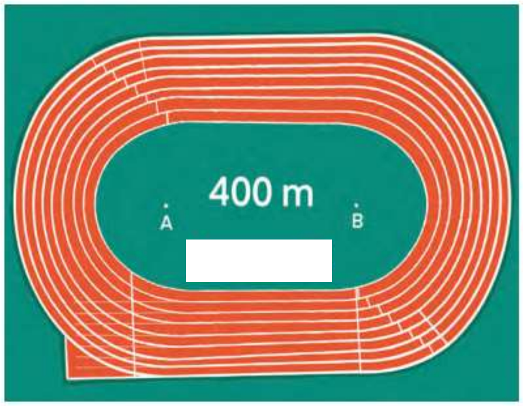
Measuring Space Perimeter and Area Class 9 Solutions Maths Ganita Manjari Chapter 6

Think and Reflect (NCERT Textbook Page No. 127)

Question 1.
What is the difference in radius between the first and second lanes? Use the Fig. 6.11 to find the stagger needed by the runner in the second lane. Will an equal stagger be needed between the third and second lanes?



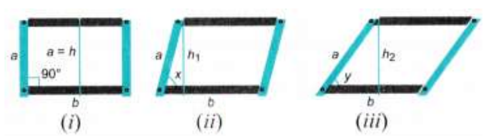
Schematic diagram of a 400 m athletics track

Solution:
The extra distance in an outer lane is created only on the curves.
The distance of a circular path is C = 2πr
For the lane 1, r_1 = 36.5 m and width, w = 1.22 m
If Lane 2 has a radius that is w (lane width) metres larger than Lane 1,
the extra distance for one full circle (360°) is 2π(r1+w)−2πr1=2πw=2×227×1.22 m = 7.67 m Yes. Because the width of each lane is constant (1.22 m), the difference in radius between Lane 3 and Lane 2 is the same as the difference between Lane 2 and Lane 1.

Think and Reflect (NCERT Textbook Page No. 131)

Question 1.
The area of a rectangle can be found when we know the lengths of its sides. Is the same true for a parallelogram? That is, can we find the area of a parallelogram when we know the lengths of its sides? Why or why not?

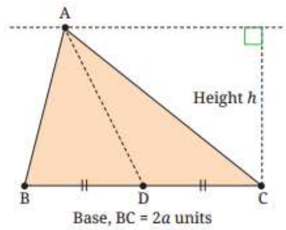
Solution:
Unlike a rectangle, knowing only the lengths of the sides of a parallelogram is not enough to determine its area. To understand why, we have to look at the “rigidity” of the shapes. Imagine we have four sticks (2 of lengths a units and 2 of lengths b units) connected by hinges at the corners.



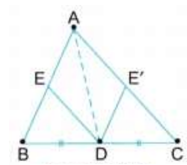
When we form a rectangle, the angles are fixed at 90°.
This locks the height to match one of the side lengths.
In Fig. (i), a = h, so area = bh sq units.
When we form a parallelogram, these hinges swing.
We keep the side lengths (a and b) the same while squashing the shape.
As the parallelogram becomes thinner (more tilted, 90° > x > y), its height decreases, even though the side lengths remain constant.
Since the area formula is Base × Height, the area changes with the height.
In Fig. (ii), $a > h_1$. So area = $bh_1 < bh$.
Further, in Fig. (iii), $a > h_1 > h_2$
So area = $bh_2 < bh_1$

Think and Reflect (NCERT Textbook Page No. 133)

Question 1.
Since ΔABD and ΔACD have equal area, you may wonder — Can we divide ΔABD using straight cuts into two or more pieces that we can then rearrange to exactly cover AACD? What do you think? Is it possible?



Solution:
We can cut one polygon into a finite number of smaller polygonal pieces and rearrange them to form the other polygon perfectly.
Since ΔABD and ΔACD have equal area. Therefore, a finite set of straight cuts exists to turn one into the other.

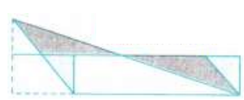


Here we can cut ΔABD into two pieces along DE || AC. Then, ΔAED will cover the area ΔDE'A, and ΔBED will cover the remaining area ΔDE'C.

Think and Reflect (NCERT Textbook Page No. 134)

Question 1.
Suppose we are given two polygons P and Q with equal area. Will it always be possible to divide one of them using straight cuts into two or more pieces and then rearrange the pieces to exactly cover the other polygon? Try this out for familiar shapes, e.g., 1. A square and non-square rectangle with equal area, 2. Two triangles with different shapes but equal area, 3. A triangle and a square with equal area. Formulate a conjecture of your own about this. Think of various rectangles with a perimeter of 40 units (the sides do not have to be integers). 1. How many such rectangles are there? 2. Among them, is there one whose area is the largest? What are its dimensions? 3. Among all these rectangles, is there one whose area is the smallest? What are its dimensions? Do either of these answers come as a surprise to you?

Solution:
Part 1: 1. Square and Non-Square Rectangle: This is the easiest to visualise. If we have a 4 × 4 square (Area = 16) and a 2 × 8 rectangle (Area = 16), we can slice the square and stack the pieces to form the rectangle. 2. Two Triangles of different shapes Even if one is an acute triangle and the other is a long, thin obtuse triangle, they can be decomposed into the same set of pieces. The most common method is to show that any triangle can be cut into pieces to form a rectangle. If both triangles can form the same rectangle, they can form each other. 3. A Triangle and a Square This is more complex but entirely possible. It usually involves a multi-step translation: Triangle → Rectangle → Square

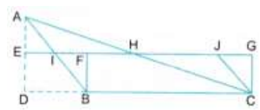


Conjecture: Any two simple polygons P and Q can be subdivided into a finite number of congruent polygonal pieces if and only if they have the same area. Part 2: Rectangles with Perimeter 40 Let the sides of the rectangle be x and y. The perimeter is 2(x + y) = 40 ⇒ x + y = 20 Therefore, y = 20 – x. The Area (A) is A = x(20 – x) 1. Since x can be any real number between 0 and 20 (not just integers), there are infinitely many such rectangles. We could have a rectangle that is 10 × 10, 19 × 1, or even 19.99 × 0.01. 2. The square shape has the largest area. Area: 100 square units Dimensions: x = 10, y = 10 3. There is no single smallest rectangle. As x gets closer to 0 (making the rectangle extremely thin), the area gets closer and closer to 0. For example, if x = 0.0001, then y = 19.9999. The area is nearly zero. Because x must be greater than 0 to form a shape, we say the area has an infimum of 0, but there is no minimum rectangle because you can always pick a thinner one. Why is this surprising? Most people expect the extremes to be balanced—a maximum at one end and a minimum at the other. However, in geometry, the square is the most efficient rectangle (enclosing the most area for the least perimeter). The least efficient rectangle doesn't really exist; it just keeps collapsing into a line segment until it disappears.

Think and Reflect (NCERT Textbook Page No. 142)

Question 1.
What procedure would you use to square a given triangle? Here, the task is to construct a square whose area is equal to the area of some given triangle. Think carefully. How would you proceed?

Solution:
To square a triangle (also known as the Quadrature of a Triangle), we use a classic straightedge-and-compass construction. Since we cannot directly turn a triangle into a square in one leap, we use a rectangle as the intermediate bridge. The logic follows this path: Triangle → Rectangle → Square Step 1: Triangle to Rectangle First, we construct a rectangle with the same area as the triangle. 1. Let the triangle ABC have base BC (b) and height AD (h). The area of ∆ABC = 1212 × b × h
2. Construct a rectangle BCGF with one side equal to base BC and the other side equal to 12h = DE. (by bisecting the height).



Here, ∆AIH and ∆BIF together exactly cover the area of ∆CGH.
Mathematically, we have a rectangle with sides x = b and y = 12h
The area of this rectangle is xy = b × 12h = 12bh which matches the triangle exactly. Step 2: Rectangle to Square Now, follow the procedure as discussed.

Exercise 6.1 Solutions

Unless stated otherwise, use the approximation 227 for π.

Question 1.
The perimeter of a circle is 44 cm. What is its radius?

