4}y^2$$

(iii) $(2.5p + 1.5q)^2$

Solution:
$(2.5p + 1.5q)^2$
$= (2.5p)^2 + 2 \times (2.5p) \times (1.5q) + (1.5q)^2$
$= 6.25p^2 + 1.5pq + 2.25\ q^2$

(iv) $(34s + 8t)^2$

Solution:
$(34s + 8t)^2$
$=(34s)^2 + 2(34s)(8t) + (8t)^2$
$= 916s^2 + 12st + 64t^2$

(v) $(x + 12y)^2$

Solution:
$(x + 12y)^2$
$= x^2 + 2(x)(12y) + (12y)^2$
$= x^2$ + x2y + 14y2

(vi) (1x+1y)2

Solution:
(1x+1y)2
= (1x)2+2(1x)(1y)+(1y)2
= 1x2+2xy+1y2

Question 2.
Using the same identity, find the values of the following:
(i) $(64)^2$

Solution:
$(64)^2$
Using the identity: $(a + b)^2 = a^2 + 2ab + b^2$
$64^2 = (60 + 4)^2$
$= 60^2 + 2 \times 60 \times 4 + 4^2 = 3600 + 480 + 16 = 4096$

(ii) $(105)^2$

Solution:
$(105)^2$
Using the identity: $(a + b)^2 = a^2 + 2ab + b^2$
$105^2 = (100 + 5)^2$
$= 100^2 + 2 \times 100 \times 5 + 5^2 = 10000 + 1000 + 25 = 11025$

(iii) $(205)^2$

Solution:
$(205)^2$
Using the identity: $(a + b)^2 = a^2 + 2ab + b^2$
$205^2 = (200 + 5)^2$
$= 200^2 + 2 \times 200 \times 5 + 5^2 = 40000 + 2000 + 25 = 42025$

Exercise Set 4.2 Solutions

Question 1. Factor completely:
(i) $9x^2 + 14xy + 16y^2$

Solution:
$9x^2 + 14xy + 16y^2$
$= (3x)^2 + 2 \times (3x) \times (4y) + (4y)^2$
$= (3x + 4y)^2$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]

(ii) $4s^2 + 20st + 25t^2$

Solution:
$4s^2 + 20st + 25t^2$
$= (2s)^2 + 2 \times (2s) \times (5t) + (5t)^2$
$= (2s + 5t)^2$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]

(iii) $49x^2 + 28xy + 4y^2$

Solution:
$49x^2 + 28xy + 4y^2$
$= (7x)^2 + 2 \times (7x) \times (2y) + (2y)^2$
$= (7x + 2y)^2$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]

(iv) $64p^2 + 323pq + 49q^2$

Solution:
$64p^2 + 323pq + 49q^2$
$= (8p)^2 + 2 \times (8p) \times (23q) + (23q)^2$
$= (8p + 23q)^2$

(v) $3a^2 + 4ab + 43b^2$

Solution:
$3a^2 + 4ab + 43b^2$
$= (\sqrt{3}a)^2 + 2 \times (\sqrt{3}a) \times (2\sqrt{3}b) + (2\sqrt{3}b)^2$
[∵ (√3) = √3 × √3 = 3×3−−−−√ = 3]
$= (\sqrt{3}a + 2\sqrt{3}b)^2$ [∵ $(a + b)^2 = a^2 + 2ab + b^2$]

(vi) $95s^2 + 6sv + 5v^2$

Solution:
$= (35\sqrt{s})^2 + 2 \times (35\sqrt{s}) \times (\sqrt{5}v) + (\sqrt{5}v)^2$
(∵ 6 = 2 × 35√ × √5)
$= (35\sqrt{s} + \sqrt{5}v)^2$
[∵ $(a + b)^2 = a^2 + 2ab + b^2$]

Question 2.
Find the values of the following using the identity $(a – b)^2 = a^2 – 2ab + b^2$.
(i) $(79)^2$

Solution:
$(79)^2 = (80 – 1)^2$
$= 80^2 – 2 (80) (1) + 1^2 = 6400 – 160 + 1 = 6241$

(ii) $(193)^2$

Solution:
$(193)^2 = (200 – 1)^2 = 200^2 – 2(200)(7) + 7^2 = 40000 – 2800 + 49 = 37249$

(iii) $(299)^2$

Solution:
$(299)^2 = (300 – 1)^2$
$= 300^2 – 2(300)(1) + 1^2 = 90000 – 600 + 1 = 89401$

Exercise Set 4.3 Solutions

Question 1.
Find the following squares using one of the above these calculations easier.
(i) 117^2

Solution:
$117^2 = (100 + 10 + 7)^2$
$= 100^2 + 10^2 + 7^2 + 2(100)(10) + 2(10)(7) + 2(7)(100)$
[∵ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2be + 2ca$] = 10000 +100 + 49 + 2000 + 140 + 1400 = 13689

(ii) 78^2

Solution:
$78^2 = (80 - 2)^2 = 80^2 - 2 \times 80 \times 2 + 2^2$
[$\because (a - b)^2 = a^2 - 2ab + b^2$] = 6400 – 320 + 4 = 6084

