Predicting What Comes Next Exploring Sequences and Progressions Class 9 Solutions Maths Ganita Manjari Chapter 8

Think and Reflect (NCERT Textbook Page No. 174)

Can you describe the pattern in each of the above sequences? Can you predict the next few numbers in these sequences?

Solution:
Sequence: 1, 2, 3, 4, 5, 6,… (Natural Numbers) Pattern: Each term of the sequence obtained by adding 1 to the previous term. Next few numbers: 7, 8, 9, 10, 11,…

Sequence: 1, 3, 5, 7, 9, 11,… (Odd Numbers) Pattern: Each term is obtained by adding 2 to the previous term. Next few numbers: 13, 15, 17, 19, 21, …
Sequence: 1, 3, 6, 10, 15, 21,… (Triangular Numbers) Pattern: Each term of the sequence is the sum of all natural numbers up to that point.

For example: 1st term: 1 2nd term: 1+2 = 3 3rd term: 1 +2 + 3 = 6 4th term: 1 + 2 + 3+ 4= 10 and so on. Next few numbers: 28, 36, 45, 55, 66, … Sequence: 1, 4, 9, 16, 25, 36,… (Square Numbers) Pattern: Each term is the square of a natural number: 1st term: $1^2 = 1$
2nd term: $2^2 = 4$
3rd term: $3^2 = 9$
4th term: $4^2 = 16$ and so on. Next few numbers: 49, 64, 81, 100, 121,…

Exercise (Page 176)

Consider the sequence 1, 4, 7, 10, 13,… Can you predict the next four terms? Can you derive the first 10 terms of the sequence obtained by adding all the terms up to a given term of this sequence (Hint: The first term is 1. The second term is 1 + 4 = 5, the third term 1+4 + 7=12 and so on.)

Solution:
The given sequence: 1, 4, 7, 10, 13,… Here the difference between two consecutive term is constant, i.e., 3 (4 – 1 = 7 – 4 = 10 – 7 = 13 – 10 = 3). Therefore, to find the next four terms, we simply add the common difference of 3 to the last known term: 13 + 3 = 16, 16 + 3 = 19, 19 + 3 = 22,22 + 3 = 25. Thus, the next four terms in the sequence are: 16, 19, 22, 25 Now, deriving the first 10 terms of the sequence obtained by adding all terms up to a given term:
$T_1 = 1$

$T_2 = 1 + 4 = 5$
$T_3 = 1 + 4 + 7 = 12$
$T_4 = 1 + 4 + 7 + 10 = 22$
$T_5 = 1 + 4 + 7 + 10 + 13 = 35$
$T_6 = 1 + 4 + 7 + 10 + 13 + 16 = 51$
$T_7 = 1 + 4 + 7 + 10 + 13 + 16 + 19 = 70$
$T_8 = 1 + 4 + 7 + 10 + 13 + 16 + 19 + 22 = 92$
$T_9 = 1 + 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25 = 117$
$T_{10} = 1 + 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 = 145$ Thus, the first 10 terms of the new sequence are: 1, 5, 12, 22, 35, 51, 70, 92, 117, 145.

Can you write t_5, t_6, t_7 and t_8, for the sequence of triangular numbers?

Solution:
We know that, in the triangular number sequence, each term represents the sum of the natural numbers up to that term.
So, the terms of the triangular numbers are:
$t_5 = 1 + 2 + 3 + 4 + 5 = 15$
$t_6 = 1 + 2 + 3 + 4 + 5 + 6 = 21$
$t_7 = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$
$t_8 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$ Thus, the 5th, 6th, 7th and 8th triangular numbers: 15, 21, 28 and 36.

Think and Reflect (NCERT Textbook Page No. 176)

Can you think of any other kinds of sequences? List out five different types of sequences and discus their properties with your friends.

Solution: Do it yourself.

Think and Reflect (NCERT Textbook Page No. 177)

Why is it useful to have an explicit formula for the nth term of a sequence?

Solution:
Easy Prediction: It allows quick calculation of any term without listing all previous terms. Pattern Understanding: It reveals how terms are related and how they change. Efficient Calculation: It provides a direct way to calculate terms, saving time.

Exercise (Page 177)

Using the explicit rule $u_n = 2n – 1$, find the 53rd term, 108th term, and the 1170th term of the odd number sequence.

Solution:
The given explicit rule $u_n = 2n - 1$
Substitute 53, 108 and 1170 for n in the expression 2n – 1.
$u_{53} = 2 \times 53 - 1 = 106 - 1 = 105$
$u_{108} = 2 \times 108 - 1 = 216 - 1 = 215$
$u_{1170} = 2 \times 1170 - 1 = 2340 - 1 = 2339$

Think and Reflect (NCERT Textbook Page No. 177)

Can you find the rule describing the nth term of the sequence of square numbers?

Solution:
The sequence of square numbers is as follows: 1, 4, 9,16, 25,36, 49, 64,81, 100,… To determine the rule for the nth term of this sequence, observe that each term corresponds to the square of a natural number: $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$,… and so on. Thus, the explicit formula for the nth term of the sequence of square numbers is: $t_n = n^2$.

Exercise (Page 178)

Consider the expression $t_n = 3n - 7$.
(i) Find its first, second, third, 12th, 18th, and 50th terms.
(ii) Which term of the sequence is 332?
(iii) Is 557 a term of this sequence? Why or why not?

Solution:
(i) Given expression for nth term: 3n – 7, so substitute
1, 2, 3, 12, 18, and 50 for n in the given expression.
First term = 3 × 1 – 7 = -4
Second term = 3 × 2 – 7= -1
Third term = 3 × 3 – 7 = 2
$t_{12} = 3 \times 12 - 7 = 36 - 7 = 29$
$t_{18} = 3 \times 18 - 7 = 54 - 7 = 47$
$t_{50} = 3 \times 50 - 7 = 150 - 7 = 143$

(ii) To find which term of the sequence correspondence to 332, we set $t_n = 332$ and solve for n 332 = 3n – 7
⇒ 332 + 7 = 3n
⇒ 3n = 339
⇒ n = 3393 = 113 Thus, the 113th term of the sequence is 332.

(iii) To check if 557 is a term of the sequence, we solve for n in the equation t_n = 557:
557 = 3n – 7
⇒ 557 + 7= 3n
⇒ 564 = 3n
⇒ n = 5643 = 188 Since n = 188 is a whole number, 557 is indeed of a term of the sequence, and it is the 188th term.

Think and Reflect (NCERT Textbook Page No. 180)

Can you predict the number of squares in Stages 5 and 6 of the sequence?
In Stages 10,11 and 12? In Stage 20? At any stage?