Solution:
Given: The circumference of a circle, C = 44 cm We know that C = 2πr ⇒ 44 = 2 × 227 × r
⇒ r = 442×722 = 7 cm

Question 2.
Calculate, correct to 3 significant figures, the circumference of a circle with: (i) radius 7 cm (ii) radius 10 cm (iii) radius 12 cm

Solution:
We know that C = 2πr (i) Given, r = 7 cm C = 2 × 227 × 7 = 44 So, the circumference of a circle 44.0 cm.

(ii) Given, r = 10 cm C = 2 × 227 × 10 = 62.8571 So, the circumference of a circle is 62.857 cm.

(iii) Given, r = 12 cm C = 2 × 227 × 12 = 75.4285 So, the circumference of a circle 75.429 cm.

Question 3.
Calculate the length of the arc of a circle if:
(i) the radius is 3.5 cm and the angle at the centre is 60°, and
(ii) the radius is 6.3 m, and the angle at the centre is 120°.

Solution:
We know that the length of the arc of a circle with radius r and central angle θ = θ360∘×2πr

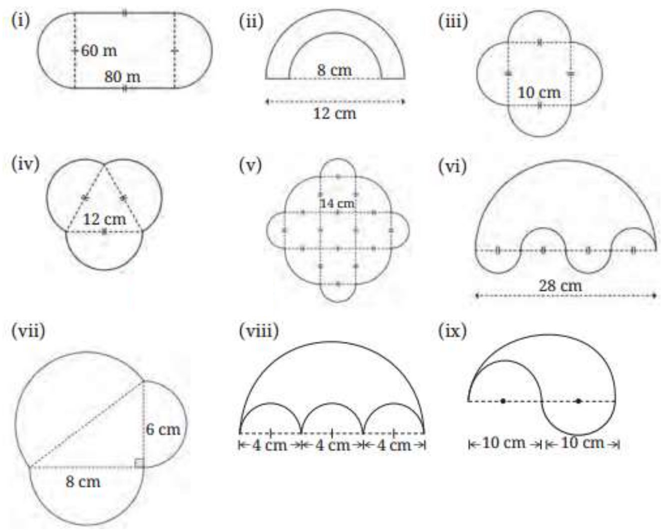
(i) Given, r = 3.5 cm and θ = 60°
l = θ360∘×2πr=60∘360∘×2×227×3.5 = 3.666 So, the length of the arc of a circle is 3.67 cm.

(ii) Given, r = 6.3 m and θ = 120° l = θ360∘×2πr=120∘360∘×2×227×6.3 = 13.2 So, the length of the arc of a circle 13.2 m.

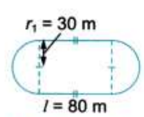
Question 4.
Find the perimeter of a sector (i.e., the curved portion as well as the two straight portions) of a circle of radius 14 cm and sector angle 75°.

Solution:
The perimeter of a sector includes the arc length plus the two radii, so P = θ360∘×2πr+2r
Here, r = 14 cm and θ = 75°
P = θ360∘×2πr+2r
= 75∘360∘×2×227×14+2×14
= 553+28
= 4613 cm

Question 5.
Find the perimeters of the following shapes (taking the arcs to be quarter, half, or three-quarters of a circle, as appropriate) (Fig. 6.14i to 6.14ix):

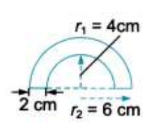


Solution:
(i) The boundary of the given figure comprises two semicircles of the same radius and two lengths of the rectangular part.



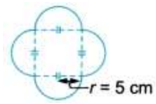
So, perimeter = 2πr + 2l
P = 2×227×30+2×80
= 18847 + 160
= 34847 m

(ii) The boundary of the given figure comprises two semicircles of different radii, 4 cm and 6 cm, and two widths of 2 cm each.



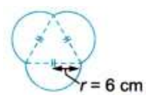
So, perimeter = $\pi r_1 + \pi r_2 + 2w$
P = 227×4+227×6+2×2
= 227(4+6)+4
= 3537 cm

(iii) The boundary of the given figure comprises four semicircles of the same radius 5 cm.



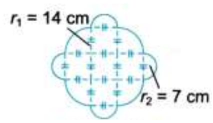
So, perimeter = 4 × πr
P = 4 × 227 × 5
= 62227 cm

(iv) The boundary of the given figure comprises three semicircles of the same radius.



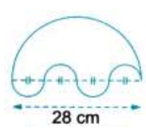
So, perimeter = 3 × πr
P = 3 × 227 × 6
= 5647 cm

(v) The boundary of the given figure comprises four quadrants of the same radius 14 cm and four semicircles of the same radius 7 cm.



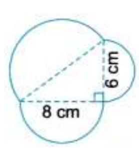
So, perimeter = 4×14×2πr1+4×12×2πr2
= 2πr1+4πr2
= 2×227×14+4×227×7 = 176 cm

(vi) The boundary of the given figure comprises one large semicircle of diameter 28 cm and four small semicircles of the same diameter 7 cm.



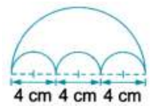
So, perimeter = 12×2πr1+4×12×2πr2
= πr1+4πr2
= 227×14+4×227×72 = 88 cm

(vii) The boundary of the given figure comprises three semicircles whose diameters are the sides of a right-angled triangle with legs 6 cm and 8 cm.



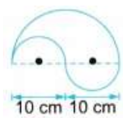
Using Baudhayana-Pythagoras theorem,
Hypotenuse = 62+82−−−−−−√=36+64−−−−−−√ = 10 cm
So, perimeter = πr1+πr2+πr3=π(r1+r2+r3)
= 227×(3+4+5)
= 227 × 12
= 3757 cm

(viii) The boundary of the given figure comprises one large semicircle of diameter 12 cm and three small semicircles of the same diameter 4 cm.



So, perimeter = $\pi r1+3\pi r2$
= $227\times6+3\times227\times2$
= 3757 cm

(ix) The boundary of the given figure comprises one large semicircle of radius 10 cm and two small semicircles of the same diameter 10 cm.



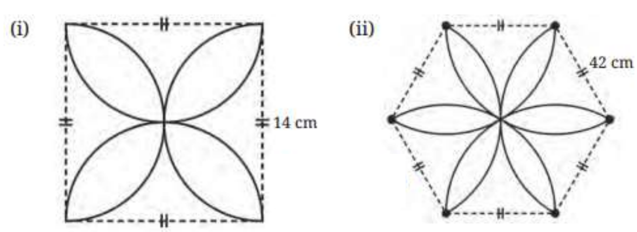
So, perimeter = $\pi r1+2\pi r2$
= $227\times10+2\times227\times5$
= 6267 cm

Question 6.
If the diameter of a car tyre is 56 cm, then:
(i) How far does the car need to travel for the tyre to complete one revolution?
(ii) How many revolutions does the tyre make if the car travels 10 km?

Solution:
(i) One full revolution of a tyre is equal to its circumference. The formula for circumference using diameter (d) is C = πd
Given: d = 56 cm C = 227 × 56 = 176 cm The car travels approximately 1.76 m for every one revolution.

(ii) To find the number of revolutions, we divide the total distance by the distance of one revolution (the circumference). First, we must ensure the units match. 10 km = 10000 m Number of revolutions = 10000 ÷ 1.76 ~ 5,681.8 The tyre makes approximately 5,682 revolutions to travel 10 km.

Question 7.
Find the total perimeter of all the petals in each of the given flowers.



A: The centres of the arcs are the midpoints of the sides of the square

B: The centres of the arcs are the vertices of the hexagon

Solution:
(i) All four petals are made by four semicircles, each with a diameter of 14 cm.
So, the total perimeter of all the petals = $4\pi r$
P = 4 × 227 × 7 = 88 cm

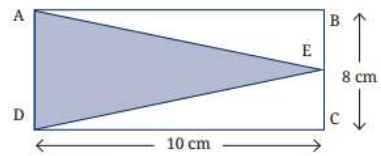
(ii) All six petals are made by six arcs of congruent circles, each with radius 42 cm and a central angle of 120°. So, the total perimeter of all the petals = $6\times\theta360\circ\times2\pi r$
= $6\times120\circ360\circ\times2\times227\times42$ = 528 cm

Question 8.
The ratio of the perimeters of two circles is 5 : 4. What is the ratio of their radii?