(iii) 198^2

Solution:
$198^2 = (200 - 2)^2$
$= 200^2 + 2^2 - 2 \times 200 \times 2$
[$\because (a - b)^2 = a^2 - 2ab + b^2$] = 40000 – 800 + 4 = 39204

(iv) 214^2

Solution:
$214^2 = (200 + 10 + 4)^2$
$= 200^2 + 10^2 + 4^2 + 2(200)(10) + 2(10)(4) + 2(4)(200)$
[$\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2be + 2ca$] = 40000 + 100 + 16 + 4000 + 80 + 1600 = 45796

(v) 1104^2

Solution:
$1104^2 = (1000+ 100 + 4)^2$
$= 1000^2 + 100^2 + 4^2 + 2(1000) (100) + 2(100)(4) + 2(4)(1000)$
[$\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2be + 2ca$] = 1000000 + 10000 + 16 + 200000 + 800 + 8000 = 1218816

(vi) 1120^2

Solution:
$1120^2 = (1000 + 100 + 20)^2$
$= 1000^2 + 100^2 + 20^2 + 2(1000)(100) + 2(100)(20) + 2(20)(1000)$
[$\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$] = 1000000 + 10000 + 400 + 200000 + 4000 + 40000 = 1254400

Question 2.
Factor using suitable identities:
(i) $16y^2 - 24y + 9$

Solution:
$16y^2 - 24y + 9 = (4y)^2 - 2 \times (4y) \times 3 + (3)^2$
$= (4y - 3)^2$
[$\because (a - b)^2 = a^2 - 2ab + b^2$]

(ii) $94s^2 + 6st + 4t$

Solution:
$94s^2 + 6st + 4t$
$= (32s)^2 + 2 \times (32s) \times (2t) + (2t)^2$
$= (32s + 2t)^2$

(iii) m29+mk3+k24 + 3nk + 2mn + $9n^2$

Solution:
= (m3)2+(k2)2 + $(3n)^2$ + 2 × (m3) × (3n) + 2 × (k2)×(m3) + 2 × (k2) × (3n)
= (m3+k2+3n)2
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

(iv) p216 – 216p2

Solution:
(p4)2 – 2 × p4×4p+(4p)2
= (p4−4p)2
$[\because (a - b)^2 = a^2 - 2ab + b^2]$

(v) $9a^2 + 4b^2 + c^2 - 12ab + 6ac - 46c$

Solution:
$9a^2 + 4b^2 + c^2 - 12ab + 6ac - 46c$
$= (3a)^2 + (-2b)^2 + (c)^2 + 2(3a)(-2b) + 2(3a)(c) + 2(-2b)(c)$
$= [3a + (-2b) + c]^2$
$= (3a - 2b + c)^2$

Question 3.
Expand the following using the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$:
(i) $(p + 3q + 7r)^2$

Solution:
$(p + 3q + 7r)^2$
$= p^2 + (3q)^2 + (7r)^2 + 2\{p)(3q) + 2(3q)(7r) + 2(p)(7r)$
$= p^2 + 9q^2 + 49r^2 + 6pq + 42\ qr + 14pr$

(ii) $(3x - 2y + 4z)^2$

Solution:
$(3x - 2y + 4z)^2$
$= (3x)^2 + (-2y)^2 + (4z)^2 + 2(3x)(-2y) + 2(-2y) + 2(-2y)(4z) + 2(3x)(4z)$
$= 9x^2 + 4y^2 + 16x^2 - 12xy - 16yz + 24xz$

Question 4.
Is this an identity? $(a + b – c)^2 + (a – b + c)^2 + (a – b – c)^2 = 2a^2 + 2b^2 + 2c^2$.

Solution:
No, this is not an identity.
Expanding the LHS of the equation, we get
$a^2 + b^2 + c^2 + 2ab – 2ac – 2bc + a^2 + b^2 + c^2 – 2ab + 2ac – 2bc + a^2 + b^2 + c^2 – 2ab – 2ac + 2bc = 3a^2 + 3b^2 + 3c^2 – 2bc – 2ab – 2\ ac$ which is not equal to RHS. Hence, the given equation is not an identity.

Exercise Set 4.4 Solutions

Question 1.
Fill in the blanks to complete the following identities:
(i) $s^2 – 11s + 24$ = (__________)(__________)

Solution:
$s^2 – 11s + 24$
Compare it with $s^2 + (a + b)s + ab$, we get a + b = -11 and ab = 24.
Clearly, these two equations can be satisfied together only when a = -8 and b = -3 or vice-versa.
So, $s^2 + [(-8) + (-3)]s + 24 = [s + (-8)][s + (-3)]$
$= (s – 8)(s – 3)$
$[x^2 + (a + b)x + ab = (x + a)(x + b)]$
Thus, $s^2 – 11x + 24 = (s – 8) (s – 3)$