Solution:
From the figure, the number of squares at each stage forms the sequence: 1, 5, 9, 13, … We observe that at each stage, 4 squares get added to the comers of the pattern in the previous stage, so number of squares at successive stages can be written as: 1, 1 + 4, 1 + 4 + 4, 1 + 4 + 4 + 4,… This may be written as 1 + 4 × 0, 1 + 1 × 4, 1 + 2 × 4, 1 + 3 × 4, and so on Similarly, number of squares in: Stage 5: 1 + 4 × 4 = 17 Stage 6: 1 + 4 × 5 = 21 Stage 10: 1 + 4 × 9 = 37 Stage 11: 1 + 4 × 10 = 41 Stage 12: 1 + 4 × 11 = 45 Stage 20: 1 + 4 × 19 = 77 So, at any stage n number of sequences given by t_n = 1 + (n – 1) × 4 = 1 + 4n – 4 = 4n – 3

Think and Reflect (NCERT Textbook Page No. 181)

Consider all the sequences we have discussed so far in this chapter. Which ones are arithmetic progressions and which ones are not? Can you justify your claim?

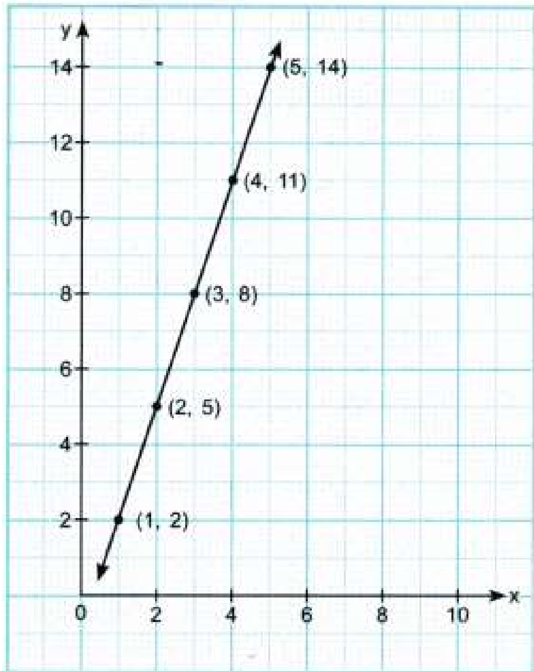
Solution: Do it yourself.

Exercise (Page 182)

Verify that the following sequences are arithmetic progressions and write their terms. What do you observe when you plot the ordered pairs emerging from them? (i) 2, 5, 8, 11,…

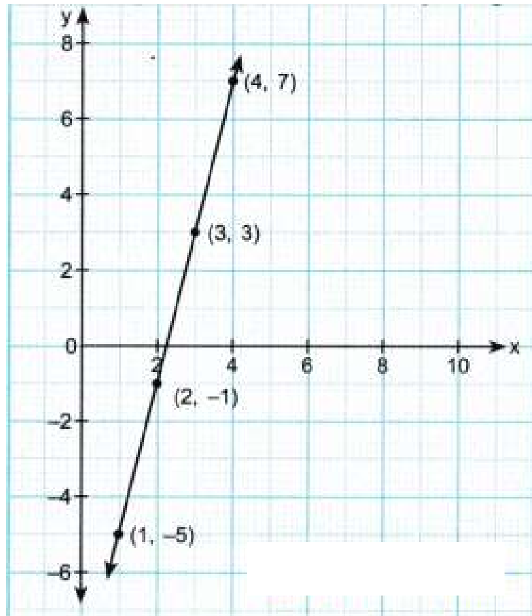
Solution:
2, 5, 8, 11,… First find the difference of two consecutive terms, 5 – 2 = 3, 8 – 5 = 3, 11 -8 = 3,… and so on. Here we observe that the difference between consecutive terms is constant (3), this is an arithmetic progression. The nth term of an arithmetic progression is given by t_n = a + (n – 1)d
Where a is first term and d is the common difference. Here a = 2 and d = 3

So, nth term = 2 + (n – 1) × 3 = 2 + 3n – 3 = 3n – 1
When we plot the ordered pairs emerging from the sequence, i.e„ (1, 2), (2, 5), (3, 8), (4, 11), (5, 14), and so on, where the x coordinate represents the term number and y coordinate represents the value of the corresponding term.



We observe that the points lie on straight lines. This is characteristic of arithmetic progressions.

(ii) -5, -1, 3, 7,… Solution: -5, -1, 3, 7, … First find the difference of two consecutive terms, -1 – (-5) = 4, 3 – (-1) = 4, 7 – 3 = 4,… and so on. Here we observe that the difference between consecutive terms is constant (4), so this is an arithmetic progression. The nth term of an arithmetic progression is t_n = a + (n – 1)d.
Where a is first term and d is the common difference. Here a = – 5 and d = 4
When we plot the ordered pairs emerging from the sequence i.e., (1, -5), (2, -1), (3, 3), (4, 7), and so on where the x coordinate represent the term number and y



We observe that the points lie on straight lines. This is characteristic of arithmetic progressions.

Exercise (Page 182)

Using the formula $t_n = a + (n – 1) × d$, find the nth term of the following arithmetic progressions.
(i) 12,52,92,132,…

Solution:
Given: 12,52,92,132,…
Here, first term (a) = 12 and common difference (d)
= 52−12=92−52=132−92
= 2
Using the formula $t_n = a + (n – 1) × d$
$t_n = 12 + (n – 1) × 2$
$t_n = 12 + 2n – 2$
⇒ $t_n = 2n – 32$

(ii) 1.5, 3.5, 5.5, 7.5,…

Solution:
Given: 1.5, 3.5, 5.5, 7.5, … Here, first term (a) = 1.5 and common difference (d) = 3.5 – 1.5 = 5.5 – 3.5 = 7.5 – 5.5 = 2 Using the formula $t_n = a + (n – 1) × d$
$t_n = 1.5 + (n – 1) × 2$
⇒ $t_n = 1.5 + 2n – 2$
⇒ $t_n = 2n – 0.5$

Exercise (Page 183)

Find recursive rules for the APs in the previous exercises.

Solution:
The formula for the recursive rules of an arithmetic progression: $t_n = t_{n-1}$ + c.d.
Where t_n = nth term, t_{n-1} = previous term, c.d. = common difference
(i) Given: 2, 5, 8, 11,… (Exercise Page 182)
Here first term (t_1) = 2
and common difference = 3
Thus the recursive rule is: $t_1 = 2$, $t_n = t_{n-1} + 3$ for $n \geq 2$

(ii) Given: -5, -1, 3, 7,… (Exercise Page 182) Here, first term (t_1) = -5
and common difference = 4
Thus the recursive rule is: $t_1 = -5$, $t_n = t_{n-1} + 4$ for $n \geq 2$.