Solution:
Let the radii of the two circles be r_1 and r_2, respectively.
Then, their perimeters (circumferences) will be $2\pi r_1$ and $2\pi r_2$, respectively.
Given: $C_1 : C_2 = 5 : 4$
$\Rightarrow 2\pi r_1 : 2\pi r_2 = 5 : 4$
$\Rightarrow r_1 : r_2 = 5 : 4$

Exercise 6.2 Solutions

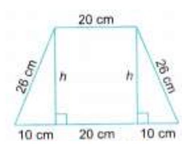
Question 1.
Find the area of triangle ADE in A Fig. 6.31.



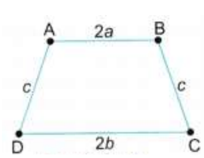
Solution:
In $\triangle ADE$, base AD = BC = 8 cm and height DC = 10 cm.
So, area of $\triangle ADE$ = 12 × bh
= 12 × 8 × 10
= 40 cm^2

Question 2.
The parallel sides of a trapezium are 40 cm and 20 cm. If its non-parallel sides are both equal, each being 26 cm, find the area of the trapezium.

Solution:



To find the area, we need the height (h).
Parallel sides: a = 40 cm, b = 20 cm.
Non-parallel sides: Both are 26 cm.
If we drop perpendiculars from the ends of the shorter side to the longer side, they create a rectangle in the middle (width 20 cm) and two identical right-angled triangles on the ends.
The base of each triangle is: 40−202 = 10 cm
Using Baudhayana-Pythagoras theorem,
$10^2 + h^2 = 26^2$
$\Rightarrow 100 + h^2 = 676$
$\Rightarrow h^2 = 576$
$\Rightarrow h = 24$ cm
Area = 12 × (a + b)h
= 12 × (40 + 20) × 24
= 720 cm^2
Alternative method:
This is an isosceles trapezium.



Applying the special case of Brahmagupta's formula, we have
Area of the isosceles trapezium = (a+b)c2−(b−a)2−−−−−−−−−−√
Here, a = 10 cm, b = 20 cm and c = 26 cm
So, area of the isosceles trapezium = (10+20)262−(20−10)2−−−−−−−−−−−−−√
= 30262−(10)2−−−−−−−−−√
= 30 × 24
= 720 cm^2

Question 3.
Find the area of a triangle, given that its sides are 8 cm and 11 cm long, and its perimeter is 32 cm.

Solution:
Sides: a = 8 cm, b = 11 cm. Perimeter: 2s = 32 cm ⇒ Semi-perimeter (s) = 16 cm. Third side (c): 32 – (8 + 11) = 13 cm. Area of triangle = s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√
= 16(16−8)(16−11)(16−13)−−−−−−−−−−−−−−−−−−−−−−−√
= 16×8×5×3−−−−−−−−−−−√
= 8√30 cm^2
= 43.82 cm^2

Question 4.
The sides of a triangular plot are in the ratio 3 : 5 : 7; its perimeter is 300 m. Find its area.

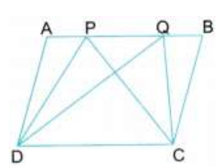
Solution:
Let sides be 3x, 5x, and 7x. Then, Perimeter: 3x + 5x + 7x = 300 ⇒ 15x = 300 ⇒ x = 20 Sides: a = 60 m, b = 100 m, c = 140 m Semi-perimeter (s): 150 m Area of a triangle = s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√
= 150(150−60)(150−100)(150−140)−−−−−−−−−−−−−−−−−−−−−−−−−−−−−√
= 150×90×50×10−−−−−−−−−−−−−−−√
= 1500√3 m^2
= 2598.08 m^2

Question 5.
One diagonal of a rhombus is twice as long as the other diagonal. If the rhombus has an area of 128 cm2, find the length of the shorter diagonal.

Solution:
Let diagonals be: d_1 = x and d_2 = 2x
Area of a rhombus = 12d1d2 = 128 cm^2
⇒ 12 × x × 2x
⇒ x^2 = 128 ⇒ x = 8√2 cm ~ 11.31 cm Therefore, the length of the shorter diagonal is 11.31 cm.

Question 6.
ABCD is a parallelogram. P and Q are any two points on side AB. What can you say about the ratio area (△PCD) : area (△QCD)?

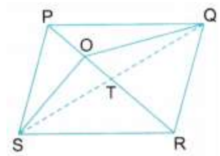
Solution:



Points P and Q both lie on side AB. Both △PCD and △QCD share the same base CD. Since AB is parallel to CD, both triangles also share the same height (the distance between the parallel lines). Therefore, the area of △PCD equals the area of △QCD ⇒ Area of △PCD : Area of △QCD = 1 : 1.

Question 7.
O is any point on the diagonal PR of a parallelogram PQRS. Prove that the areas of triangles PSO and PQO are equal.

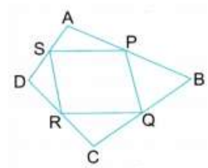
Solution:
In parallelogram PQRS, draw diagonal QS that intersects PR at T.



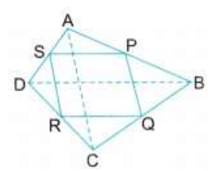
In APSQ, PT is the median Area (∆PST) = Area (∆PQT) …… (i) (Median of a triangle divides it into two triangles of equal area) Also, in ∆OSQ, OT is the median. Area (∆OST) = Area (∆OQT) ……(ii) (Median of a triangle divides it into two triangles of equal area) Subtracting eqn. (ii) from eqn. (i), Area (∆PST) – Area (∆OST) = Area (∆PQT) – Area (∆OQT) ⇒ Area (∆PSO) = Area (∆PQO)

Question 8.
If the midpoints of the sides of a 4-gon (also known as a quadrilateral, but we prefer to call it a '4-gon') are joined in order, prove that the area of the parallelogram thus formed will be half of the area of the given 4-gon. (You may wonder whether the 4-gon thus formed is always a parallelogram, and if so, why? These questions will be tackled and answered in the chapter on quadrilaterals.)

Solution:
Let the original 4-gon (quadrilateral) be ABCD, with midpoints P, Q, R, S of sides AB, BC, CD, DA, respectively. Join P, Q, R, S to form a 4-gon, i.e., parallelogram PQRS.

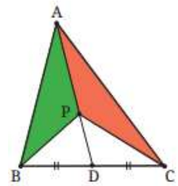


To prove: Area of parallelogram PQRS = 12(Area of 4-gon ABCD)
Proof: To prove the area relationship, we look at the four triangles cut off from the corners of the 4-gon (ABCD) to leave the parallelogram (PQRS).



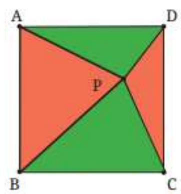
P is the midpoint of AB, and Q is the midpoint of BC.
So, PQ || AC and PQ = 12AC
Also, the height of ∆PBQ is half the height of ∆ABC (relative to base AC).
Therefore, Area (∆PBQ) = 14Area (∆ABC) …..(i)
Looking at the opposite corner and applying the same logic,
Area (∆SDR) = 14 Area (∆ADC) …..(ii)
Combining equations. (i) and (ii), we have:
Area (∆PBQ) + Area (∆SDR) = 14 Area (∆ABC) + 14 Area (∆ADC) = 14 Area (4-gon ABCD) …..(iii)
Similarly, from the other two corners:
Area (∆PAS) + Area (∆QCR) = 14 Area (∆BAD) + 14 Area (∆BCD) = 14 Area (4-gon ABCD) ……(iv)
Combining equations. (iii) and (ii), we have:
Area (∆PBQ) + Area (∆SDR) + Area (∆PAS) + Area (∆QCR) = 14 Area (4-gon ABCD) + 14 Area (4-gon ABCD) = 12 Area (4-gon ABCD)
Area (PQRS) = Total Area – Area of corner triangles = Area (4-gon ABCD) – 12 Area (4-gon ABCD)
Thus, Area (PQRS) = 12 Area (4-gon ABCD)

Question 9.
In ∆ABC, the midpoint of BC is D (Fig. 6.32). Median AD is drawn. P is any point on AD. Show that area (∆ABP) = area (∆ACP).