(ii) (__________) (x + 1) = $(3x^2 – 4x – 7)$

Solution:
(__________) (x + 1) = $(3x^2 – 4x – 7)$
We have, $3x^2 – 4x -7$
Multiply the coefficient of x^2 (which is 3) by the constant term (which is -7): 3 x (-7) = -21
Now, we need to find two numbers that multiply to -21 and add up to the coefficient of x (which is -4).
The two numbers that satisfy this condition are -7 and 3 because (-7) x 3 = —21,
(-7) + 3 = -4
Therefore, $(3x^2 – 4x -7) = 3x^2 – (7 – 3)x – 7$
$= 3x^2 + 3x – 7x – 7$
$= 3x(x + 1) – 7(x + 1)$
$= (3x – 7) (x + 1)$
Thus, $(3x – 7) (x + 1) = (3x^2 – 4x – 7)$

(iii) $10x^2 - 11x - 6 = (2x - \_\_)(\_\_ + 2)$

Solution:
$10x^2 - 11x - 6 = (2x - \_\_)(\_\_ + 2)$
We have, $10x^2 - 11x - 6$
Multiply the coefficient of x^2 (which is 10) by the constant term (which is -6): 10 x (-6) = -60.
Now, we need to find two numbers that multiply to -60 and add up to the coefficient of x (which is -11). The two numbers that satisfy this condition are -15 and 4 because (-15) x 4 = -60,
(-15)+ 4 = -11.
Therefore, $10x^2 - 11x - 6 = 10x^2 + [4 + (-15)]x - 6$
$= 10x^2 + 4x - 15x - 6$
$= 2x(5x + 2) - 3(5x + 2)$
$= (2x - 3)(5x + 2)$
Thus, $10x^2 - 11x - 6 = (2x - 3)(5x + 2)$

(iv) $6x^2 + 7x + 2 = (\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_\_\_)$

Solution:
$6x^2 + 7x + 2 = (\_\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_\_\_)$
Multiply the coefficient of x^2 (which is 6) by the constant term (which is 2): 6 × 2 = 12.
Now, we need to find two numbers that multiply to 12 and add up to the coefficient of x (which is 7). The two numbers that satisfy this condition are 3 and 4 because 3 × 4 = 12 and 3 + 4 = 7.
Therefore, $6x^2 + 7x + 2 = 6x^2 + 3x + 4x + 2$
$= 3x(2x + 1) + 2(2x + 1)$
$= (3x + 2)(2x + 1)$.
Thus, $6x^2 + 7x + 2 = (3x + 2)(2x + 1)$

Question 2.
Select and use the identity that will help you to find the following products without multiplying directly:
(i) $(41)^2$

Solution:
$(41)^2 = (41 + 1)(41 - 1) + 1^2$
$= 42 \times 40 + 1$
$= 1681$
$[\because a^2 = (a + b)(a - b) + b^2]$

(ii) $(27)^2$

Solution:
$(27)^2 = (30 – 3)^2 = 302^2 × 30 × 3 + 32$
$=900 – 180 + 9$
$=729$
[∵ $(a – b)^2 = a^2 – 2ab + b^2$]

(iii) (23 × 17)

Solution:
$(23 × 17) = (20 + 3) (20 – 3) = 20^2 – 3^2$
$= 400 – 9$
$= 391$
[∵ $a^2 – b^2 = (a + b)(a – b)$]

(iv) $(135)^2$

Solution:
$(135)^2 = (140 – 5)^2$
$= 140^2 – 2 × 140 × 5 + 5^2$
$= 19600 – 1400 + 25$
$= 18225$
[∵ $(a – b)^2 = a^2 – 2ab + b^2$]

(v) $(97)^2$

Solution:
$(97)^2 = (97 + 7) (97 – 7) + 7^2$
$= 104 × 90 + 49$
$= 9409$
[∵ $a^2 = (a + b)(a – b) + b^2$]

(vi) (18 × 29)

Solution:
$(18 × 29) = (20 – 2) (20 + 9) = 20^2 + (-2 + 9)20 + (-2) × 9$
[∵ $(x + a)(x + b) = x^2 + (a + b)x + ab$] $= 400 + 140 – 18 = 522$

(vii) (34 × 43)

Solution:
$= (40 – 6) (40 + 3) = 40^2 + (-6 + 3)40 + (-6) × 3$
[∵ $(x + a)(x + b) = x^2 + (a + b)x + ab$]
$= 40^2 + (-6 + 3)40 + (-6) × 3 = 1600 – 120 – 18 = 1462$

(viii) $(205)^2$

Solution:
$(205)^2 = (205 + 5)(205 – 5) + 5^2$
$= 210 \times 200 + 25$
$= 42025$
$[\because a^2 = (a + b)(a – b) + b^2]$

Question 3.
Factor the following:
(i) $9a^2 + b^2 + 4c^2 – 6ab + 12ac – 4bc$

Solution:
$9a^2 + b^2 + 4c^2 – 6ab + 12ac – 4be$
$= (3a)^2 + (-b)^2 + (2c)^2 + 2(3a)(-b) + 2(3a)(2c) + 2(-b)(2c)$
$= (3a – b + 2c)^2$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

(ii) $16s^2 + 25t^2 – 40st$?

