The Mathematics of Maybe Introduction to Probability Class 9 Solutions Maths Ganita Manjari Chapter 7

Think and Reflect (NCERT Textbook Page No. 156)

Question 1.
Such unpredictability can be useful sometimes! For example, in a cricket match, the fact that a coin is tossed to decide which team will bat first is considered to be a fair method. Can you explain why?

Solution:
A coin toss is used in cricket to decide which team bats or bowls first. It is considered fair because of some basic ideas from probability. Equal chances: A coin has two sides-Heads and Tails. In a fair coin, both sides have an equal chance of landing face up. Probability of Heads = 12 (50%)
Probability of Tails = 12 (50%) So, both teams have an equal chance of winning the toss. Random result: A coin toss is a random experiment. This means we cannot predict the result before the coin is tossed. Because of this, no team has any advantage. Only one outcome at a time: The result can be either Heads or Tails, but not both. This makes the decision clear—one team wins the toss. Independent Event: Each toss is independent. Previous results do not affect the next one. Even if Heads appears many times, the next toss still has a 50-50 chance. Fair and open process: The toss is done in front of both captains and the referee. This makes the process transparent and fair. Conclusion: Since both teams have the same probability, the coin toss is a fair and unbiased method.

Think and Reflect (NCERT Textbook Page No. 157)

Question 1.
Ask your friend to predict the outcome of a ₹ 1 coin you toss. Do you see that your friend could guess heads or tails but could not know for certain? That's randomness! All possible results are known, but each try is unpredictable.

Solution:
When we ask our friend to predict the outcome of a coin toss, we notice that our friend can guess either Heads or Tails, but cannot be certain about the result. This is called randomness. All possible outcomes are known, but the result of each trial cannot be predicted in advance.

Think and Reflect (NCERT Textbook Page No. 163)

Question 1.
If I have rolled a -4 on a die 8 times in succession, the probability of rolling a 4 again is still only ~ 0.16 (assuming the die is fair). Probability does not tell you what will happen next but predicts what will happen in the long run.

Solution:
Even if we roll eight 4s in a row, the probability of rolling another 4 on the 9th try is still 16 (approx. 0.16)
Here is why:
Each roll is an independent event. The die doesn't remember previous results.
The conditions remain the same every time.
Sample Space (S) = {1, 2, 3, 4, 5, 6}, i.e., 6 outcomes
Favourable Outcome = {4}, i.e., 1 outcome
Probability: P(4) = 16 ~ 0.16
This does not change, no matter how many times we roll.
The gambler's fallacy is the false belief that a streak must end.
The myth: "A 4 is overdue to stop."
The reality: The probability is still 16.
Short run: Streaks like repeated 4s can happen randomly.
Long run: Over many trials, each number appears about 16 of the time.
Conclusion: Each roll is independent, so the probability of getting a 4 remains 16.

Think and Reflect (NCERT Textbook Page No. 167)

Question 1.
When we used the sample space {Rain, No Rain} in Example 1, we focused only on whether it will rain or not. However, if we want to include different amounts of rainfall, such as drizzle, light rain, or heavy rain, we need to expand the sample space to {No Rain, Drizzle, Light Rain, Heavy Rain} to better match the level of detail required by the question. It is important to ensure that the sample space is sufficiently detailed to suit the specific problem being studied.

Solution:
Expanding the sample space from {Rain, No Rain} to {No Rain, Drizzle, Light Rain, Heavy Rain} ensures that it aligns with the level of detail required for the situation being studied. It is important to note that the sample space must be

comprehensive enough to cover all possible outcomes relevant to the situation. This allows for more accurate prediction or analysis based on the variability of rainfall.

Think and Reflect (NCERT Textbook Page No. 169)

Question 1.
Can you calculate the probability of getting one head and one tail?

Solution:
When a coin is tossed two times, Sample space S = {HH, HT, TH, TT} Total outcomes n(S) = 4 Favourable outcomes for getting one head and one tail, E = {HT, TH} Number of favourable outcomes = n(E) = 2 P(one head and one tail)
= Number of favourable outcomes Number of all possible outcomes
= 24
= 12 = 0.5 or 50%

Exercise 7.1 Solutions

Question 1.
Rank the following events on a scale from 0 (Impossible) to 1 (Certain). Label each event: Impossible, less likely, equally likely (even chance), more likely, certain. Give reasons why you gave each event its ranking. (i) The next Monday will come after Sunday. (ii) It will snow in Mumbai in July. (iii) An elephant will walk through your classroom today. (iv) You will greet at least one friend at school tomorrow.