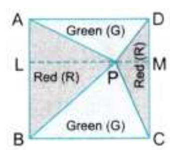


Solution:
Given: BD = DC To prove: Area (∆ABP) = area (∆ACP) Proof: Since AD is the median of ∆ABC. Area (∆ABD) = Area (∆ACD) …..(i) (Median of a triangle divides it into two triangles of equal area) Also, in ∆PBC, PD is the median. Area (∆PBD) = Area (∆PCD) ….(ii) (Median of a triangle divides it into two triangles of equal area) Subtracting eqn. (ii) from eqn. (i), Area (∆ABD) – Area (∆PBD) = Area (∆ACD) – Area (∆PCD) ⇒ Area (∆ABP) = Area (∆ACP)

Question 10.
Given a square ABCD, let P be a point within it. Join PA, PB, PC, PD (Fig. 6.33). What is the ratio of the areas of the red region (∆PAB and ∆PCD) and the green region (∆PBC and ∆PDA)?

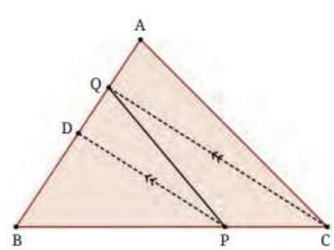


Solution:
We are given a square ABCD, and point P is located within the square.
By joining PA, PB, PC, and PD, four triangles are formed.

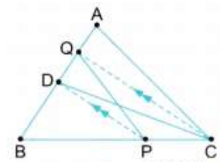


To find: Total areas of the red region (∆PAB and ∆PCD) : Total areas of the green region (∆PBC and ∆PDA).
Construction: Through P, draw LM || BC to divide the square ABCD into two rectangles ADML and LMCB.
Now, ∆PBC and rectangle LMCB lie between the same parallels and on the same base BC.
So, area (∆PBC) = 12 Area (4-gon LMCB) …..(i)
Similarly, ∆PAD and rectangle LMDA lie between the same parallels and on the same base AD.
So, area (∆PAD) = 12 Area (4-gon LMDA) ……(ii)
Combining (i) and (ii), we have:
area (∆PBC) + area (∆PAD) = 12 Area (4-gon LMCB) + 12 Area (4-gon LMDA)
⇒ Total areas of the green region (∆PBC and ∆PDA) = 12 Area (square ABCD) …..(iii)
By drawing a line parallel to AB, we can have
⇒ Total areas of the red region (∆PAB and ∆PCD) = 12 Area (square ABCD) …..(iv) Thus, the ratio of the areas of the red region to the green region is 1 : 1.

Question 11.
In ∆ABC, D is the midpoint of AB. P is any point on BC, and Q is a point on AB such that CQ || PD. PQ is joined (Fig. 6.34). Prove that Area (∆BPQ) = 12 Area (∆ABC).



Solution:
Given: D is the midpoint of AB, and the line segment PD is parallel to CQ.



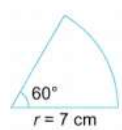
To prove: Area (∆BPQ) = 12 area (∆ABC)
Construction: Join DC.
Proof: Since CD is the median of ∆ABC.
So, area (∆BDC) = 12 area (∆ABC) ……(i)
Further, ∆DPQ and ∆DPC are on the same base DP and between the same parallels DP || CQ.
So, area (∆DPC) = area (∆DPQ) ……(ii)
Also, area (∆BDC) = area (∆BPD) + area (∆DPC) = area (∆BPD) + area (∆DPQ) (using (ii))
⇒ area (∆BDC) = area (∆BPQ) …..(iii)
From equations (i) and (iii),
area (∆BPQ) = 12 area (∆ABC) Thus, the area of ∆BPQ is half the area of ∆ABC.

Exercise 6.3 Solutions

Unless stated otherwise, use the approximation 227 for π.

Question 1.
Find the area of a sector of a circle with radius 7 cm if the angle of the sector is 60°.

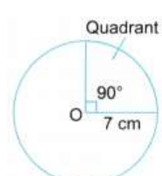
Solution:
Given radius, r = 7 cm, and angle of the sector, θ = 60°.



So, the area of the sector
A = θ360∘×πr2
= 60∘360∘×227×72
= 773
= 2523 cm^2

Question 2.
Find the area of a quadrant of a circle whose circumference is 44 cm.

Solution:

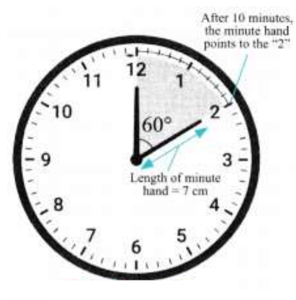


Given the circumference of a circle, C = 44 cm
⇒ 2πr = 44
⇒ 2 × 227 × r = 44
⇒ r = 7 cm
So, the area of the quadrant
A = 14×πr2

= 14×227×72
= 772
= 38.5 cm^2

Question 3.
The length of the minute hand of a clock is 7 cm. Find the area swept by the minute hand in 10 minutes.

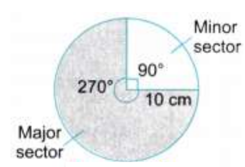
Solution:
Given radius, r = the length of the minute hand of a clock = 7 cm Angle swept by the minute hand in 10 minutes, θ = 1060 × 360° = 60°



So, the area of the sector A = θ360∘×πr2
= 60∘360∘×227×72
= 773
= 2523 cm^2
Thus, the area swept by the minute hand in 10 minutes is 2523 cm^2

Question 4.
A chord of a circle of radius 10 cm subtends 90° at the centre. Find the area of the corresponding: (i) minor sector (that subtends 90° at the centre), and (ii) major sector (that subtends 270° at the centre). (Use π = 3.14.)

Solution:
Given: radius, r = 10 cm

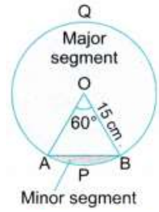


(i) Angle subtended by a chord at the centre, θ = 90°
So, the area of the minor sector
A = θ360∘×πr2
= 90∘360∘×3.14×102
= 1572
= 78.5 cm^2

(ii) Angle of the major sector, θ = 270° So, the area of the major sector A = θ360∘×πr2
= 270∘360∘×3.14×102
= 4712
= 235.5 cm^2

Question 5.
A chord of a circle of radius 15 cm subtends an angle of 60° at the centre of the circle. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Solution: Given: radius, r = 15 cm

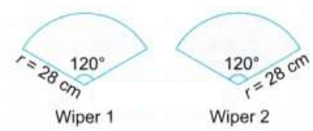


(i) Angle subtended by a chord at the centre, θ = 60°
Since isosceles ∆OAB has the vertical angle 60°, it is an equilateral triangle.
Area of the minor segment APB = Area of the minor sector OAPB – Area of equilateral ∆OAB
A=θ360∘×πr2−3–√4a2=60∘360∘×3.14×152−3–√4×152=16×3.14×225−1.734×225=(1.573−1.734)×225=(6.28−5.1912)×225=1.0912×225
= 20.44 cm^2

(ii) Area of the major segment AQB = Area of the circle – Area of the minor segment APB A = πr2−(θ360∘×πr2−3√4a2)
= 3.14 × 152 – 20.44
= 706.5 – 20.44
= 686.06 cm^2

Question 6.
A car has two wipers that do not overlap. Each wiper has a blade of length 28 cm and sweeps through an angle of 120°. Find the total area cleaned at each sweep of the blades.

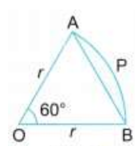
Solution:
Given: radius, r = 28 cm Angle swept at the centre by the wiper, θ = 120°



So, the area of the sector swept by each blade
A = θ360∘×πr2
= 120∘360∘×227×282
= 24643
= 82113 cm^2
Thus, the total area cleaned at each sweep of the blades = 2 × 82113 = 164223 cm^2

Question 7.
A chord of a circle of radius r subtends an angle of 60° at the centre of the circle. Show that the area of the corresponding minor segment of the circle is equal to πr2(16−3√4π).