(iii) Given: 12,… (Exercise Page 183)
Here, first term (t_1) = 12
and common difference = 2
Thus the recursive rule is:
$t_1 = 12$
$t_n = t_{n-1} + 2$ for $n \geq 2$

(iv) Given: 1.5, 3.5, 5.5, 7.5, … Here first term (t_1) = 1.5 and c.d. = 2
Thus the recursive rule is: $t_1 = 1.5$, $t_n = t_{n-1} + 2$ for $n \geq 2$

Think and Reflect (NCERT Textbook Page No. 184)

Can the same approach be used to find the sum of 1 + 2 + 3 + … + 100?

Solution:
Let S represent the sum of the first 100 natural numbers: S= 1 + 2 + 3 + … + 100 …(i) Now, write the sum in reverse order: S = 100 + 99 + 98 + … + 1 …(i) Add equation (i) and (ii), we get: S = 1 + 2 + 3 + … + 100 S = 100 + 99 + 98 + … + 1 Adding the two equations term by term: 2S = (1 + 100) + (2 + 99) + (3 + 98) + … + (100 + 1) Each pair of corresponding numbers adds up to 101. Since there are 100 terms, and each pair adds to 101. Thus, the sum is: 2S = 100 × 101 = 10100 ⇒ S = 101002 = 5050 Therefore, the sum of the first 100 natural numbers is 5050.

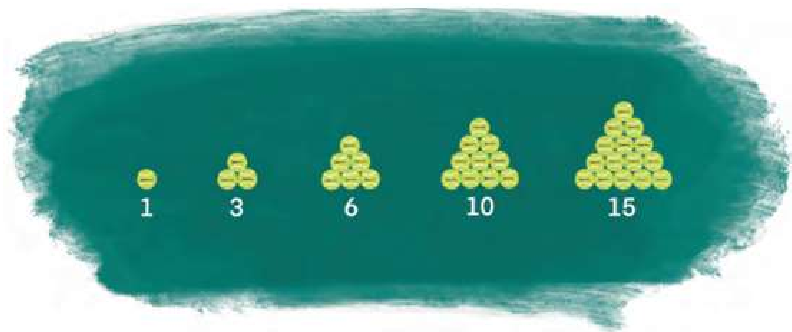
Think and Reflect (NCERT Textbook Page No. 185)

Can you use this formula to find S_{20}, S_{50} or S_{1000}?

Solution:
We know that S_n = W–y—
S_{20} = 20(20+1)2=20×212 = 210

S_2 = 50(50+1)2=50×512 = 1275
And S,ooo = 1000(1000+1)2=1000×10012 = 500500

Let us revisit the sequence t„ of triangular numbers 1, 3, 6, 10, 15,… shown in Figure. Note that the nth term of this sequence is the sum of the first n natural numbers. Thus t_n = n(n+1)2



Can you use this to find the 10th, 17th and 80th triangular numbers?

Solution:
Given, nth term of triangular number: t_n = n(n+1)2
10th triangular number: Substitute n = 10 into the given formula.
t_{10} = 10(10+1)2=10×112 = 55
17th triangular number: Substitute n = 17 into the given formula.
t_{17} = 17(17+1)2=17×182 = 153
80th triangular number: Substitute n = 80 into the given formula.
t_{80} = 80(80+1)2=80×812 = 3240
Thus, 10^{th}, 17^{th}, and 80^{th} triangular numbers are 55, 153, and 3240 respectively.

Think and Reflect (NCERT Textbook Page No. 186)

Can you predict the number of squares in Stages 5 and 6 of the pattern?
In Stages 10, 11 and 12?
In Stage 20?
At any stage?
How is this different from the growing pattern in Fig. 8.3?

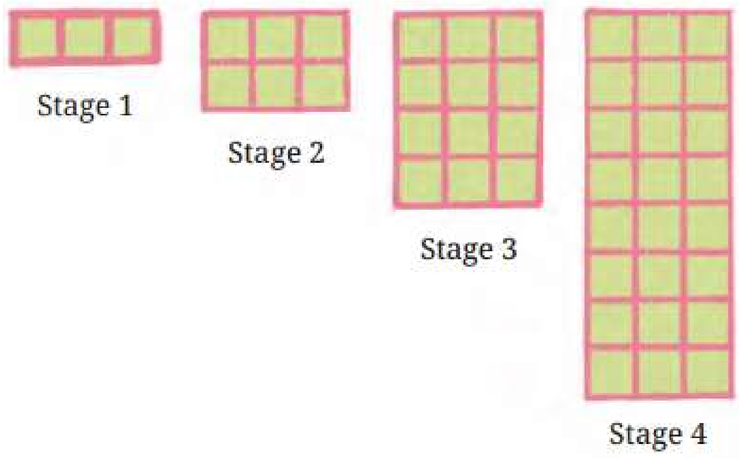


Figure: A growing pattern of squares

Solution:
We observe that the number of squares in each stage is increasing in a consistent pattern. Each number is double the previous number:
Number of squares in each stage:
Stage 1: 3 squares
Stage 2: 6 squares (3 × 2)
Stage 3: 12 squares (3 × 2 × 2 = 3 × 22)
Stage 4: 24 squares (3 × 2 × 2 × 2 = 3 × 23)
This suggests that the number of squares in each stage is multiplying by 2.
Following this pattern, we can predict the next terms:
Stage 5: $3 \times 2^4 = 3 \times 16 = 48$ squares
Stage 6: $3 \times 2^5 = 3 \times 32 = 96$ squares
Stage 10: $3 \times 2^9 = 3 \times 512 = 1536$ squares
Stage 11: $3 \times 2^{10} = 3 \times 1024 = 3072$ squares
Stage 12: $3 \times 2^{11} = 3 \times 2048 = 6144$ squares
Stage 20: $3 \times 2^{19} = 3 \times 524288 = 1572864$ squares Now, In Figure, the growth does not follow a strict multiplication rule. Instead, the pattern develops by adding squares to specific positions, rather than simply multiplying the number of squares. As the stages progress, new squares are symmetrically added to the existing ones, resulting in a unique form of growth that differs from the simple doubling observed in Figure

In conclusion, while both patterns exhibit growth, the one in Figure follows an exponential growth pattern, with the number of squares doubling at each stage. In contrast, the pattern in Figure involves a more intricate arrangement and addition of squares, rather than a straightforward multiplication process.