Solution:
(i) The next Monday will come after Sunday. Rank: 1 Label: Certain Reason: The text defines a probability of 1 as “certain to happen.” Since the sequence of days in a week is a fixed, repeating pattern where Monday always follows Sunday, this event is guaranteed.

(ii) It will snow in Mumbai in July. Rank: 0 Label: Impossible Reason: The text uses 0 to represent impossible events. Given Mumbai’s geographical location and tropical climate, it does not reach the freezing temperatures required for snow, especially during the monsoon month of July. This is similar to the book’s example of “getting a number greater than 6 on a die.”

(iii) An elephant will walk through your classroom today. Rank: 0 Label: Impossible Reason: Classrooms are structured, urban environments where large wild animals do not have access. Unless your school is specifically located inside a wildlife sanctuary or circus grounds, the probability of this occurring during a normal school day is zero.

(iv) You will greet at least one friend at school tomorrow. Rank: Close to 1 (e.g., 0.8 or 0.9) Label: More likely Reason: The text explains that “more likely” applies to events that have a high chance of occurring but are not 100% guaranteed. Since school is a social setting where you consistently interact with peers, the likelihood is very high.

Exercise 7.2 Solutions

Question 1.
A teacher mixes a large bag of sweets of different colours and randomly selects a sample of 30 sweets. She counts the number of sweets of each colour: 10 red sweets | 8 green sweets | 7 yellow sweets | 5 blue sweets (i) Calculate the probability that a randomly picked sweet from the sample is green. (ii) If there are 600 sweets in total in the large bag, estimate how many are likely to be yellow, based on the sample results.

Solution:
Number of all possible outcomes in sample = 10 + 8 + 7 + 5 = 30 (i) Number of favourable outcomes = 8 (green sweets) P(picking a green sweet) = 830 = 0.266… = 0.267 or 26.7%

(ii) Number of favourable outcomes = 7 (yellow sweets in sample) P(picking a yellow sweet from the sample) = 730 = 0.2333… or 23.3%.
An estimate of the probability that sweets are likely to be yellow is 730.
So approximately 140 (730 of 600) sweets are yellow.

Question 2.
A survey is conducted at a school where a random sample of 40 students is asked about their favourite club. The responses are: 14 students: Science Club | 11 students: Arts Club | 9 students: Sports Club | 6 students: Debate Club Assume there are 800 students in the whole school. (i) What is the probability that a randomly chosen student from the sample prefers the Arts Club? (ii) Using the sample results, estimate how many students in the whole school are likely to prefer the Sports Club.

Solution:
Number of all possible outcomes in sample = 14 + 11 + 9 + 6 = 40 (i) Number of favourable outcomes = 11 (students preferring the Arts club) P(A student prefers Arts Club) = 1140 = 0.275 or 27.5%

(ii) Number of favourable outcomes = 9 (students preferring the Sports Club)
P(A student prefers Sports Club) = 940
Estimate for 800 students = 940 × 800 = 180 students likely to prefer sports

club.

Question 3.
Toss a coin 20 times and record the result each time (heads or tails). (i) How many times did you get heads? (ii) How many times did you get tails? (iii) Calculate the experimental probability of getting heads. (iv) If you toss the coin once more, what is the probability of getting tails?

Solution: Do it yourself.

Question 4.
Toss a paper cup into the air 100 times. After each toss, record whether the cup lands on its bottom, upside down on its top or on its side (See Fig. 7.5). Assign probabilities to the outcomes by using experimental probability.

Paper cup landing positions (left to right)—bottom, top and side

Solution: Do it yourself.

Question 5.
What is the probability of getting an even number when rolling a fair 6-sided die?

Solution:
Number of all possible outcomes = 6 (numbers 1, 2, 3, 4, 5, 6) Number of favourable outcomes = 3 (even numbers 2, 4, 6) P(getting an even number)
= Number of favourable outcomes Number of all possible outcomes
= 36 = 0.5 or 50%

Question 6.
Suppose you roll a 6-sided die 12 times and get a ‘3’ three times. (i) What is the experimental probability of rolling a ‘3’? (ii) What is the theoretical

probability of rolling a '3'? (iii) Why might these probabilities be different? What would you expect to happen if you rolled the die 60, 600, or 6000 times?

Solution:
(i) Number of all outcomes = 12 (Total rolls in experiment) Number of favourable outcomes = 3 (times '3' actually appeared) P(rolling a 3)
= Number of favourable outcomes Number of all outcomes
= 312 = 0.25 or 25%

(ii) Number of all possible outcomes = 6 (numbers 1 through 6) Number of favourable outcomes = 1 (only the number 3) P(rolling a 3)
= Number of favourable outcomes Number of all outcomes
= 16 = 0.167 or 16.7%

(iii) Difference between Experimental and Theoretical probabilities: The difference exists because experimental probability is based on evidence from a limited sample, while theoretical probability is based on the ideal mathematical outcome. As the number of trials increases, the experimental probability will likely get closer to the theoretical one. This is why a larger sample size makes our estimates more reliable.