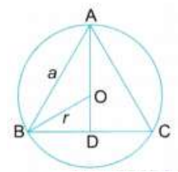
Solution:
Angle subtended by a chord at the centre, θ = 60°



Since isosceles ∆OAB has the vertical angle 60°,
So, base angle OAB = angle OBA
= 180∘−60∘2
= 60°
Thus, ∆OAB is an equilateral triangle.
Area of the minor segment APB = Area of the minor sector OAPB – Area of equilateral ∆OAB
A = θ360∘×πr2−3√4r2
= 60∘360∘×πr2−3√4×r2
= πr2(16−3√4π)

Question 8.
An equilateral triangle is inscribed in a circle of radius r. Show that the ratio of the area of the triangle to the area of the circle is equal to 33√4π≈0.413.

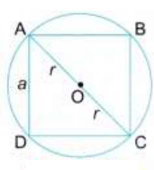
Solution:
Let ∆ABC be an equilateral triangle inscribed in a circle of radius r and centre O.



The centre O divides the median AD into a ratio 2 : 1.
32 OA = 32r
3√2AB
⇒ 32r=3√2a
⇒ a = √3r
Now area of ∆ABC = 3√4a2=3√4(3–√r)2=33√4r2
Area of circle = πr^2
So, the ratio of the area of the triangle to the area of the circle = 33√4r2πr2=33√4π≈3×1.734×3.14≈5191256≈0.413

Question 9.
A square is inscribed in a circle of radius r. Show that the ratio of the area of the square to the area of the circle is equal to 2π≈0.637.

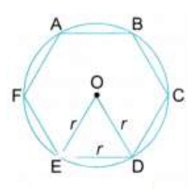
Solution:
Let ABCD be a square inscribed in a circle of radius r and centre O.



Diagonal of the square = Diameter of the circle = 2r
Also, Diagonal of the square = √2 × side
⇒ 2r = √2× a
⇒ a = √2r
Now area of the square = $(\sqrt{2}r)^2 = 2r^2$
Area of circle = πr^2
So, the ratio of the area of the triangle to the area of the circle = 2r2πr2=2π≈0.637.

Question 10.
A hexagon is inscribed in a circle of radius r. Show that the ratio of the area of the hexagon to the area of the circle is equal to 33√2π≈0.827. Can you see why the answer is exactly twice the answer to Question 8?

Solution:
Let ABCDEF be a regular hexagon inscribed in a circle of radius r and centre O.



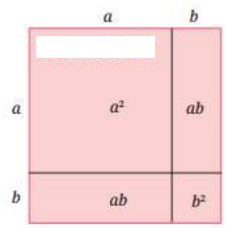
The regular hexagon inscribed in a circle can be divided into 6 equilateral triangles, each of side r.
Now area of the hexagon = 6×3√4r2=33√2r2
Area of circle = πr^2

So, the ratio of the area of the triangle to the area of the circle = 33√2r2πr2=33√2π≈0.827. An equilateral triangle (Question 8) is made of 3 such smaller triangles meeting at the centre, whereas a hexagon (Question 10) is made of 6. Since both are inscribed in the same circle of radius r, the hexagon simply covers exactly double the area that the inscribed equilateral triangle does.

End of Chapter Exercise Solutions

In the problems below, unless stated otherwise, use the approximation 227 for π.

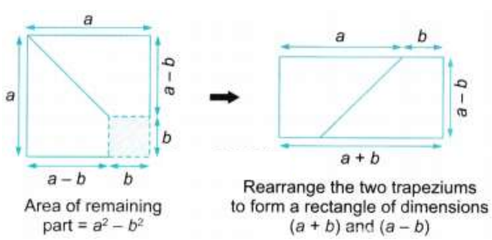
Question 1.
Identities in algebra can sometimes be shown as area relationships. For example:



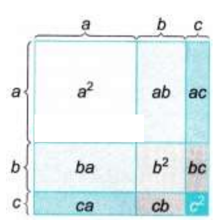
Area model of an identity

The figure shown corresponds to the identity $(a + b)^2 = a^2 + 2ab + b^2$. Do you see how?
Draw figures corresponding to the identities $(a + b)(a – b) = a^2 – b^2$ and $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.

Solution:
In the given area model, a square of side (a + b) has been divided into four parts,
i.e., two squares of dimensions a × a and b × b, and two rectangles of dimensions ax b.
Therefore, the area of the big square = the sum of the areas of the four pieces
i.e., $(a + b)^2 = a^2 + b^2 + ab + ab = a^2 + 2ab + b^2$
For the identity $(a + b)(a – b) = a^2 – b^2$, the figure can be a rectangle with dimensions (a + b) and (a – b).
Let us take a square of side a and remove a square of side b from it.
Then split the remaining part into two trapeziums and rearrange them to form the required rectangle as shown below.



Area of the rectangle = $(a + b)(a – b) = a^2 – b^2$
The identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$ can be visualised as a larger square with side length (a + b + c), divided into smaller squares and rectangles that represent the terms in the expanded form of the square.
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$



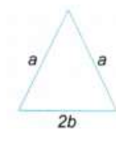
A square with side length (a + b + c) is divided into nine parts.
Total area = $a^2 + b^2 + c^2 + ab + ab + ac + bc + ac + bc = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Question 2.
An isosceles triangle has a perimeter of 40 cm; the equal sides are 15 cm each. Find the area of the triangle.

Solution:
Equal Sides: a = 15 cm, b = 15 cm Perimeter: 2s = 40 cm ⇒ Semi-perimeter (s) = 20 cm Third side (c): 40 – (15 + 15) = 10 cm Area of triangle = s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√
= 20(20−15)(20−15)(20−10)−−−−−−−−−−−−−−−−−−−−−−−√
= 20×5×5×10−−−−−−−−−−−−√
= 50√2 cm^2
= 70.7 cm^2

Question 3.
An isosceles triangle has a base of 10 cm, and its area is 60 cm2. What are the lengths of the equal sides?

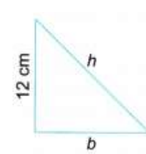
Solution:
Using the special case of Heron's formula, we have



Area of an isosceles triangle, A = ba2−b2−−−−−−√, a = equal side and b = half of the base
Base, 2b = 10 cm
⇒ b = 5 cm
Area A = 60 cm^2
Putting these values in A = ba2−b2−−−−−−√,
⇒ 60 = 5a2−52−−−−−−√
⇒ 12 = a2−52−−−−−−√
⇒ $12^2 = a^2 – 5^2$
⇒ $a^2 = 144 + 25 = 169 = 13^2$ ⇒ a = 13 cm

Question 4.
The area of a right-angled triangle is 54 sq. cm. One of its legs has a length of 12 cm. Find its perimeter.

Solution:
Given: Area = 54 sq. cm, Let height = 12 cm



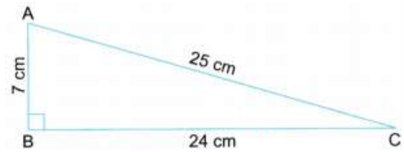
Area of a right triangle, A = 12 × base × height
⇒ 54 = 12 × b × 12
⇒ b = 9 cm
Using Baudhayana-Pythagoras theorem,
h = 122+92−−−−−−−√=225−−−√ = 15 cm So, perimeter of the triangle = 12 + 9 + 15 = 36 cm.

Question 5.
The sides of a triangle are in the ratio 2 : 3 : 4, and its perimeter is 45 cm. Find its area.

Solution:
Let the sides be 2x, 3x, and 4x. Perimeter of a triangle = sum of the sides of the triangle ⇒ 45 = 2x + 3x + 4x ⇒ 9x = 45 ⇒ x = 5 So, the sides are 10 cm, 15 cm, and 20 cm. Using Heron's Formula: Semi-perimeter (s) = 452 = 22.5 cm.
Area of triangle = s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√
= 22.5(22.5−10)(22.5−15)(22.5−20)−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−√
= 22.5×12.5×7.5×2.5−−−−−−−−−−−−−−−−−−√
= 18.75√15
= 72.62 cm^2

Question 6.
The sides of a triangle have lengths 7 cm, 24 cm, and 25 cm. Find the area of the triangle in two different ways.