Solution:
$16s^2 + 25t^2 – 40st$
$= (4s)^2 + (5t)^2 – 2 \times (4s)(5t)$
$= (4s – 5t)^2$
$[\because (a – b)^2 = a^2 + b^2 – 2ab]$

(iii) $r^2 – r – 42$

Solution:
$r^2 – r – 42$
$= r^2 + (-7 + 6)r + (—7)(6)$
$= [r+ (-7)](r + 6)$
$[\because (x + a)(x + b) = x^2 + (a + b)x + ab] = (r – 7)(r + 6)$

(iv) $49g^2 + 14gb + b^2$

Solution:
$49g^2 + 14gb + b^2$
$= (7g)^2 + 2(7g)b + h^2$
$= (7g + h)^2$ $[\because (a + b)^2 = a^2 + b^2 + 2ab]$

(v) $64u^2 + 121v^2 + 4W^2 – 176uv – 32uw + 44vw$

Solution:
$64u^2 + 121v^2 + 4W^2 – 176uv- 32uw + 44vw$
$= (-8u)^2 + (11v)^2 + (2w)^2 + 2(-8u)(11v) + 2(-8u)(2v) + 2(11v)(2w)$

= $[(-8)u + 11v + 2w]^2$
[∵ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]
= $(11v + 2w – 8u)^2$

Exercise Set 4.5 Solutions

Question 1.
Simplify the following rational expressions assuming that the expressions in the denominators are not equal to zero:
(i) 3p2−3pq−18q2p2+3pq−10q2

Solution:
3p2−3pq−18q2p2+3pq−10q2
We have,
$3p^2 – 3pq – 18q^2 = 3 (p^2 – pq – 6q^2)$
= $3(p^2 – 3pq + 2pq – 6q^2)$
(∵ -6 = -3 × 2 and -3 + 2 = -1)
= 3[p(p – 3q) + 2q(p – 3q)]
$3p^2 – 3pq – 18q^2 = 3(p + 2q)(p – 3q)$ …(i)
Now, $p^2 + 3pq – 10q^2 = p^2 + 5pq – 2pq – 10q^2$
[∵ -10 = 5 × (-2) and 5 + (-2) = 3]
= p(p + 5q) – 2q(p + 5q)
Therefore, $p^2 + 3pq – 10q^2 = (p – 2q)(p + 5q)$ …(ii)
From (i) and (ii), we get
3p2−3pq−18q2p2+3pq−10q2=3(p+2q)(p−3q)(p−2q)(p+5q)

(ii) n3−3n2m+3nm2−m35m2−10mn+5n2

Solution:
Given the expression: n3−3n2m+3nm2−m35m2−10mn+5n2
Step 1: Factorise the numerator and denominator.
Numerator: The expression $n^3 – 3n^2m + 3nm^2 – m^3$ is a perfect cube expansion and factors as (n – mf
Denominator: The expression $5m^2 – 10mn + 5n^2$ factors as $5(m – n)^2$
Step 2: Simplify the expression.
Now, the expression becomes (n−m)35(m−n)2
Since (n – m) = -(m – n), we get
−(m−n)35(m−n)2=−(m−n)5=n−m5

(iii) w3−v3+x3+3wvxw2+v2+x2−2wv−2vx+2wx

Solution:
w3−v3+x3+3wvxw2+v2+x2−2wv−2vx+2wx
We have, $w^3 – v^3 + x^3 + 3wvx$

= (w – v + x)[w^2 + $(-v)^2$ + x^2 – xv(—v) – (—v)x – wx]
= (w – v + x)(w^2 + v^2 + x^2 + vw + vx – wx)
[∵ $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - xz - yz)$]
Now, $w^2 + v^2 + x^2 - 2wv - 2vx + 2wx = (w - v + x)^2$
[∵ $(a + b + c)^2 = a^2 + b^2 + c^2$ + 2ab + 2 be + 2ca]

(iv) 4y2−20yz+25z2(25z2−4y2)

Solution:
4y2−20yz+25z2(25z2−4y2)
$4y^2 - 20yz + 25z^2 = (2y - 5z)^2$
$25z^2 - 4y^2$ = (5z – 2y)(5z + 2y) = (2y−5z)2(5z−2y)(5z+2y)
Since (2y – 5z) = -(5z – 2y), $(2y - 5z)^2 = (5z - 2y)^2$
So, (5z−2y)2(5z−2y)(5z+2y)=5z−2y5z+2y

(v) (x2+x−6)(x2−7x+12)(x2−6x+8)(x2−9)

Solution:
(x2+x−6)(x2−7x+12)(x2−6x+8)(x2−9)
We have,
x^2 + x – 6- (x + 3)(x – 2)
x^2 – 7x + 12 = (x – 3)(x – 4)
x^2 – 6x + 8 = (x – 2)(x – 4)
x^2 – 9 = (x – 3)(x + 3)
Therefore, (x2+x−6)(x2−7x+12)(x2−6x+8)(x2−9)
= (x+3)(x−2)(x−3)(x−4)(x−2)(x−4)(x−3)(x+3) = 1