Exercise 7.3 Solutions

Question 1.
When a single 6-sided die is rolled, what is the total number of possible outcomes in the sample space?

Solution:
Sample Space (S): {1, 2, 3, 4, 5, 6} Number of all possible outcomes: 6
Description: Since each face of the die represents a unique result, there are six distinct possibilities.

Question 2.
For the following experiments, write down the sample space S.
(i) Rolling a die and tossing a coin together.
(ii) Choosing a random integer between -5 and +5.
(iii) A box containing 5 green and 7 red balls. One ball is drawn at random.

Solution: (i) In the case of rolling a die and tossing a coin together, we multiply the possibilities of the die (1, 2, 3, 4, 5, 6) by the possibilities of the coin (H, T). Each outcome is a pair consisting of one number and one side of the coin.

.

Sample Space (S): {(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T), (5, H), (5, T), (6, H), (6, T)} Number of all possible outcomes: 12

(ii) Choosing a random integer between -5 and +5 excludes the endpoints (-5 and +5). The sample space consists of all the whole numbers located strictly between those two values. Sample Space (S): {-4, -3, -2, -1, 0, 1, 2, 3, 4} Number of all possible outcomes: 9

(iii) A box containing 5 green and 7 red balls. One ball is drawn at random. Sample Space (S): {Green, Red} Number of all possible outcomes: 12 (Total balls: 5 + 7) While there are only two types of outcomes (Green or Red), the weight or likelihood of the sample space is determined by the total number of items. Number of favourable outcomes for Green: 5 Number of favourable outcomes for Red: 7 Although there are 12 balls in total, the sample space includes only the types of outcomes, not their quantities. Also, if individual balls are considered, then Sample space (S) = {G_1, G_2, G_3, G_4, G_5, R_1, R_2, R_3, R_4, R_5, R_6, R_7}

Question 3.
In a village fair, there are 3 popular snacks available: Samosa, Pakora, and Bhaji. For drinks, villagers can choose either Chai or Lassi. (i) List the sample space of all possible snack and drink combinations a person could choose at the fair. (ii) List the event 'Selecting Samosa as a snack.'

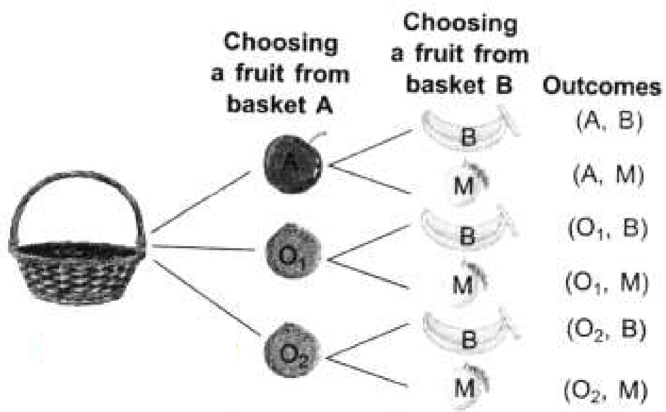
Solution:
(i) To find the sample space, we pair each snack with every possible drink. There are 3 snacks (Samosa, Pakora, Bhaji) and 2 drinks (Chai, Lassi). Sample Space (S): {(Samosa, Chai), (Samosa, Lassi), (Pakora, Chai), (Pakora, Lassi), (Bhaji, Chai), (Bhaji, Lassi)} Each outcome represents one complete meal choice. Number of all possible outcomes: 6

(ii) An event is a specific subset of the sample space that satisfies a given condition. In this case, we only look for pairs where the snack is a Samosa. Event (E): Selecting Samosa as a snack ={(Samosa, Chai), (Samosa, Lassi)} Number of favourable outcomes: 2

Exercise 7.4 Solutions

Question 1.
There are two fruit baskets, A and B. Basket A has one apple and two oranges. Basket B has one banana and one mango. You randomly pick one fruit from each basket. (i) Draw a tree diagram showing all possible pairs of fruits. (ii) List the sample space. (iii) What is the probability of picking one apple and one banana?