Solution:
First, notice that $7^2 + 24^2$ = 49 + 576 = 625, and 25^2 = 625.
Since $a^2 + b^2 = c^2$, this is a right-angled triangle.



Method 1: Using Base and Height
In a right triangle, the two shorter sides are the base and height.
Area of a right triangle, A = 12 × base × height
= 12 × 24 × 7
= 84 cm^2
Method 2: Using Heron's Formula
Semi-perimeter (s) = 7+24+252 = 28 cm.
Area of a triangle = s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√
= 28(28−7)(28−24)(28−25)−−−−−−−−−−−−−−−−−−−−−−−−√
= 28×21×4×3−−−−−−−−−−−−√
= 3 × 4 × 7
= 84 cm^2

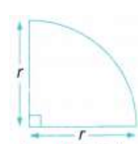
Question 7.
If the wheel of a bicycle has a diameter of 60 cm, find how far a cyclist will have travelled after the wheel has rotated 100 times.

Solution:
When a wheel rotates once, it covers a distance equal to its circumference.

The diameter (d) = 60 cm.
C = πd = 227 × 60 = 188.5 cm The wheel rotates 100 times. So, total distance = 188.5 cm × 100 = 18,850 cm or 188.5 m

Question 8.
Find the area of a quadrant of a circle whose circumference is 66 cm.

Solution:
A quadrant is exactly one-fourth of a circle. To find the area, we first need the radius.



The circumference (C) = 66 cm.
⇒ 2πr = 66
⇒ 2 × 227 × r = 66
⇒ r = 66×72×22=212 cm
Now, area of quadrant = 14πr2
= 14×227×(212)2
= 6938
= 8658 cm^2

Question 9.
The wheel of a car has an outer radius of 28 cm. Calculate how far the car travels after one complete turn of the wheel, and how many times the wheel turns during a journey of 1 km.

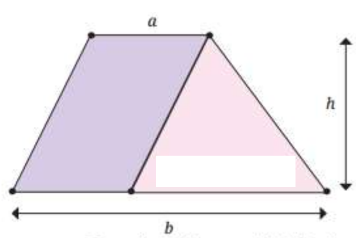
Solution:

Distance in one turn (Circumference):
C = 2πr
= 2 × 227 × 28
= 176 cm
Now, 1 km = 1,00,000 cm
So, number of turns = Total Distance Circumference
= 100000176
= 568211

Question 10.
Two rectangles have the same area and the same perimeter. Does this mean that they are congruent with each other?

Solution:
Yes. If two rectangles have the same area (A) and the same perimeter (P), they must be congruent (meaning their lengths and widths are identical). Congruent Dimensions: For rectangles, the Area (A = l × b) and Perimeter [P = 2(l + b)] systems of equations have only one unique solution for l and b. Unique Shape: If both $l_1 \times b_1 = l_2 \times b_2$ and $2(l_1 + b_1) = 2(l_2 + b_2)$, the rectangles must be identical. Counter-Example Logic: While a 4 × 4 square (A = 16 and P = 16) and a 2 × 6 rectangle (A = 12 and P = 16) have the same perimeter, their areas differ. If the areas are also forced to be 16, only the 4 × 4 square works, forcing congruence.

Question 11.
You know that the area of a parallelogram is base × height. Using this and the figure, show that the area of a trapezium is half the sum of the parallel sides × height, i.e., 12(a + b)h.



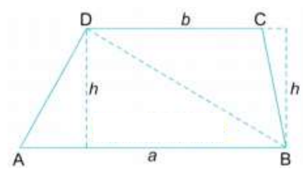
Trapezium: sides a and b, height h

Solution:
Looking at the figure, we observe two parts as follows:
1. The Parallelogram:
The top side is a.
Since opposite sides of a parallelogram are equal, its base on the bottom line is also a.
Its height is h.
2. The Triangle:
The total bottom base of the trapezium is b.
The part of that base belonging to the parallelogram is a.
Therefore, the base of the triangle is the remaining portion: (b – a).
Its height is also h.
Area of Parallelogram, A_1 = base × height = ah
Area of Triangle, A_2 = 12 × base × height
A_2 = 12 × (b – a) × h

Now, we add the two areas together to get the total area of the trapezium:
$A = A_1 + A_2$
= ah + 12 × (b – a) × h
= 12 × (2a + b – a) × h
= 12 × (a + b) × h = Area of trapezium

Question 12.
By dividing a trapezium into two triangles, show that its area is half the sum of the parallel sides multiplied by the height (the same formula as the one given above).

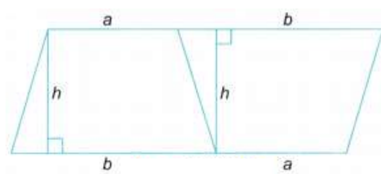
Solution:
This is the standard geometric derivation.



Take a trapezium ABCD with parallel sides a and b and height h.
Draw a diagonal (BD) to split it into two triangles: ∆ABD and ∆BCD.
In ∆ABD, base AB = a and the height = h.
Area of ∆ABD, i.e., A_1 = 12 × base × height = 12ah
In ∆BCD, even though it is oriented differently, the perpendicular distance between the parallel lines (the height) remains h, and base DC = b.
Area of ∆BCD, i.e., A_2 = 12 × base × height = 12bh
Adding them together gives:
$A = A_1 + A_2$
= 12ah + 12bh
= 12(a + b)h

Question 13.
Show how we can use two identical copies of a trapezium to make a parallelogram. How will this give us the formula for the area of a trapezium?

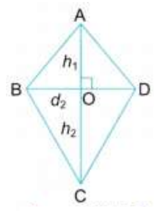
Solution:
This is a very intuitive way to see the formula. Take two identical copies of a trapezium with sides a, b, and height h. Rotate one copy 180° and place it next to the first one so that the side of length a is adjacent to the side of length b.



Together, they form a large parallelogram.
The base of this parallelogram is (a + b).
The height of this parallelogram is h.
Area of Parallelogram A = base × height
A = (a + b) × h
Since the parallelogram is made of two identical trapeziums:
So, area of one trapezium = 12 Area of Parallelogram = 12 (a + b)h

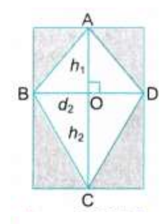
Question 14.
Show that the area of a kite is half the product of its diagonals. Show this:
(i) using algebra, and (ii) using geometry.

Solution:
Let the diagonals be d_1 and d_2.
Note that in a kite, the diagonals intersect at 90°.
(i) Using Algebra



The longer diagonal (d_1) divides the kite into two triangles. o
The shorter diagonal (d_2) is the base for both, and d1 is split int two heights, h_1 and h_2, such that $h_1 + h_2 = d_1$.
Area (ΔABD) = 12BD×AO=12d2h1
Area (ΔCBD) = 12BD×CO=12d2h2
Total area A = 12d2h1+12d2h2=12d2(h1+h2) = 12d2d1

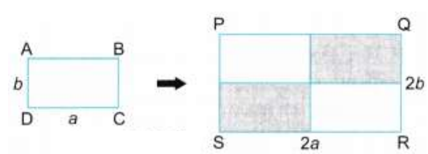
(ii) Using Geometry (Rectangular Bound)



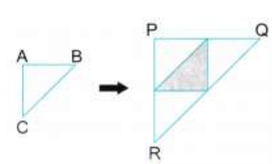
If you draw a rectangle around the kite, the area of the rectangle is $d_1 \times d_2$.
You can see that the kite occupies exactly half of the rectangle's space because the four outer triangles are congruent to the four inner triangles of the kite.
A = 12d2d1

Question 15.
Three problems about fitting congruent shapes together: (i) Rectangle ABCD has sides a, b, and rectangle PQRS has sides 2a, 2b. Show that PQRS has 4 times the area of ABCD. Does this mean that 4 copies of rectangle ABCD will fit into rectangle PQRS? Check and see! (ii) ΔABC has sides a, b, c, and ΔPQR has sides 2a, 2b, 2c. Show that ΔPQR has 4 times the area of ΔABC. Does this mean that 4 copies of ΔABC will fit into ΔPQR? Check and see! (iii) ΔABC has sides a, b, c, and ΔPQR has sides 3a, 3b, 3c. Show that ΔPQR has 9 times the area of ΔABC. Does this mean that 9 copies of ΔABC will fit into ΔPQR? Check and see!