(vi) p4−16p2−4p+4

Solution:

$$\frac{p^4-16}{p^2-4p+4} = \frac{(p^2+4)(p^2-4)}{(p-2)^2}$$
$$= \frac{(p^2+4)(p-2)(p+2)}{(p-2)^2}$$
$$= \frac{(p^2+4)(p+2)}{(p-2)}$$

End of Chapter Exercises Solutions

Question 1.
Use suitable identities to find the following products:
(i) $(-3x + 4)^2$

Solution:
$(-3x + 4)^2$
We can expand this using the identity
$(a + b)^2 = a^2 + 2ab + b^2$
$(-3x + 4)^2 = (-3x)^2 + 2(-3x)(4) + 4^2$
$= 9x^2 – 24x + 16$

(ii) (2s + 7) (2s – 7)

Solution:
(2s + 7) (2s – 7) We can use the identity $(a + b)(a – b) = a^2 – b^2$
$(25 + 7) (25 – 7) = (25)^2 – (7)^2$
$= 4s^2 – 49$

(iii) $(p^2 + 12)(p^2 – 12)$

Solution:
$(p^2 + 12)(p^2 – 12)$
Using the identity: $(a + b)(a – b) = a^2 – b^2$
$(p^2 + 12)(p^2 – 12)$
$= (p^2)^2 – (12)^2$
$= p^4 – 14$

(iv) (2n + 7) (2n – 7)

Solution:
(2n + 1)(2n – 1) Using the identity: $(a + b)(a – b) = a^2 – b^2$
$(2n + 7) (2n – 7) = (2n)^2 – 7^2$
$= 4n^2 – 49$

(v) $(s – 2t) (s^2 + 2st + 4t^2)$

Solution:
$(s – 2t) (s^2 + 2st + 4t^2)$
Here, we use the-identity
$(x – y)(x^2 + xy + y^2) = x^3 – y^3$
$(s – 2t) (s^2 + 2st + 4t^2) = s^2 – (2t)^2$
$= s^3 – 8t^3$

(vi) $(12r – 4r)^2$

Solution:
$(12r – 4r)^2$
We simplify this by using the identity

$(a – b)^2 = a^2 – 2ab + b^2$
$(12r)^2 – 2(12r)(4r) + (4r)^2$
$= 14r2 – 4 + 16r^2$

(vii) $(-3m + 4k – l)^2$

Solution:
$(-3m + 4k – l)^2$
We can expand using the identity:
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
$(-3m + 4k – l)^2 = (—3m)^2 + (4k)^2 + (-l)^2 + 2(-3m)(4k) + 2(-3m)(-l) + 2(4k)(-l)$
$= 9m^2 + 16k^2 +12 — 24mk + 6ml – 8kl$

(viii) $(x – 13)^3$

Solution:
$(x – 13)^3$
To expand this, we use the identity:
$(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$
$= a^3 – b^3 – 3ab(a – b)$
$(x – 13)^3 = x^3 – 127y^3 – 3x \text{ x } 13y(x – 13y)$
$= x^3 – 127y – x^2y + x3$

(ix) $(72k – 23m)^3$

Solution:
To expand this, we use the identity:
$(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$

$$\left(\frac{7}{2}k-\frac{2}{3}m\right)^3 = \left(\frac{7}{2}k\right)^3 - 3\left(\frac{7}{2}k\right)^2\left(\frac{2}{3}m\right) + 3\left(\frac{7}{2}k\right)\left(\frac{2}{3}m\right)^2 - \left(\frac{2}{3}m\right)^3$$
$$= \frac{343}{8}k^3 - 3\times\frac{49}{4}k^2\times\frac{2}{3}m + 3\times\frac{7}{2}k\times\frac{4}{9}m^2 - \frac{8}{27}m^3$$
$$= \frac{343}{8}k^3 - \frac{49}{2}k^2m + \frac{14}{3}km^2 - \frac{8}{27}m^3$$

Question 2.
Find the values using suitable identities:
(i) 17 × 21

Solution:
17 × 21 Here, 17 = 20 – 3 and 21 = 20 + 1, so: 17 × 21 = (20 – 3)(20 + 1)
Using the identity $(x + a)(x + b) = x^2 + (a + h)x + ab$
$= 20^2 + (-3 + 1)20 + (-3)(1) = 400 – 40 – 3 = 357$

(ii) 104 × 96

Solution:
104 × 96 Here, 104 = 100 + 4 and 96 = 100 – 4, So, 104 × 96 = (100 + 4)(100 – 4) Using the identity $(a + b)(a – b) = a^2 – b^2$
$= 100^2 – 4^2 = 10000 – 16 = 9984$

(iii) 24 × 16

Solution:
24 × 16 Here, 24 = 20 + 4 and 16 = 20 – 4 So, 24 × 16 = (20 + 4)(20 – 4) Using the identity $(a + b)(a – b) = a^2 – b^2$
$= 20^2 – 4^2 = 400- 16 = 384$

(iv) 147^3

Solution:
147^3
We can use the identity for cubes:
$(a – b)^3 = a^3 – 3a2b + 3\ ab2 – b^3$
Here, 147 = 150 – 3,
So, $147^3 = (150 – 3)^3$
$= 150^3 – 3 × 150^2 × 3 + 3 × 150 × 3^2 – 3^3$ Simplifying: = 3375000 – 202500 + 4050 – 27 = 3176523