Exercise (Page 188)

Check whether the Jollowing sequences are geometric progressions and find their nth terms.
(i) 2, 10, 50, 250, ………

Solution:
Given sequence: 2, 10, 50, 250,… Here, t_1 = 2, t_2 = 10, t_3 = 50, t_4 = 250
Now, ratio of consecutive pairs of terms:
t2t1=102=5,t3t2=5010=5,t5t4=25050=5
Therefore, the common ratio is 5
And the n^{th} term = a × $r^{(n-1)}$ = 2 × $5^{(n-1)}$

(ii) 4, 83,169,3227,…

Solution:
Given sequence:
Here, t_1 = 4, t_2 = 83, t_3 = 169,t_4 = 3227
Now, ratio of consecutive pairs of terms.
12
Therefore, the common ratio is −12.
Therefore, the n^{th} term = a × $r^{(n-1)}$ = 2 × $(12)^{n-1}$

(iii) 3, −32,34,−38,…

Solution:
Given sequence: 3, −32,34,−38,…
Here, t_1 = 3, t_2 = 12, t_3 = 12, t_4 = 12
Now, ratio of consecutive pairs of terms:
12
Therefore, the common ratio is −12
And the n^{th} term = a × $r^{(n-1)}$ = 2 × $(23)^{n-1}$

Exercise (Page 188)

Can you find a recursive rule for the formula t_n = 3 × 10^{n-1} that generates the geometric progression 3, 30, 300, 3000, ?

Solution:
The given sequence is: 3, 30, 300, 3000, …
This is a geometric progression where:
The first term (q) is 3.
The common ratio (r) between consecutive terms is 303 = 10
Thus, each term is obtained by multiplying the previous term by 10.
In a geometric progression, the recursive formula follows the general pattern:
$t_n = t_{n-1} \times r$

Where: t_n is nth term
t_{n-1} is the previous term and r is the common ratio.
For the given sequence r = 10.
Therefore the recursive rule for this sequence will be:
$t_n = t_{n-1} \times 10$ for n > 1

Think and Reflect (NCERT Textbook Page No. 189)

Observe the Sierpinski triangle and try to answer the following questions (a) How many black triangles are there in Stages 0 to 3 of Fig. 8.7? (b) Can you predict the number of black triangles at Stages 4 and 5? (c) Can you find a rule for the number of black triangles at the nth stage? (d) Suppose the area of the triangle (that is, the black region) in Stage 0 is 1 square unit. What is the area of the black region in Stages 1,2 and 3? What will be the area of the black region in Stages 4 and 5? Find a rule for the area of the black region at the nth stage. What happens to this area as n, the number of stages, goes on increasing?

Solution:
(a) Let's count the number of black triangles in each stage: Stage 0: 1 = 3° = 1 black triangle. Stage 1: (1 × 3) = 3^1 = 3 black triangles.
Stage 2: (3 × 3) = 3^2 = 9 black triangles.
Stage 3: (9 × 3) = 3^3 = 27 black triangles. Now, total number of black triangles from Stage 0 to Stage 3: 1 + 3 + 9 + 27 = 40 black triangles. Thus, there are 40 black triangles in Stages 0 to 3.

(b) The number of black triangles follows a pattern where each stage has 3 times the number of triangles in the previous stage. Stage 4: 27 × 3 = 3^4 = 81 black triangles.
Stage 5: 81 × 3 = 3^5 = 243 black triangles. Thus, the number of black triangles at Stage 4 is 81 and at Stage 5 is 243.

(c) The number of black triangles at each stage forms a geometric progression where the first term is 1 (for Stage 0) and the common ratio is 3. Thus, the number of black triangles at the nth stage is given by: $t_n = 3^n$.

(d) Stage 0: The area of the black triangle is given as 1 sq. unit. Stage 1: The area of the black triangles is 34 of the area of Stage 0, so the area is 34 sq. unit.
Stage 2: The area of the black triangles is 34 area of Stage 1, so the area is 34×34=(34)2 sq. unit.

Stage 3: The area of the black triangles is 34 of the area of Stage 2, so the area is (34)3 sq. unit.
Therefore, the area at Stage n is given by: Area at Stage (A_n) = (34)n As n increases, the area of the black region becomes smaller and approaches 0 as the common ratio This shows that the area of the black region in the Sierpinski triangle decreases exponentially as the number of stages increases.

Exercise 8.1 Solutions

Question 1.
Find the first five terms of the sequence in which the nth term is given by
(i) $t_n = 3n – 4$,
(ii) $t_n = 2 – 5n$, and
(iii) $t_n = n^2 – 2n + 3$ for n > 1.

Solution:
(i) $t_n = 3n – 4$,
Substitute 1,2, 3,4, and 5 for zz in the given equation.
$t_1 = 3 × 1 – 4 = 3 – 4 = -1$
$t_2 = 3 × 2 – 4 = 6 – 4 = 2$
$t_3 = 3 × 3 – 4 = 9 – 4 = 5$
$t_4 = 3 × 4 – 4 = 12 – 4 = 8$
$t_5 = 3 × 5 – 4 = 15 – 4= 11$ Thus, first five terms of the given sequence are: -1, 2, 5, 8, 11

(ii) Given $t_n = 2 – 5n$,
Substitute 1, 2, 3, 4, and 5 for n in the given equation.
$t_1 = 2 – 5(1) = 2 – 5 = -3$
$t_2 = 2 – 5(2) = 2 – 10 = -8$
$t_3 = 2 – 5(3) = 2 – 15 = -12$
$t_4 = 2 – 5(4) = 2 – 20 = -18$
$t_5 = 2 – 5(5) = 2 – 25 = -23$ Thus, first five terms of the given sequence: -3, -8,-13,-18, -23

(iii) Given: $t_n = n^2 – 2n + 3$
$t_1 = 1^2 – 2(1) + 3 = 2$
$t_2 = 2^2 – 2(2) + 3 = 3$
$t_3 = 3^2 – 2(3) + 3 = 6$
$t_4 = 4^2 – 2(4) + 3 = 11$
$t_5 = 5^2 – 2(5) + 3 = 18$ Thus, first five terms of the given sequence are: 2, 3, 6, 11, 18

Question 2.
Find the 10th and 15th terms of the sequence $t_n = 5n – 3$ for n ≥ 1.

Solution:
Given: $t_n = 5n – 3$
$t_{10} = 5(10) – 3 = 47$
And $t_{15} = 5(15) – 3 = 72$ Thus, 10,h term = 47 and 15th term = 72.

Question 3.
Determine whether 97 and 172 are terms of the sequence $t_n = 5n – 3$ for $n \geq 1$.

Solution:
Given: $t_n = 5n – 3$
Substitute 97 for tn
97 = 5n – 3
⇒ 100 = 5n
⇒ n = 20
Thus, 97 is the 20th term of the given sequence.
Now, substitute t_n – 172 ∴ 172 = 5n – 3 ⇒ 175 = 5n ⇒ n = 35 Thus, 172 is the 35th term of the given sequence.

Question 4.
Which term of the sequence $t_n = 5n – 3$ for $n \geq 1$ is 607?

Solution:
Given: $t_n = 5n – 3$
Substitute 607 for t_n 607 = 5n — 3 ⇒ 610 = 5n Thus, 607 is 122th term of the given sequence.