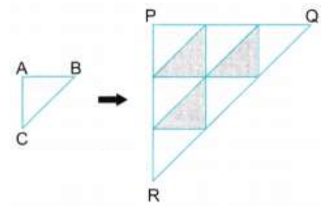
Solution:
(i) Rectangles Area ABCD = a × b = ab = A (let) Area PQRS = 2a × 2b = 4ab = 4 Fitting: Yes, 4 copies of ABCD will fit perfectly into PQRS as shown.



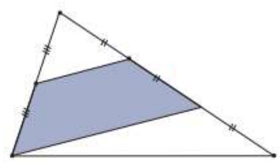
(ii) Area of ΔABC with sides a, b, c using Heron's formula, A = s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√
Area of ΔPQR with sides 2a, 2b, 2c = 2s(2s−2a)(2s−2b)(2s−2c)−−−−−−−−−−−−−−−−−−−−−−−−√
= 2×2×2×2s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−−−−−−−−−−−√
= 4A
Fitting: Yes. You can fit 4 copies of ΔABC into ΔPQR.
Three triangles are placed normally (one at each vertex), and the 4th one fits upside down in the middle.



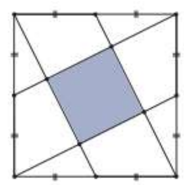
(iii) Area of ΔABC with sides a, b, c using Heron’s formula, A = s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−√
Area of ΔPQR with sides 3a, 3b, 3c = 3s(3s−3a)(3s−3b)(3s−3c)−−−−−−−−−−−−−−−−−−−−−−−−√
= 3×3×3×3s(s−a)(s−b)(s−c)−−−−−−−−−−−−−−−−−−−−−−−−−−−√
= 9A
Fitting: Yes. 9 copies of ΔABC can be fit into ΔPQR.
You would have 1 triangle in the first row, 3 in the second, and 5 in the third (totaling 1 + 3 + 5 = 9).



Question 16.

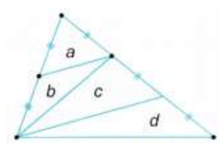


What fraction of the triangle is shaded?

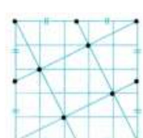


What fraction of the square is shaded?

Solution:
In Fig. 6.43, let the area of the whole triangle be A.
Draw a line to split the shaded portion into two parts, b and c.
The median divides a triangle into two parts with equal area.

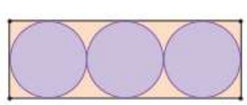


So, area a = area b and area c = area d (since one side is bisected and another side is trisected)
⇒ area b + area c = area a + area d
Also, area a + area b = area c = area d = A3
and area a = area b = A6
area b + area c = A6+A3=A2
Shaded traction = A2 A=12

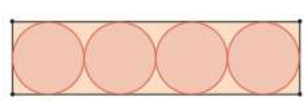


In Fig. 6.44, draw lines parallel to the sides of the square and passing through each vertex of the shaded region.
Thus, they split the whole square into 25 equal parts, out of which the shaded area covers 5 parts.
That means, the fraction of the shaded portion of the square = 525=15

Question 17.



What fraction of the rectangle is covered by the circles?



What fraction of the rectangle is covered by the circles?

Solution:
In Fig. 6.45, let the radius of each circle be r.
Then the rectangle has a length of 6r and a breadth of 2r.
So, the area of each circle = πr^2
There are 3 circles, so the total area covered by the circles = $3\pi r^2$
Area of rectangle = $6r \times 2r = 12r^2$
Thus, the fraction of the rectangle covered by the circles = $\frac{3\pi r^2}{12r^2} = \frac{\pi}{4}$
In Fig. 6.46, let the radius of each circle be r.
Then the rectangle has a length of 8r and a breadth of 2r.
So, the area of each circle = πr^2
There are 4 circles, so the total area covered by the circles = $4\pi r^2$
Area of rectangle = $8r \times 2r = 16r^2$
Thus, the fraction of the rectangle covered by the circles = $\frac{4\pi r^2}{16r^2} = \frac{\pi}{4}$

Question 18.
Use the above to make a conjecture about the area occupied by circles fitted into a rectangle in the manner shown. Test your conjecture for particular cases: 10 circles; 20 circles; 50 circles. Then prove your conjecture!

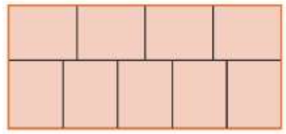
Solution:
Conjecture: From the previous problems, we observed that the area occupied by circles in a rectangle follows a certain pattern. The circles are arranged such that their diameters fit along the length of the rectangle, and their total area increases as more circles are added. Based on this observation, we can make a conjecture about the area occupied by the circles in the given arrangement. The conjecture is that: Fraction of the rectangle occupied by circles = $\frac{\pi}{4}$

This result implies that no matter how many circles are added (as long as they fit neatly into the rectangle), the fraction of the rectangle's area covered by the circles approaches $\frac{\pi}{4}$.
Testing the Conjecture:
Let's test this conjecture for specific cases.
Case 1: 10 Circles
Each circle has radius r, and the diameter is 2r.
The total area of 10 circles is $10 \times \pi r^2$.
The area of the rectangle, with length $10 \times 2r = 20r$ and breadth 2r, is:
Area of rectangle = $20r \times 2r = 40r^2$
The fraction of the rectangle occupied by the circles = $\frac{10\pi r^2}{40r^2} = \frac{\pi}{4}$

Case 2: 20 Circles Total area of 20 circles = $20 \times \pi r^2 = 20\pi r^2$
The area of the rectangle, with length $20 \times 2r = 40r$ and breadth 2r, is:
Area of rectangle = $40r \times 2r = 80r^2$
The fraction of the rectangle occupied by the circles = $\frac{20\pi r^2}{80r^2} = \frac{\pi}{4}$

Case 3: 50 Circles Total area of 50 circles = $50 \times \pi r^2 = 50\pi r^2$
The area of the rectangle, with length $50 \times 2r = 100r$ and breadth 2r, is:
Area of rectangle = $100r \times 2r = 200r^2$
The fraction of the rectangle occupied by the circles = $\frac{50\pi r^2}{200r^2} = \frac{\pi}{4}$
Conclusion from Testing: In all three cases (10 circles, 20 circles, 50 circles), the fraction of the rectangle occupied by the circles is $\frac{\pi}{4}$, which confirms the conjecture.
Proof of Conjecture:
Let's now prove the conjecture. We know that:
Each circle has radius r, so the diameter is 2r.
If n circles are fitted into a rectangle, the total area of the circles is $n \times \pi r^2 = n\pi r^2$.
The length of the rectangle is 2nr (since n circles fit side-by-side along the length), and the breadth is 2r.
Thus, the area of the rectangle = $(2nr) \times (2r) = 4nr^2$
The fraction of the rectangle covered by the circles = $\frac{n\pi r^2}{4nr^2} = \frac{\pi}{4}$
This shows that, regardless of the number of circles n, the fraction of the rectangle covered by the circles is always $\frac{\pi}{4}$, proving the conjecture.
Thus, the area occupied by circles fitted into a rectangle in this manner will always be $\frac{\pi}{4}$ of the area of the rectangle.

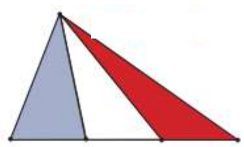
Question 19.
The figure shows nine identical rectangles fitted together to make a large rectangle whose area is 72 cm^2. Find the perimeter of each small rectangle.