Solution:
(i) Tree Diagram showing all possible pairs of fruits

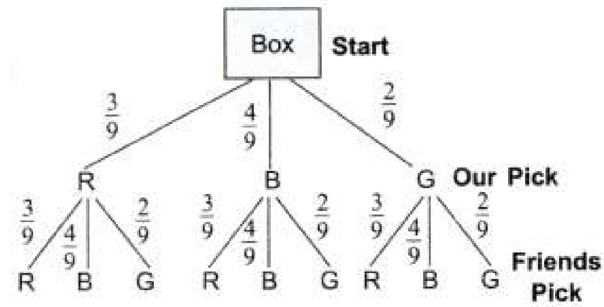


Each fruit from Basket A is paired with each fruit from Basket B. Therefore, total outcomes = 3 × 2 = 6.

(ii) Sample Space (S) = {(A, B), (A, M), (O_1, B), (O_1, M), (O_2, B), (O_2, M)}
Number of all possible outcomes = 6
(iii) Probability of picking one apple and one banana
Number of favourable outcomes = 1 (only the pair (A, B))
Number of all possible outcomes = 6
P(Apple and Banana)
= Number of favourable outcomes Number of possible outcomes
= 16 = 0.167 or 16.7%

Question 2.
Let us say that you have a box containing 3 red pens, 4 black pens, and 2 green pens. You pick a pen (without looking) from the box and put it back. Then your friend does the same. (i) What are the possible outcomes of the pen colours? Can you draw a tree diagram representing the possible outcomes? (ii) Can you use the tree diagram to guess the probability that both you and your friend pick pens of the same colour?

Solution:
(i) Since you put the pen back, the second person has the same choices as the first. The possible colours are Red (R), Black (B), and Green (G).



The possible outcome pairs are: (R, R), (R, B), (R, G), (B, R), (B, B), (B, G), (G, R), (G, B), (G, G)
Since the pen is replaced after the first pick, the number of outcomes remains the same for both selections.
Sample Space (S) = {(R, R), (R, B), (R, G), (B, R), (B, B), (B, G), (G, R), (G, B), (G, G)}
Number of all possible outcomes = 9
P(Red) = 39
P(Black) = 49
P(Green) = 29

(ii) Yes, to find the probability that both of us pick the same colour, we look at the three paths where the colours match: (R, R), (B, B), and (G, G). We multiply the probabilities along those specific branches and then add them together. Both Red: Your 3 choices × Friend's 3 choices = 9 ways P(R, R) = 39×39=981
P(B, B) = 49×49=1681
P(G, G) = 29×29=481
Total probability = 981+1681+481=2981
P(Same colour) = 2981 = 0.358 or 35.8% The tree diagram helps you see that while there are only 3 "matching colour" scenarios, the likelihood of each scenario depends on how many pens of that colour were in the box to begin with.

End of Chapter Exercise Solutions

Question 1.
Fill in the blanks.
(i) The probability of an impossible event is _____________ (ii) The set of all possible outcomes of a random experiment is called the _____________ (iii) The probability of an event that is certain to happen is _____________ (iv) Tossing a fair coin has a probability of _____________ for getting heads.

Solution:
(i) The probability of an impossible event is 0. (ii) The set of all possible outcomes of a random experiment is called the sample space. (iii) The probability of an event that is certain to happen is 1. (iv) Tossing a fair coin has a probability of 12 (or 0.5) for getting heads.

Question 2.
In a survey of 50 students, 15 students said they liked football. The number of students who like football is 15, and the ______________ (frequency/relative frequency) is ______________ (fill in the fraction or decimal).

Solution:
The number of students who like football is 15, and the relative frequency is 1550=310 (or 0.3).

Question 3.
Which of the following experiments has equally likely outcomes? Explain. (i) A driver attempts to start a car. The car starts or does not start. (ii) Tossing a fair coin once. (iii) Rolling a fair 6-sided die. (iv) Choosing a marble randomly from a bag that contains 3 red marbles and 7 blue marbles. (v) A baby is bom. It is a boy or a girl.

Solution:
Outcomes are equally likely when each possible result has the same theoretical probability. (i) A driver attempts to start a car. Not equally likely. Reason: Starting a car depends on factors like battery health, fuel, and engine condition. It is not a random process where “starting” and “failing” have a 50/50 chance.

(ii) Tossing a fair coin once. Equally likely. Reason: The word “fair” implies the coin is balanced. Number of all possible outcomes = 2 (H, T) Number of favourable outcomes = 1 for each. Both have a probability of 12.

(iii) Rolling a fair 6-sided die. Equally likely. Reason: Each of the 6 faces is identical in shape and weight. Number of all possible outcomes = 6 Number of favourable outcomes = 1 for each number. All have a probability of 16.

(iv) Choosing a marble from a bag with 3 red and 7 blue marbles. Not equally likely. Reason: There are more blue marbles than red ones. P(Red) = 310 (30%)
P(Blue) = 710 (70%)

(v) A baby is born. Equally likely. Reason: Biologically, the chance of a baby being a boy or a girl is treated as 12 (50%) each in a large population.