Question 5.
A sequence is given by the recursive rule $t_1 = – 5$, $t_{n+1} = t_n + 3$ for $n \geq 1$. Find the first five terms of the sequence. Is 52 a term of this sequence? If so, which term is it?

Solution:
Given: $t_n = -5$, $t_{n+1} = t_n + 3$
First five terms of the sequence:
$t_1 = -5$
$t_2 = t_1 + 3 = – 5 + 3 = -2$
$t_3 = t_2 + 3 = -2 + 3 = 1$
$t_4 = t_3 + 3 = 1 + 3 = 4$
$t_5 = t_4 + 3 = 4 + 3 = 7$
Thus, first five terms are: -5, -2, 1, 4, 7
Now, check whether 52 is a term of this sequence:

General form: t_n = —5 + (n – 1).3
Substitute 52 for t_n 52 = -5 + (n – 1) × 3 ⇒ 57 = 3 (n – 1) ⇒ n – 1 = 19 ⇒ n = 20
So, 52 is a term and it is the 20th term of the given sequence.

Question 6.
Let T_1 = 1, T_2 = 2, T_3 = 4, and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for n > 4. Find T4, T5, T6, T7, and T8.

Solution:
Given: T_1 = 1, T_2 = 2, T_3 = 4
and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for n > 4
Now, $T_4 = T_3 + T_2 + T_1$
= 4 + 2 + 1
= 7
$T_5 = T_4 + T_3 + T_2$
= 7 + 4 + 2
= 13
$T_6 = T_5 + T_4 + T_3$
= 13 + 7 + 4
= 24
$T_7 = T_6 + T_5 + T_4$ = 24 + 13 + 7 = 44
$T_8 = T_7 + T_6 + T_5$ = 44 + 24 + 13 = 81
Hence, T_4 = 7, T_5 = 13, T_6 = 24, T_7 = 44, T_8 = 81

Exercise 8.2 Solutions

Question 1.
Find the 10^{th} and 26^{th} terms of the AP: 3, 8, 13, 18, …

Solution:
The given arithmetic progression (AP) is: 3, 8, 13, 18,…
Here, the first term (a) = 3
and the common difference (d) = 8 – 3 = 5.
The formula for the 77^{th} term of an AP is given by: t_n = a + (n – 1).d
For 10th term:
Substitute n = 10. a = 3, and d = 5 into the formula:
t_{10}= 3 + (10 – 1) × 5 = 3 + 9 × 5 = 3 + 45 = 48 Thus, the 10,h term is 48.

For 26th term: Substitute n = 26, a = 3, and d = 5 into the formula: t_{26} = 3 + (26 – 1) × 5 = 3 + 25 × 5 = 3 + 125 = 128 Thus, the 26th term is 128.

Question 2.
Which term of the AP : 21, 18, 15,… is – 81? Also, is 0 a term of this AP? Give reasons for your answer.

Solution:
The given AP is: 21, 18, 15, … Here, a = 21 and d = 18 – 21 = -3. The formula for the w,h term of an AP is given by: t„= a + (n – 1).d To find which tenn is -81, substitute t_n = -81, a = 21, and d = -3 into the formula: -81 = 21 + (n – 1) × (-3) Solving for n: n = 34 + 1 =35 Thus, the 35th term of the AP is -81.

To find if 0 is a term of this AP, substitute t_n = 0, a = 21, and d = – 3 into the formula:
∴ 0 = 21 + [(n – 1) × (-3)]
⇒ 0 – 21 = (n – 1) × (-3)
Solving for n:
n – 1 = 0−21−3 = 7 n = 7 + 1 = 8 Thus, the 8th tenn of the AP is 0. Therefore, 0 is a term of this AP.

Question 3.
Find the 77^{th} term of the AP: 11, 8, 5, 2 … Write the recursive rule for this AP.

Solution:
The given AP is: 11, 8, 5, 2,…
Here, a = 11 and d = 8 – 11 = -3.
The formula for the n^{th} term of an AP is given by: t„= a + (n – \).d
Substitute a = 11 and d = – 3 into the formula:
t_n = 11 + (n – 1) × (-3)
= 11 – 3n + 3
= 14 – 3n
Thus, the 77^{th} term of the AP is:
t_n = 14 – 3n
⇒ t_1 = 11
t_2 = 14 – 6 = 8 = 11 – 3
t_3 = 14 – 9 = 5 = 8 – 3
∴ The recursive rule for this AP is: t_1 = 11
$t_n = t_{n-1} + (-3) = t_{n-1} - 3$ for n ≥ 2.

Question 4.
An AP consists of 50 terms in which the 3rd term is 12 and the last term is 106. Find the 29th term. (Hint: If 'a' is the first term and 'd' the common difference, then we arrive at the equations a + 2d = 12 and a + 49d = 106. Solve this pair of linear equations for 'a' and 'd'.)

Solution:
Let the first term be a, and the common difference be d. Given: The 3rd term t_3 = 12, and the 50th term t_{50} = 106.

We can use the formula for the nlh term of an AP: $t_n = a + (n - 1).d$
From $t_3 = 12$, we get the equation:
a + 2d = 12 ….(i)
From $t_{50} = 106$, we get the equation:
a + 49d= 106 …(ii)
Subtract (i) from (ii)
(a + 49d) – (a + 2d) = 106 – 12
⇒ 47d = 94
⇒ d = 9447 = 2
Substitute d= 2 into (i):
a + 2 × 2 = 12 ⇒ a = 12 – 4 = 8
Thus, a = 8 and d = 2.
Now, substitute a = 8 and d = 2, n = 29 into the formula for tn:
⇒ t_{29} = 8 + (29 – 1) × 2 = 8 + 28 × 2 = 8 + 56 = 64 Thus, the 29th term is 64.

Question 5.
How many 2-digit numbers are divisible by 3? What is the sum of all these 2-digit numbers?

Solution:
The 2-digit numbers divisible by 3 form an arithmetic progression, where the first 2-digit number divisible by 3 is 21. And the sequence is: 12, 15, 18,…, 99.
The first tenn (a) = 12, the last term (f) = 99 and common difference (d) = 3.
We use the formula for the n^{th} term of an AP:
$t_n = a + (n - 1)d$
Substitute tn = 99, a = 12, and d = 3 into the formula:
99 = 12 + (n – 1) × 3
⇒ 99 – 12 = (n – 1) × 3
⇒ 87 = (n – 1) × 3
⇒n – 1 = 873 = 29 ⇒ n = 29 + 1 = 30 Thus, there are 30 terms, i.e., numbers.
Since there are 30 terms in the sequence, so. 15 pairs can be made. The sum of each pair is (12 + 99) + (15 + 96) + (18 + 93) + … Each pair sum to 111, and there are 15 such pairs. Thus, the total sum is: 15 × 111 = 1665 Therefore, the sum of all the 2-digit numbers divisible by 3 is 1665.