(v) 199^3

Solution:
199^3
We can use the identity for cubes:
$(a – b)^3= a^3 – 3a^2b + 3ab^2 – b^3$
Here, 199 = 200- 1,
So, $199^3 = (200 – 1)^3$
$= 2003 – 3 × 200^2 × 1 + 3 × 200 × 1^2 – 1^3 = 8000000 – 120000 + 600 – 1 = 7880599$

(vi) 127^3

Solution:
127^3
We can use the identity for cubes:
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
Here, 127 = 130 – 3,
So, $127^3 = (130 - 3)^3$
$= 130^3 - 3 \times 130^2 \times 3 + 3 \times 130 \times 3^2 - 3^3 = 2197000 - 152100 + 3510 - 27 =$
2048383

(vii) $(-107)^3$

Solution:
$(-107)^3$
We can use the identity for cubes:
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
Here, -107 =-100 – 7,
So, $(-107)^3 = (-100 - 7)^3$
$= (-100)^3 - 3 \times (-100)^2 \times 7 + 3 \times (-100) \times 7^2 - 7^3 = -1000000 - 210000 +$
$(-14700) - 343 = -1225043$

(viii) $(-299)^3$

Solution:
$(-299)^3$
We can use the identity for cubes:
$(a + b)^3 = a^3 + 3a^2b + 3\ ab^2 + b^3$
Here, -299 = -300 + 1,
So, $(-299)^3 = (-300 + 1)^3$
$= (-300)^3 + 3 \times (-300)^2 \times 1 + 3 \times (-300) \times 1^2 + 1^3 = -27000000 + 270000 - 900$
$+ 1 = 26730899$

Question 3.
Factor the following algebraic expressions:
(i) $4y^2 + 1 + 116y2$

Solution:
$4y^2 + 1 + 116y2$
$= (2y)^2 + 2 \times (2y) \times (14y)+(14y)2$
$= (2y + 12y)^2$
$[\because (a + b)^2 = a^2 + 2ab + b^2]$

(ii) $9m^2 - 125n2$

Solution:

$9m^2$ – 125n2

= $(3m)^2$ – $(15n)^2$

= (3m + 15n)(3m – 15n)

[∵ $a^2 - b^2 = (a + b)(a - b)$]

(iii) $27b^3$ – 164b3

Solution:

$$27b^3 - \frac{1}{64b^3} = (3b)^3 - \left(\frac{1}{4b}\right)^3$$
$$= \left(3b - \frac{1}{4b}\right)\left[(3b)^2 + (3b)\left(\frac{1}{4b}\right) + \left(\frac{1}{4b}\right)^2\right]$$
$$= \left(3b - \frac{1}{4b}\right)\left(9b^2 + \frac{3}{4} + \frac{1}{16b^2}\right)$$

(iv) x^2 + 5x6+16

Solution:

$$x^2 + \frac{5x}{6} + \frac{1}{6} = \frac{6x^2 + 5x + 1}{6} = \frac{6x^2 + 2x + 3x + 1}{6}$$
$$= \frac{2x(3x+1) + 1(3x+1)}{6}$$
$$= \frac{(3x+1)(2x+1)}{6}$$

(v) $27u^3$ – 1125−27u25+9u25

Solution:

$$27u^3 - \frac{1}{125} - \frac{27u^2}{5} + \frac{9u}{25}$$
$$= (3u)^3 - \left(\frac{1}{5}\right)^3 - 3(3u)^2\left(\frac{1}{5}\right) + 3(3u)\left(\frac{1}{5}\right)^2$$
$$= \left(3u - \frac{1}{5}\right)^3 \quad [\because (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3]$$

(vi) $64y^3$ – $1125z^3$

Solution:

$$64y^3 + \frac{1}{125}z^3 = (4y)^3 + \left(\frac{1}{5}z\right)^3$$

$$= \left(4y + \frac{1}{5}z\right)\left[(4y)^2 + \left(\frac{1}{5}z\right)^2 - (4y)\left(\frac{1}{5}z\right)\right]$$

$$[\because a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$$

$$= \left(4y + \frac{1}{5}z\right)\left(16y^2 + \frac{1}{25}z^2 - \frac{4}{5}yz\right)$$

(vii) $p^3 + 27q^3 + r^3 - 9pqr$

Solution:
$p^3 + 27q^3 + r^3 - 9pqr$
We have, $x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^3 + y^3 + z^3 - xy - xz - yz)$
$= (p + 3q + r)(p^3 + 9q^3 + r^3 - 3pq - 3qr - pr)$

(viii) $9m^2 - 12m + 4$

Solution:
$9m^2 - 12m + 4$
$= (3m)^2 - 2(3m)(2) + (2)^2$
$= (3m - 2)^2$
$[\because (a - b)^2 = a^2 - 2ab + b^2]$