Question 4.

Write the sample space and calculate the probability based on the given information. (i) Two coins are tossed at the same time. What is the probability of getting at least one head? (ii) Ten identical cards numbered 1 to 10 are placed in a box. One card is drawn at random. What is the probability of drawing a card with an even number? (iii) A die is rolled once. What is the probability of getting a number greater than 4? (iv) A bag contains 3 red balls, 2 blue balls, and 1 green ball. One ball is picked at random. What is the probability that it is not red? (v) Three coins are tossed simultaneously. What is the probability of getting exactly two heads?

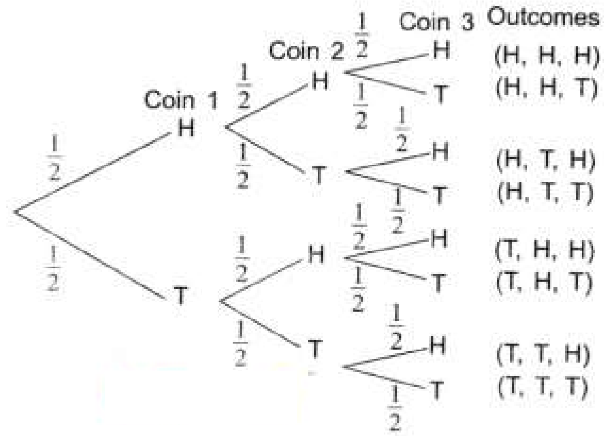
Solution:
(i) Two coins are tossed at the same time. Sample Space (S): {HH, HT, TH, TT}
Number of all possible outcomes = 4 Event (At least one head): {HH, HT, TH}
Number of favourable outcomes = 3 P(At least one head)
= Number of favourable outcomes Number of all possible outcomes
= 34 = 0.75 or 75%

(ii) Ten cards numbered 1 to 10. Sample Space (S): {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
Number of all possible outcomes = 10 Event (Even number): {2, 4, 6, 8, 10}
Number of favourable outcomes = 5 P(Even number)
= Number of favourable outcomes Number of all possible outcomes
= 510
= 12 = 0.5 or 50%

(iii) A die is rolled once. Sample Space (S): {1, 2, 3, 4, 5, 6} Number of all possible outcomes = 6 Event (Number > 4): {5, 6} Number of favourable outcomes = 2 P(Number > 4)
= Number of favourable outcomes Number of all possible outcomes
= 26
= 13 = 0.333 or 33.3%

(iv) A bag contains 3 red, 2 blue, and 1 green ball. Number of all possible outcomes = 6 (Total: 3 + 2 + 1) Event (Not red): {Blue, Blue, Green} Number of favourable outcomes = 3 P(Not red)
= Number of favourable outcomes Number of all possible outcomes
= 36
= 12 = 0.5 or 50%

(v) Three coins are tossed simultaneously. Using a tree diagram



Sample Space (S): {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
Number of all possible outcomes = 8
Event (Exactly two heads): {HHT, HTH, THH}
Number of favourable outcomes = 3
P(Exactly two heads)
= Number of favourable outcomes Number of all possible outcomes
= 38 = 0.375 or 37.5%

Question 5.
A bag has 3 candies: strawberry, lemon, and mint. One is picked at random. What is the probability of picking a strawberry candy?

Solution:
Sample Space (S): {strawberry, lemon, mint} Number of all possible outcomes = 3 Number of favourable outcomes = 1 (only the strawberry candy) P(picking a strawberry candy) = 13 = 0.333 or 33.3%

Question 6.
A child has 2 shirts (one red and one blue) and 3 types of pants (jeans, khakis, and shorts). List all the possible combinations of outfits consisting of one shirt and one pair of pants. Display your answer in a table format.

Solution:
A child has 2 choices for shirts and 3 choices for pants. To find all possible combinations, we pair each shirt with every type of pants.

Shirts	Pants	All Possible Outcomes
Red	Jeans	Red, Jeans
	Khakis	Red, Khakis

	Shorts	Red, Shorts
Blue	Jeans	Blue, Jeans
	Khakis	Blue, Khakis
	Shorts	Blue, Shorts

Number of all possible outcomes = Number of choices for shirts × number of choices for pants = 2 × 3 = 6 outfits

Question 7.
A tyre company records distances before replacement in 1000 cases.

Distance (km)	Less than 4000	4001 to 9000	9001 to 14000	More than 14000
Number of cases	20	210	325	445

Find the probability that a randomly chosen tyre lasts:
(i) Less than 4000 km.
(ii) Between 4000 and 14000 km.
(iii) More than 14000 km.