Nine identical rectangles stacked together

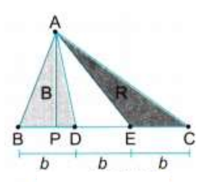
Solution:
Let the length and breadth of each small rectangle be a and b, respectively.
Then, the area of each small rectangle = ab
Total area = 9ab = 72 cm^2
⇒ ab = 8 cm^2 …..(i)
From the given figure,
4a = 5b
⇒ b = 45a
Putting the value in eqn (i),
a × 45a = 8
⇒ a^2 = 10
⇒ a = √10 cm
So, b = 4510−−√ cm
Now, the required perimeter = 2(a + b)
= 2(10−−√+4510−−√)
= 18510−−√ cm

Question 20.
Show that the areas of the shaded blue (B) triangle and the shaded red (R) triangle are equal.



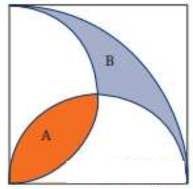
Lines from a vertex to the points of trisection of the opposite side

Find a way of cutting up the blue (B) triangle into some number of pieces and rearranging the pieces to cover the red (R) triangle.



Solution:
1st part: Let us look at the base and height of the two parts, red (R) and blue (B).
The Base: The problem states that the opposite side is trisected.
This means the base of the blue triangle and the base of the red triangle are the same length.
Let's call this length b.
The Height: Both triangles share the same top vertex.
Therefore, they both have the same perpendicular height (h) dropped from that vertex to the line containing the base.
Area of Blue Triangle = 12bh
Area of Red Triangle = 12bh Since the values of b and h are identical for both, their areas are equal. 2nd part: Do it yourself.

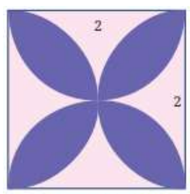
Question 21.
The figure shows a quarter circle in a square. Its centre is at one vertex, and it passes through two adjacent vertices. There are two semicircles on two adjacent sides as diameters. They create the shaded regions A and B. Show that A and B have equal area.



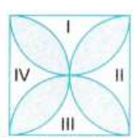
A quarter circle and two semicircles

Solution:
The figure shows a quadrant inside a square.
The centre of the quarter circle is at one vertex, and the quadrant passes through two adjacent vertices.
So, its radius is equal to the side length of the square.
Let the side length of the square be s.
Then, area of the quadrant = 14πs2
There are two semicircles, each on one of the adjacent sides of the square, with diameters along the sides of the square.
Therefore, the radius of each semicircle is half the side length of the square. i. e., s2
So, the area of each semicircle = 12π(s2)2=18πs2
The shaded areas A and B are the regions between the quadrant and the semicircles.
Area of B = Area of the quadrant – (Total area of two semicircles – Area of A)
⇒ Area of B = 14πs2−(18πs2+18πs2− Area of A)
⇒ Area of B = 14πs2−14πs2 + Area of A ⇒ Area of B = Area of A

Question 22.
In Fig. 6.50, four semicircles have been drawn within the given square whose side is 2 units. The centres of these semicircles are the midpoints of the sides. They create a 4-petalled flower (shown in blue). Find the perimeter and the area of this flower.

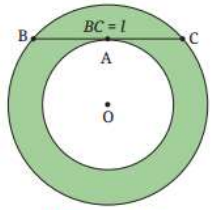


Solution:
All four petals are made by four semicircles, each of diameter 2 units, then the radius is 1 unit.
So, the total perimeter of all the petals = 4πr
⇒ P = 4×227×1=887=1247 units

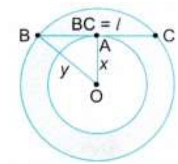


In the given figure, the total area of regions I and III together = Area of square – Total areas of two opposite semicircles
= s2−2×12πr2
= 22−227×12
= 4 – 227
= 67 sq.unit
Also, the total area of regions I and III together = the total areas of regions IV and II together = 67 sq. unit
Now, the total area of all the petals = Area of square – (total area of regions I, II, III, and IV)
= 4−2×67=167=227 sq. units

Question 23.
In Fig. 6.51, we see two concentric circles with a common centre O. A chord BC of the larger circle is drawn, touching the smaller circle at A. The length of BC is l. Show that the area of the green region enclosed between the two circles is 14πl2.

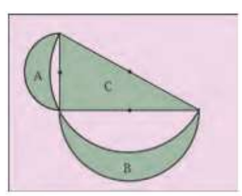


Solution:
Join O to A and B.
Let OA = x and OB = y
Notice that BC is a chord of the bigger circle and a tangent to the smaller circle.
Therefore, OA ⊥ BC and BA = CA = l2



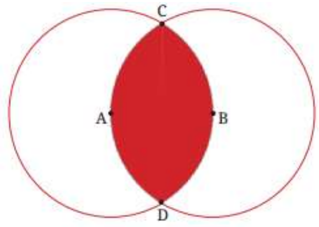
In the right-angled ΔOAB,
$OA^2 + AB^2 = OB^2$
⇒ x2+(l2)2=y2
⇒ y2−x2=l24
Now, Area of the shaded region = Area of the larger circle – Area of the smaller circle
= $\pi y^2 - \pi x^2$
= $\pi(y^2 - x^2)$
= 14πl2

Question 24.
In Fig. 6.52, semicircles have been drawn on all the sides of a right-angled triangle as shown. Show that Area (A) + Area (B) = Area (C).



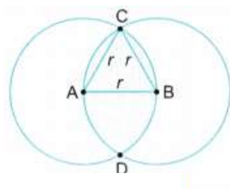
Solution:
Let the sides of the right triangle be 2a, 2b, and 2c, such that from the figure,
Area (A) + Area (B) = Total area of (two smaller semicircles + Area of triangle) – Area of the largest semicircle
= 12πa2+12πb2+Area(C)−12πc2
= 12π(a2+c2)+Area(C)−12πc2
= 12πc2+Area(C)−12πc2 Thus, Area (A) + Area (B) = Area (C)

Question 25.
Fig. 6.53 shows two circles passing through each other’s centres. Find the area of the region enclosed by the two circles in terms of the common radius r.



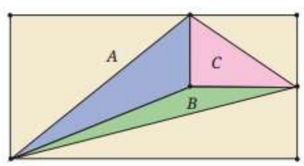
Two congruent circles, radius r

Solution:
∆ABC is an equilateral triangle, as AC, BC, and AB are radii of two congruent circles.

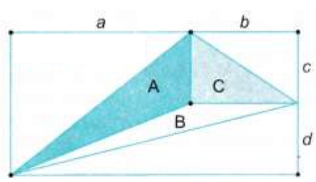


Area of the half-shaded portion = Sum of sectors with centre A and B – Area of equilateral triangle ABC
= 60∘360∘×πr2+60∘360∘×πr2−3√4r2
= 2×16×πr2−3√4r2
= (π3−3√4)r2
So, the total area of the shaded portion is (π3−3√4)r2.

Question 26.
In Fig. 6.54, we see three triangles within a rectangle. The areas of the triangles are A, B, and C, as marked. Show that the area of the rectangle is 2(A+C)(B+C)C.



Solution:
Let adjacent sides of the rectangle be (a + b) and (c + d) as shown in the figure.



So, area of rectangle = (a + b)(c + d)
For triangle A, base = c and height = a
So, Area A = 12ac
For triangle C, base = b and height = c
So, Area C = 12bc
For triangle B, base = b and height = d
Area B = 12bd
Now, 2(A+C)(B+C)C=2(12ac+12bc)(12bd+12bc)12bc
= 2×12c(a+b)×12b(d+c)12bc = (a + b)(c + d) = Area of rectangle

Question 27.
In the figure, we see two shaded regions formed by a quarter circle, a semicircle, and a triangle.

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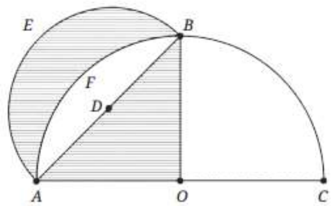

Show that the areas of the two shaded regions are equal.

Solution:
Let OA = OB = r.
Then, AB = √2r (Using Baudhayana-Pythagoras theorem)
The area of △OAB = 12 × OA × OB = $12r^2$
The area of the shaded portion AFBE = Area of the semicircle with centre D + Area of the triangle OAB -Area of the quarter circle with centre O
= 12×π(2√r2)2+12r2−14πr2
= 14πr2+12r2−14πr2
= $12r^2$ Thus, the area of triangle OAB = the area of the shaded portion AFBE.