Question 6.
Harish started work at an annual salary of ₹ 5,00,000 and received an increment of ?20,000 each year. After how many years did his income reach ₹ 7,00,000?

Solution:
The salary increases by ₹ 20,000 each year, so this forms an arithmetic progression. The first term (a) = ₹ 5,00,000 and the common difference (d) =

₹20,000. The formula for the 77^{th} term is: $t_n = a + (n – 1)d$
Substitute t„ = ₹ 7,00,000, a = ₹ 5,00,000,
and d = ₹ 20,000: 7,00,000 = 5,00,000 + (n – 1) × 20,000
⇒ 2,00,000 = (n – 1) × 20,000
⇒ n – 1 = 20000020000 = 10 ⇒ n = 10 + 1 = 11 Thus, Harish's income will reach ₹ 7,00,000, in 11th year, after 10 years.

Question 7.
A child arranges marbles in rows so that the first row has 1 marble, the second has 2 marbles, the third has 3, and so on up to 25 rows. How many marbles does the child use in all?

Solution:
The number of marbles in each row forms an arithmetic progression. The first term (a) = 1, the common difference (d) = 1, number of terms (n) = 25 So the arithmetic progression is: 1 + 2 + 3 + …+ 25 This is the sum of first 25 natural numbers. The sum of the first n natural number is given by: n(n+1)2
Substitute n = 25,
S_{25} = 25(25+1)2=25×262 = 325 Thus, the child uses 325 marbles in all.

Exercise 8.3 Solutions

Question 1.
Find the 12^{th} term of a GP with common ratio 2, whose 8th term is 192.

Solution:
We know that nth term of GP is: $t_n = a \times r^{(n-1)}$
Given: 8th term = 192 and common ratio = 2
$192= a \times 2^{8-1}$
$\Rightarrow 192 = a \times 2^7$
⇒ 192 = a × 128
⇒ a = 12
Now, 12^{th}term
$t_{12} = 32 \times 2^{12-1} = 12 \times 2^{11}$
$=3 \times 2^{10}$
$= 3 \times 10^{24} = 3072$

Question 2.
Find the 10^{th} and n^{th} terms of the GP: 5, 25, 125,….

Solution:
Given GP: 5, 25, 125,…
We know that n^{th} term of GP is: $t_n = a \times r^{(n-1)}$
For 10th term, substitute n = 10 in the above formula.

tl0 = $5 \times 5^{10-1} = 5 \times 5^{9} = 9765625$ [∵ a = 5, r = 5]
The n^{th} term is given by: $t_n = 5 \times 5^{n-1} = 5^{n}$
Thus, the n^{th} term is 5^{n}.

Question 3.
A sequence is given by the recursive rule $t_1 = 2$, $t_{n+1} = 3t_{n-2}$ for n ≥ 1. Which term of the sequence is 730?

Solution:
We are given the recursive rule:
$t_1 = 2$, $t_{n+1} = 3t_{n-2}$ for n ≥ 1
To find which term is 730.
We will find the terms of the sequence to find t_n.
$t_1 = 2$
$t_2 = t_{1+1} = 3t_1 - 2 = 3 \times 2 - 2 = 4$
$t_3 = t_{2+1} = 3t_2 - 2 = 3 \times 4 - 2= 10$
$t_4 = t_{3+1} = 3t_3 - 2 = 3 \times 10 - 2 = 28$
$t_5 = t_{4+1} = 3t_4 - 2 = 3 \times 28 - 2 = 82$
$t_6 = t_{5+1} = 3t_5 - 2 = 3 \times 82 - 2 = 244$
$t_7 = t_{6+1} = 3t_6 - 2 = 3 \times 244 - 2 = 730$ Thus, the term 730 is the 7th term of the sequence.

Question 4.
Which term of the GP: 2, 6, 18, … is 4374? Write the explicit formula as well as the recursive formula for the 17th term.

Solution:
The given geometric progression (GP) is: 2, 6, 18,… Here, the first term (a) = 2, common ratio (r) = 3. The nth term formula for a geometric progression is: $t_n = a \times r^{(n-1)}$
We need to find the term $t_n = 4374$.
Substitute a = 2 and r = 3 into the formula:
$4374 = 2 \times 3^{n-1}$
$\Rightarrow 2187 = 3^{n-1}$
$\Rightarrow 37 = 3^{n-1} \Rightarrow n - 1 = 7 \Rightarrow n = 8$ Thus, the term 4374 is the 8th term of the given GP.

Question 5.
A ball is dropped from a height of 80 metres. After hitting the ground, it bounces back to 60% of the height from which it fell. It continues bouncing in this way — each time rising to 60% of the previous height. (i)

What height does the ball reach after the 5th bounce? (ii) What is the total vertical distance the ball has travelled by the time it hits the ground for the 6th time?

Solution:
(i) The height after each bounce forms a geometric progression. The first term is a = 80 metres, and the common ratio is r = 0.6. The nth term of the GP: $t_n = 80 \times (0.6)^{n-1}$
Therefore, the height of the ball after 5th bounce $t_6 = 80 \times (0.6)^{6-1}$ = 80 × (0.6)5 = 6.2208 m. Thus, the ball reaches a height of approximately 6.22 metres after the 5th bounce.

(ii) The total distance travelled is the sum of the distances fallen and risen. The ball falls 80 metres initially, then rises to 60% of the previous height, and so on.
$t_1 = 80$
$t_2 = 80 \times 0.6 = 48$ (after first bounce)
$t_3 = 48 \times 0.6 = 28.8$ (after second bounce)
$t_4 = 28.8 \times 0.6 = 17.28$ (after third bounce)
$t_5 = 17.28 \times 0.6 = 10.368$ (after fourth bounce)
$t_6 = 10.368 \times 0.6 = 6.2208$ (after fifth bounce) The total distance is given by the sum of the first 6 terms of a geometric series: 80 + 2 × (48 + 28.8 + 17.28 + 10.368 + 6.2208) = 301.3376 m. So, the total vertical distance the ball has travelled is approx. 301.34 m.

Question 6.
Which term of the sequence 2, 2√2, 4, … is 128?

Solution:
Given sequence: 2, 2√2, 4,… Here, first term (a) = 2 and common ratio (r) = 22√2=422√ = √2
Therefore, the nth term of GP: $t_n = a \times r^{(n-1)}$
Now, $t_n = 128$
⇒ $128 = 2 \times (\sqrt{2})^{n-1}$
⇒ $64 = (\sqrt{2})^{n-1}$
⇒ (2–√)12=(2–√)n−1 ⇒ n – 1 = 12 ⇒ n = 13 Thus, the term 128 is the 13th term of the sequence.