Solution:
This problem uses experimental probability because it is based on evidence collected from 1000 actual cases.
Number of all possible outcomes (Total cases) = 1000
(i) Event: The tyre lasts less than 4000 km
Number of favourable outcomes = 20
P(< 4000 km) = Number of times the event occurred Total number of cases = 201000 = 0.02 or 2%

(ii) Event: The tyre lasts between 4000 and 14000 km Number of favourable outcomes = 210 + 325 = 535 P(between 4000 and 14000 km)
= Number of times the event occurred Total number of cases
= 5351000 = 0.535 or 53.5%

(iii) Event: The tyre lasts more than 14000 km Number of favourable outcomes = 445 P(> 14000 km)
= Number of times the event occurred Total number of cases
= 4451000 = 0.445 or 44.5%

Question 8.
The letters of the word 'PEACE' are placed on cards. Leela draws a card without looking.
(i) What is the probability that it is a P, E, or C?
(ii) What is the probability that it is not an E?

Solution:
The letters are P, E, A, C, E.
Sample Space (S): {P, E, A, C, E}
Number of all possible outcomes = 5
(i) Event: The card drawn is a P, E, or C
Favourable outcomes: {P, E, C, E} (E is counted twice)
Number of favourable outcomes = 4
P(P, E, or C)
= Number of favourable outcomes Number of all possible outcomes
= 45 = 0.8 or 80%

(ii) Event: The card drawn does not have an E. Favourable outcomes: {P, A, C}
Number of favourable outcomes = 3 P(not E) =
= Number of favourable outcomes Number of all possible outcomes
= 35 = 0.6 or 60%

Question 9.
A game of chance consists of spinning an arrow (see Fig. 7.7), which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, and these are equally likely outcomes. What is the probability that it will point at

(i) 8?
(ii) An odd number?

(iii) A number greater than 2?
(iv) A number less than 9?
(v) A multiple of 3?

Solution:
The numbers are 1, 2, 3, 4, 5, 6, 7, 8.
Sample Space (S): {1, 2, 3, 4, 5, 6, 7, 8}
Number of all possible outcomes = 8
(i) Event: The arrow will point at 8
Favourable outcomes: {8}
Number of favourable outcomes = 1
P(8) = Number of favourable outcomes Number of possible outcomes
= 18 = 0.125 or 12.5%

(ii) Event: The arrow will point to an odd number. Favourable outcomes: {1, 3, 5, 7} Number of favourable outcomes = 4 P(odd)
= Number of favourable outcomes Number of possible outcomes
= 48
= 12 = 0.5 or 50%

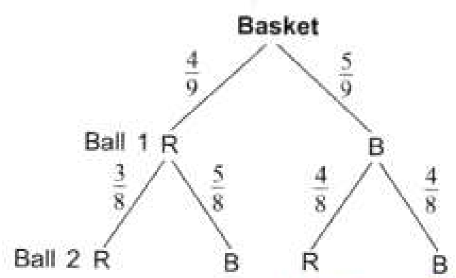
(iii) Event: The arrow will point to a number greater than 2 Favourable outcomes: {3, 4, 5, 6, 7, 8} Number of favourable outcomes = 6 P(2)
= Number of favourable outcomes Number of possible outcomes
= 68
= 34 = 0.75 or 75%

(iv) Event: The arrow will point to a number less than 9. Favourable outcomes: {1, 2, 3, 4, 5, 6, 7, 8} Number of favourable outcomes = 8 P(< 9)
= Number of favourable outcomes Number of possible outcomes
= 88 = 1 (certain event)

(v) Event: The arrow will point to a multiple of 3 Favourable outcomes: {3, 6}
Number of favourable outcomes = 2 P(multiple of 3)
= Number of favourable outcomes Number of possible outcomes
= 28
= 14 = 0.25 or 25%

Question 10.
A basket contains 4 red balls and 5 blue balls. One ball is drawn and laid aside, and a second ball is drawn. Draw a tree diagram to represent the possible outcomes and probabilities. Use the tree diagram to answer the following questions. (i) What is the probability of drawing a red ball and then a blue ball? (ii) What is the probability of drawing 2 blue balls?

Solution:
There are 4 Red (R) and 5 Blue (B) balls. Total = 9. Because the first ball is laid aside, the total becomes 8 for the second draw.