(ix) $9x^3 - 83\ y^3 + z33 + 6xyz$

Solution:
$9x^3 - 83\ y^3 + z33 + 6xyz$
$= 13(27x^3 - 8y^3 + z^3 + 18xyz)$
$= 13[(3x)^3 + (-2y)^3 + (z)^3 - 3(3x)(-2y)(z)]$
$= 13(3x - 2y + z)(9x^3 + 4y^3 + z^3 + 6xy + 2yz - 3xz)$
$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(x) $4x^2 + 9y^2 + 36z^2 + 12xz + 36yz + 24xy$

Solution:
$4x^2 + 9y^2 + 36z^2 + 12xz + 36yz + 24xy$
$= (2x)^2 + (3y)^2 + (6z)^2 + 2(2x)(3y) + 1(3y)(6z) + 2(2x)(6z)$
$= (2x + 3y + 6z)^2$
$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)$

(xi) $27u^3 - 1216-9u22+u4$

Solution:
$27u^3 - 1216-9u22+u4$
$= (3u)^3 - (16)^3 - 3(3u)^2(16) + 3(3u)(16)^2$

$= (3u – 16)^3$
[$\because (a – b)^3 = a^3 – b^3 – 3a^2b + 3ab^2$]

Question 4.
Simplify the following:
(i) 4x2+4x+14x2−1
(ii) 9(3a3−24b3)9a2−36b2
(iii) s3+125t3s2−2st−35t2
Note: Assume that the denominators are not equal to 0.

Solution:

$$(i)\ \frac{4x^2+4x+1}{4x^2-1} = \frac{4x^2+2x+2x+1}{(2x-1)(2x+1)}$$
$$= \frac{2x(2x+1)+1(2x+1)}{(2x-1)(2x+1)}$$
$$= \frac{(2x+1)(2x+1)}{(2x-1)(2x+1)} = \frac{(2x+1)}{(2x-1)}$$
$$(ii)\ \frac{9(3a^3-24b^3)}{9a^2-36b^2} = \frac{9\times3(a^3-8b^3)}{9(a^2-4b^2)}$$
$$= \frac{3(a-2b)(a^2+4b^2+2ab)}{(a-2b)(a+2b)}$$
$$= \frac{3(a^2+4b^2+2ab)}{(a+2b)}$$
$$(iii)\ \frac{s^3+125t^3}{s^2-2st-35t^2} = \frac{(s+5t)(s^2+25t^2-5st)}{(s+5t)(s-7t)}$$
$$= \frac{(s^2+25t^2-5st)}{(s-7t)}$$

Question 5.
Find possible expressions for the length and breadth of each of the followingxectangles whose areas are given by the following expressions in square units.
(i) $25a^2 – 30ab + 9b^2$

Solution:
$25a^2 – 30ab + 9b^2$
$= (5a)^2 – 2(5a)(3b) + (3b)^2$
$= (5a – 3b)^2$
[$\because (a – b) = a^2 – 2ab + b^2$] = (5a – 3b)(5a – 3b) Area of a rectangle = length × breadth Possible dimensions: Length = (5a – 3b) units Breadth = (5a – 3b) units

(ii) $36s^2 – 49t^2$

Solution:
$36s^2 – 49t^2$
$= (6s)^2 – (7t)^2$
$= (6s + 7t)(6s – 7t)$
[$\because a^2 – b^2 = (a + b)(a – b)$] Area of a rectangle = length × breadth Possible dimensions: Length = $(6s + 7t)$ units Breadth = $(6s – 7t)$ units

Question 6.
Find possible expressions for the length, breadth, and heights of each of the following cuboids whose volumes are given by the following expressions in cubic units.
(i) $6a^2 – 24b^2$

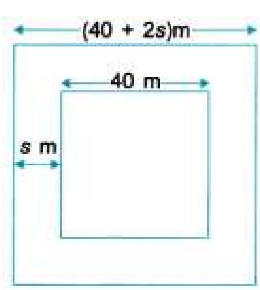
Solution:
$6a^2 – 24b^2$
$= 6(a^2 – 4b^2)$
$= 6[a^2 -(2b)^2]$
$= 6(a + 2 b)(a – 2b)$ [$\because a^2 – b^2 = (a + b)(a – b)$] Volume of a cuboid = length × breadth × height Possible dimensions: Length = 6 units Breadth = $(a + 2b)$ units Height = $(a – 2b)$ units

(ii) $3ps^2 – 15ps +12p$

Solution:
$3ps^2 – 15ps + 12p$
$= 3p(s^2 – 5s + 4)$
$= 3p(s^2 – 4s – s + 4) = 3p[s(s – 4) – 1(s – 4)] = 3p(s – 1)(s – 4)$ Volume of a cuboid = length × breadth × height Possible dimensions: Length = $3p$ units Breadth = $(s – 1)$ units Height = $(s – 4)$ units

Question 7.
The village playground is shaped as a square of side 40 metres. A path of width s metres is created around the playground for people to walk. Find an expression for the area of the path in terms of s.