Question 7.
Figure shows Stages 0 to 3 of the Sierpinski square carpet. Stage 0 of this fractal is a square sheet of paper. To construct Stage 1, each side of the square is trisected and the points of trisection of opposite sides are joined to obtain nine smaller squares. The centre square is then removed

and the 8 smaller squares are retained, leaving a square hole in the centre. The same process is repeated on the eight smaller shaded squares to obtain Stage 2 and so on.

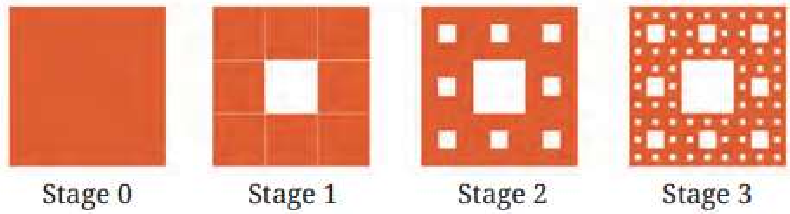


Figure : Stages 0, 1, 2 and 3 of the Sierpiński square carpet
Look at Figure and try to answer the following questions.
(i) How many red squares are there in Stages 0 to 3?
(ii) Can you predict the number of red squares in Stages 4 and 5?
(iii) Can you find a rule for the number of red squares at the n^{th} stage? Write the explicit formula as well as the recursive formula for the number of red squares at any stage. (iv) Suppose the area of the square in Stage 0 is 1 square unit. What is the area of the red region in Stages 1, 2 and 3? What will be the area of the red region in Stages 4 and 5? Find the explicit as well as the recursive formula for the area of the red region at the nth stage. What happens to this area as n, the number of stages, goes on increasing?

Solution:
(i) Stage 0: There is 1 red square. Stage 1: The square is divided into 9 smaller squares, and 1 square is removed from the center, leaving 8 red squares. Stage 2: Each of the 8 smaller red squares in Stage 1 is divided further into 9 smaller squares, and 1 square is removed from each, leaving 64 red squares. Stage 3: Each of the 64 smaller red squares in Stage 2 is further divided into 9 smaller squares, and 1 square is removed from each, leaving 512 red squares. Now, we sum up the number of red squares from Stages 0 to 3: 1 (Stage 0) + 8 (Stage 1) + 64 (Stage 2) + 512 (Stage 3) = 585 Thus, there are 585 red squares in Stages 0 to 3.

(ii) The pattern observed is that the number of red squares at each stage is increasing by a factor of 8: Stage 0: 1 red square. Stage 1: 8 red squares. Stage 2: 64 red squares. Stage 3:512 red squares. Thus, we observe that the number of red squares is being multiplied by 8 at each stage. We can predict the following as: Stage 4:512 × 8 = 4096 red squares. Stage 5: 4096 × 8 = 32768 red squares. Thus, the number of red squares at Stage 4 is 4096 and at Stage 5 is 32768.

(iii) Based on the pattern observed, the number of red squares at each stage is given by the formula: t„ = 8″ Where t„ is the number of red squares at Stage n. This is because the number of red squares at each stage is being multiplied by 8 compared to the previous stage. Thus, the recursive formula is: t_0 = 1 (Stage 0)
$t_{n+1} = t_n \times 8$ for $n \geq 0$

(iv) The total area of the square in Stage 0 is 1 square unit. As the process of forming the Sierpiriski carpet progresses, each red square occupies a fraction of the previous stage's area.

In Stage 1, 8 smaller red squares are left from the initial square. The area of each red square is 19 the original area (since the square is divided into 9 smaller squares). Thus, the total area of the red region in stage 1 is:
Area of red region in Stage 1 = 8 × 19=89 square unit.

In Stage 2, each of the 64 smaller red squares in stage 1 is divided into 9 smaller squares, and 1 square is removed. The area of each red square is 19 of the area of the each previous red square. Thus, the total area of the red region in stage 2 is:
Area of red region in stage 2 = 64 × ((19)2)
= 6481 square unit = (89)2 sq. units.

In Stage 3, each of the 512 smaller red squares in Stage 2 is divided into 9 smaller squares. The area of each red square is 19 of the area of the each previous red square. Thus, the total area of the red region in Stage 3 is:
Area of red region in stage 3
= 512 × (19)3=512729=(89)3 square units

- Explicit formula = A_n = (89)n
- Recursive formula = A_n = 89A_{n-1} where A_0 = 1.

The area of the red region at each stage follows the same pattern, decreasing by a factor of as we move from one stage to the next. As the number of stages n goes to infinity, the term (89)n approaches to zero.

End of Chapter Exercise Solutions

Question 1.
Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.

Solution:
We are given an arithmetic progression (AP) where: The 11^{th} term t_{11} = 38,

The 16^{th} term t_{16} = 73.
We need to find the 31 st term.
Let a be the first term and d be the common difference.
t_{11} = a + (11 – 1)d
⇒ 38 = a + 10d …(i)
And = a + (16 – 1)d …….(ii)
⇒ 73 = a+ 15d …(iii)
Subtract equation (i) from equation (ii):
(a+ 15d) – (a + 10d) = 73 – 38
⇒ 5d = 35
⇒ d = 7
Substitute d = 7 into the equation (i)
t_n = a + 10d = 38
⇒ a + 10 × 7= 38
⇒ a + 70 = 38
⇒ a = 38 – 70 = -32
Thus, the first term a = -32 and the common difference d = 7.
t_{31} = a + (31 – 1) × d = -32 + 30 × 7 = -32 + 210 = 178. Thus, the 31st term is 178.

Question 2.
Determine the AP whose third term is 16 and whose 7th term exceeds the 5th term by 12.

Solution:
Let a be the first term and d be the common difference. Since, = a + (3 – 1)d
⇒ 16 = a + 2d …(i) And 7th term exceeds the 5th term by 12 (Given) So, $t_7 - t_5$ = 12
Now, t_5 = a + (5 – 1)d = a + 4d
And, t_7 = a + (7 – 1)d = a + 6d ⇒ (a + 6d) – (a + 4d) = 12 ⇒ 2d = 12 ⇒ d = 6
Substitute the value of d in equation (i), we get 16 = a + 2 × 6 ⇒ a = 4 Thus, the required AP is: 4, 10, 16, 22, 28, 34, 40,…

Question 3.
How many three-digit numbers are divisible by 7? (Hint: All three-digit numbers divisible by 7 form an AP. Find the smallest and largest such three-digit numbers.)

Solution:
The smallest three-digit number divisible by 7 is 105 and the largest three-digit number divisible by 7 is 994. The terms form an arithmetic progression: Here, first term (a) = 105, common difference (d) = I and last term (t_n) = 994.
n_{th} term of an AP: t_n = a + (n – 1)d

Substitute the value of a, d and t_n: ⇒ 994 = 105 + (n – 1) × 7 ⇒ 994 – 105 = (n – 1) × 7 ⇒ n – 1 = 127 ⇒ n = 128 Thus, there are 128 three-digit numbers divisible by 7.