First draw: P(R) = 49, P(B) = 59
Second draw if first is Red: remaining 3
Red and 5 Blue, so P(R) = 38, P(B) = 58
Second draw if first is Blue: remaining 4 Red and 4 Blue,
so P(R) = 48, P(B) = 48
(i) Probability of Red then Blue (RB) = 49×58=2072
= 518 = 0.278

(ii) Probability of two Blue balls (BB) = 59×48=2072
= 518 = 0.278

Question 11.
I throw a pair of 6-sided dice. Write down an event that has a probability of 0 and an outcome that has a probability of 1.

Solution:
Probability of 0 (Impossible Event): getting a sum of 13, since the maximum is 12. Probability of 1 (Certain Event): getting a sum less than 14, since all sums are from 2 to 12.

Question 12.
Write the sample space and calculate the probability based on the given information. (i) Two dice are rolled. What is the probability that the sum is a prime number greater than 5? (ii) A bag contains 4 red, 3 green, and 2 blue balls. Two balls are drawn without replacement. What is the probability that both are of different colours? (iii) Three coins are tossed. What is the probability that the first coin shows heads and exactly two heads occur in total? (iv) A four-digit number is formed using the digits 1, 2, 3, and 4 with no repetition. What is the probability that the number is

even? (v) A student takes a multiple-choice test with 3 questions, each having 4 options (A, B, C, D), with only one correct answer. What is the probability that the student guesses and gets exactly 2 answers correct?

Solution:
(i) Two dice are rolled Sample space (S): {(1, 1), (1,2), (1,3), (1,4), (1, 5), (1,6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3,4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} Sum is a prime number greater than 5. Possible sums: 7 and 11 Favourable outcomes = {(2, 5), (1, 6), (3, 4), (4, 3), (6, 1), (5, 2), (5, 6), (6, 5)} Number of favourable outcomes = 8 Total number of outcomes = 36 Probability
= Number of favourable outcomes Number of all possible outcomes
= 836
= 29 = 0.222

(ii) 4 Red, 3 Green, 2 Blue (without replacement): different colours Since balls are distinct, we label them R_1, R_2, R_3, R_4, G_1, G_2, G_3, B_1, B_2
Sample space (S): Set of all combination of 2 balls from the 9 available {(R_1, R_2), (R_1, R_3), (R_1, R_4), (R_1, G_1, (R_1, G_2), (R_1, G_3), (R_1, B_1), (R_1, B_2), (R_2, R_3), (R_2, R_4), (R_2, G_1),….(G_2, G_3), (G_2, B_1), (G_2, B_2), (G_3, B_1), (G_3, B_2), (B_1, B_2)
Total outcomes = 8 + 7 + 6 + 5+ 4 + 3 + 2 + 1 = 36
P(same colour) = P(R, R) + P(G, G) + P(B, B)
= (49×38)+(39×28)+(29×18)
= 12+6+272
= 2072
= 518
P(different colours) = 1 – P(same colour)
= 518
= 1318 = 0.722

(iii) Three coins are tossed. Sample space (S) = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} Event: First is Heads and exactly two heads Favourable outcomes = {HHT, HTH} No. of Favourable outcome = 2 Total no outcomes = 8 Probability = Number of favourable outcomes Number of possible outcomes
= 28
= 14 = 0.25

(iv) Four-digit numbers formed using 1, 2, 3, 4 without repetition. Event: Number is even Sample space (S) = {1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 3412, 3421, 4123, 4123, 4213, 4231, 4312, 4321} Total number outcomes = 24 An even number must end in 2 or 4. Favourable outcomes = 12 Probability

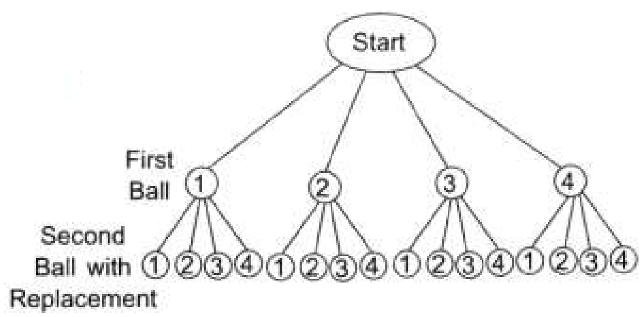
= Number of favourable outcomes Number of possible outcomes
= 1224
= 12 = 0.5

(v) Data: 3 questions, 4 options (1 correct (C), 3 wrong (W)) For 3 questions,
Sample space (S) = {CCC, CCW, CWC, WCC, CWW, WCW, WWC, WWW}
For each question, P(C) = 14, P(W) = 34
Favourable (outcomes) for exactly 2 correct answers = {CCW, CWC, WCC}
Calculation for one path (for example, CCW):
14×14×34=364
Total probability (sum of all 3 paths):
P(E) = 3 × 364
= 964 = 0.141 or 14.1%

Question 13.