Solution:



Area of the playground = 40 m × 40 m
$= (40)^2 m^2$
Area of the playground along with the path
= (40 + 2s)m × (40 + 2s)m
$= (40 + 2s)^2 m^2$
Area of the path $= (40 + 2s)^2 – (40)^2$
$= (40 + 2s + 40)(40 + 2s-40)$ [v $a^2 – b^2 = (a + b)(a – b)$]
= (80 + 2s)(2s)
$= 160s + 4s^2$
So, the area of the path is $(4s^2 + 160s) m^2$.

Question 8.
If a number plus its reciprocal equals 103, find the number.

Solution:
Let the number be x. Then its reciprocal is 1x
According to question,
x + 1x=103
⇒ x2+1x=103
⇒ $3(x^2 + 1) = 10x$
⇒ $3x^2 + 3 = 10x$
⇒ $3x^2 – 10x + 3 = 0$
⇒ $3x^2 – 9x – x + 3 = 0$
⇒ 3x(x – 3) – 1(x – 3) = 0
⇒ (3x – 1)(x – 3) = 0
⇒ x = 13 or x = 3

Question 9.
A rectangular pool has area $2a^2 + 7x + 3$ square hastas. If its width is 2x + 1 hastas, find its length. Hasta was a unit used to measure length.

Solution:
Area of the pool $= 2x^2 + 7x + 3$ square hastas
$= 2x^2 + 6x + x + 3$ square hastas = 2x(x + 3) + 10c+ 3) square hastas = (2x + 1)(x + 3) square hastas Area of the pool = length × breadth Width of the pool = (2x + 1) hastas (given) Therefore, length of the pool = (x + 3) hastas

Question 10.
If both x – 2 and x – 12 are factors of $px^2 + 5x + r$, show 2 that p = r.

Solution:
$px^2 + 5x + r$
Factors = x — 2, x – 12

⇒ x = 2 or 12
For x = 2,
$p(2)^2 + 5(2) + r = 0$ $4p + 10 + r = 0$ …(1)

For x = 12
$p(12)^2 + 5(12) + r = 0$
p4+52 + r = 0 …….(2)

Equating (1) and (2), we get

$$4p + 10 + r = \frac{p}{4} + \frac{5}{2} + r$$
$$4p - \frac{p}{4} + 10 - \frac{5}{2} + r - r = 0$$
$$\frac{15p}{4} + \frac{15}{2} = 0$$
$$\frac{15p}{4} = \frac{-15}{2}$$
$$p = \frac{-15}{2} \times \frac{4}{15}$$

p = -2 Substituting p = -2 in equation (1), we get 4(-2) + 10 + r = 0 ⇒ -8+ 10 + r = 0 ⇒ 2 + r = 0 ⇒ r = -2 ∴ p = r = – 2 Hence, p = r

Question 11.
If a + b + c = 5 and ab + bc + ca – 10, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

Solution:
We know that,
$a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= (a + b + c) [a^2 + b^2 + c^2 - (ab + be + ca)]$
$= 5(a^2 + b^2 + c^2 - (ab + bc + ca))$
$= 5(a^2 + b^2 + c^2 - 10)$ …(1)
Now, a + b + c = 5
Squaring both sides, we get
$(a + b + c)^2 = 5^2$
⇒ $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25$
⇒ $a^2 + b^2 + c^2 + 2(10) = 25$
⇒ $a^2 + b^2 + c^2 = 25 - 20 = 5$
Substituting $a^2 + b^2 + c^2 = 5$ in equation (1) 5(5 – 10) = 5(-5) = — 25 Hence proved.

Question 12.
By factoring the expression, check that $n^3 - n$ is, always divisible by 6 for all natural numbers n. Give reasons.

Solution:
$n^3 – n = n(n^2 – 1)$
Now using the identity $a^2 – b^2 = (a + b)(a – b)$
$n^3 – n = n(n – 1)(n + 1)$ This is the product of three consecutive natural numbers. We know that, among any three consecutive numbers:

- One number is divisible by 3 (because every third number is a multiple of 3)
- At least one number is even (divisible by 2)

So their product is divisible by 2 × 3 = 6 Therefore, $n^3 – n$ is always divisible by 6 for all natural numbers n. Hence, proved.

Question 13.
Find the value of
(i) $x^3 + y^3 – 12xy + 64$, when $x + y = – 4$

Solution:
$x^3 + y^3 – 12xy – 64$
Using the identity: $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
$\Rightarrow (-4)^3 = x^3 + y^3 + 3xy(—4)$
$[\because x +y = —4]$
$\Rightarrow -64 = x^3 + y^3 – 12xy$
$\Rightarrow x^3 + y^3 — 12xy + 64 = 0$

(ii) $x^3 – 8y^3 – 36xy – 216$, when $x = 2y + 6$

Solution:
We are given, $x = 2y + 6$
$\Rightarrow x – 2y = 6$
Now, $x^3 – 8y^3 – 36xy – 216$
Using the identity: $(a – b)^3 = a^3 – b^3 – 3ab(a – b)$
$(x – 2y)^3 = x^3 – (2y)^3 – 3x(2y)(x – 2y)$
$(6)^3 = x^3 – 8y^3 – 6xy(6)$
$216 = x^3 – 8y^3 – 36xy$
$\Rightarrow x^3 – 8y^3 – 36xy – 216 = 0$