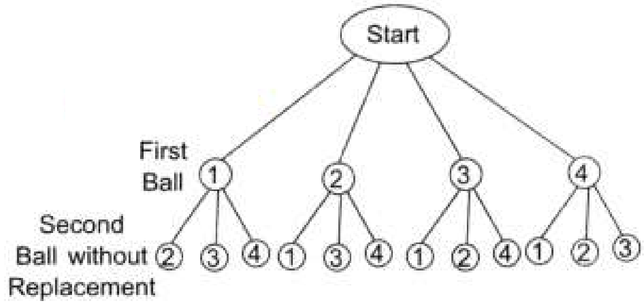
A box contains 4 balls numbered 1 to 4. Record a sample space using a tree diagram for the following experiments: (i) A ball is drawn, and the number is recorded. Then the ball is returned, and a second ball is drawn and recorded. (ii) A ball is drawn and recorded. Without replacing the first ball, the experimenter draws and records a second ball. (iii) What are the sizes of these two sample spaces?

Solution:
(i) With Replacement A ball is drawn, recorded, and returned before the second draw. Since the ball is returned, the options for the second draw are the same as the first draw: {1, 2, 3, 4}.



Sample Space (S1): S1 = {(1, 1), (1,2), (1, 3), (1,4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}

(ii) Without Replacement A ball is drawn and recorded. The second ball is drawn without replacing the first. If we pick ball 1 first, it is no longer in the box. Therefore, the second draw can only be {2, 3, 4}. So, we cannot have outcomes like (1, 1) or (2, 2).



Sample Space (S2): S2 = {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}

(iii) Sizes of the Sample Spaces The size of a sample space is denoted by n(S). 1. Size of S1 (With Replacement) Calculation: 4 (first pick) × 4 (second pick) = 16 n(S1) = 16 2. Size of S2 (Without Replacement) Calculation: 4 (first pick) × 3 (remaining options) = 12 n(S2) = 12

Question 14.
List the elements of a sample space for the simultaneous tossing of a coin and drawing of a card from a set of 6 cards numbered 1 through 6.

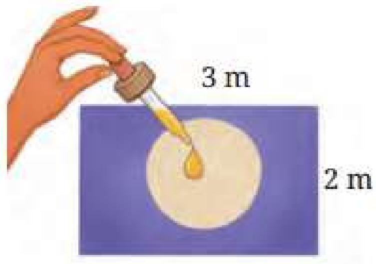
Solution:
For this experiment, we are combining two independent actions: tossing a coin and drawing a numbered card. To list the sample space systematically, we pair each possible result of the coin with each possible result of the card. 1. Identity Individual Outcomes Coin: {Heads (H), Tails (T)} → 2 outcomes Cards: {1, 2, 3, 4, 5, 6} → 6 outcomes 2. Form the Pairs (Sample Space S) We list every combination, starting with Heads and then repeating the process for Tails. S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)} Number of elements n(S) = 2 × 6 = 12

Question 15.
Three coins are tossed, and the number of heads is recorded. Which of the following lists is a sample space for this experiment? Why do the other lists fail to qualify as a sample space? (i) {1, 2, 3} (ii) {0, 1, 2} (iii) {0, 1, 2, 3, 4} (iv) {0, 1, 2, 3}

Sol.
When three coins are tossed, we count how many times heads appear. The possible results are 0, 1, 2, or 3 heads. Correct sample space: {0, 1, 2, 3} Explanation of incorrect options: Option (i) {1, 2, 3} is incomplete because it does not include 0 heads, which is possible when all coins show tails. Option (ii) {0, 1, 2} is incomplete because it does not include 3 heads, which is possible when all coins show heads. Option (iii) {0, 1, 2, 3, 4} is incorrect

because 4 heads is impossible when only three coins are tossed. Remember a sample space must include all possible outcomes and only those outcomes that can actually occur.

Question 16.
Suppose you drop a dye at random on the rectangular region shown in Fig. 7.8. What is the probability that it will land inside the circle with a diameter of 1 m?



Solution:
This problem uses geometric probability, where probability is found using area instead of counting outcomes.
Step 1: Find the total area
The dye can land anywhere in the rectangle.
Length = 3 m, Breadth = 2 m
Area of rectangle = 3 × 2 = 6 m^2
Step 2: Find the favourable area
The favourable region is the circle.
Diameter = 1 m, so radius = 0.5 m
Area of circle = π × (0.5)2 = 0.25π m^2
Step 3: Calculate the probability
Probability = Number of favourable outcomes Number of possible outcomes
= Area of circle Area of rectangle
= 0.25π6
∴ Probability of landing a dye inside the circle = π24 = 0.13 or 13%.