Question 4.
How many multiples of 4 lie between 10 and 250? (Hint: All multiples of 4 form an AP. Find the smallest and largest multiples of 4 between 10 and 250.)

Solution:
Smallest multiple of 4 greater than 10: 12 And largest multiple of 4 less than 250: 248 Thus, the AP is: 12, 16, 20, …,248. Now, t_n = a + (n – 1).d
Here, tn = 248, a = 12
and d = 4
∴ 248 = 12 + (n – 1) x 4
⇒ 248 – 12 = (n – 1) x 4
⇒ 2362 = n – 1 ⇒ n = 59 + 1 = 60 Thus, there are 60 multiples of 4 between 10 and 250.

Question 5.
Find a GP for which the sum of the first two terms is – 4 and the fifth term is 4 times the third term.

Solution:
Let the first term of GP is a and common ratio is r. :. t_n = a,t_2 = ar, t_3= ar^2, and t_5 = ar^4
Given: a + ar =-A …(i)
And ar^4 = 4 × ar^2 …(ii)
⇒ r^2 = 4
⇒ r = -2 or 2
Substitute r = 2 in equation (;):
a + a × 2 = -4
⇒ a(1 + 2) = —4
⇒ 3a = -4
⇒ a = –43
Thus, the GP is, −43,−83,−163………… Now, substitute r = -2 in equation (i):
a + a × (-2) = -4 ⇒ a – 2a = -4 ⇒ -a = -4 ⇒ a = 4 Thus, the GP is, 4,-8, 16,….

Question 6.
Find all possible ways of expressing 100 as the sum of consecutive natural numbers.

Solution:
Let the consecutive natural numbers be a, a + 1, a + 2,…, a + (k- 1). So, a + (a + 1) + (a + 2) + … + a + (k — 1) = 100 ⇒ ka + k(k−1)2 = 100 ⇒ 2ka + k(k- 1)= 200 ⇒ k(2a + k – 1)= 200 If k is odd, then (2a + k – 1) is even + even = even If, k is even, then (2a + k – 1) is odd. Writing 200 = odd × even (combination): ⇒ 200 = 2 × 2 × 2 × 5 × 5 = 8 × 25 or 5 × 40 or 1 × 200 When k = 8, (2a + k – 1) = 25 ∴ 2a + 8 – 1 = 25 ⇒ a = 9 Thus, the numbers are: (9, 10, 11, 12, 13, 14, 15, 16) When k= 5, (2a + k – 1) = 40. 2a + k – 1 = 40 ⇒ a = 18 Thus, the number are: (18, 19, 20, 21,22) When k = 1, (2a + it- 1) = 200 2a + k – 1 = 200 ⇒ a = 100 Thus, the only number is 100. Thus, the possible combinations are: (9, 10, 11, 12, 13, 14, 15, 16) and (18, 19, 20,21,22).

Question 7.
The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of the 2nd hour, 4th hour and «th hour?

Solution:
This is a geometric progression (GP) with the first term (a) = 30 and the common ratio (r) = 2. The number of bacteria after n hours is: $a \times r^n$
The number of bacteria after 2 hours:
30 × 22 = 30 × 4= 120.
The number of bacteria after 4 hours:
30 × 24= 30 × 16 = 480.
Therefore, the number of bacteria after n hours: 30×2^n.

Question 8.
The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Solution:
Let the first term be a and the common difference be d. The general formula for the n^{th} term of an AP is: tn = a + (n – 1).d
Given: $a_4 + a_8 = 24$ …(i)
and $a_6 + a_{10} = 44$ …(ii)
Since, $a_4 = a + 3d$
$a_8 = a + 7d$
and $a_6 = a + 5d$
$a_{10} = a + 9d$ Substitute these values in equation (i) and (ii) a + 3d + a + 7d = 24 ⇒ 2a + 10d= 24 ⇒ a + 5d = 12 …(iii) Now, a + 5d + a + 9d = 44 ⇒ 2a + 14d= 44 ⇒ a + 7d = 22 … (iv) Subtract equation (iii) from equation (iv), we get a + 7d

– a – 5d = 22 – 12 ⇒ 2d = 10 ⇒ d = 5 Substitute the value of d in equation (iii), a + 5 × 5= 12 ⇒ a + 25= 12 ⇒ a = -13 Thus, the first three terms of the AP are: -13, -8, -3.

Question 9.
Find the smallest value of n such that the sum of the first n natural numbers is greater than 1,000.

Solution:
The sum of first n natural number: S_n = n^n + ^ ^
Sum of first n natural number greater than 1000.
So, $S_n > 1000$
⇒ 12 > 1000
⇒ n(n+ 1) > 2000
Since n(n + 1) is very close to n^2 for larger value of n, we can estimate by finding the square root of 2000.
We know that: $40^2 = 1600$, $45^2 = 2025$, the value n must be very close to 45.
Now we test the integers around our estimate of 45:
If n = 44: S_{44} = 12 = 990
If n = 45: S_{45} = 12 = 1035 This is the first sum that exceeds 1000. So, the smallest value of n is 45.

Question 10.
Which term of the GP: 2, 8, 32, … is 131072? Write the explicit formula as well as the recursive formula for the n^{th} term.

Solution:
The given geometric progression (GP) is: 2, 8, 32,…
Here, the first term a = 2,
and the common ratio r = 82 = 4.
The explicit formula for the «th term of a GP is given by:
$T_n = a.r^{n-1}$
Substitute the value of a and r:
$T_n = 2 \times 4^{n-1}$
Thus, the explicit formula for «th term: $T_n = 2 \times 4^{n-1}$
Now, Let $T_n = 131072$
=> $131072 = 2 \text{ x } 4^{n-1}$
⇒ $4^{n-1} = 65536$
⇒ $4^{n-1} = 4^8$ ⇒ n – 1 = 8 ⇒ n = 9. Thus, the 9th term of the given GP is 131072.

Recursive formula for a GP is: $T_n = r \times T_{n-1}$
Substitute the common ratio r = 4.
$T_n = 4 \times T_{n-1}$, where $T_1 = 2$, and n > 1.

Question 11.
The sum of the first three terms of a GP is 1312 and their product is -1. Find the common ratio and the terms.

Solution:
Let the first term of GP be a and common ratio be r.
So, first three terms of GP are: a, ar, ar^2
We are given that: a + ar + ar^2 = 1312
⇒ a(1 + r + r^2) = 1312 ….. (i)
And a.ar.ar^2 = -1
⇒ $(ar)^3$ = -1
⇒ a = – 1r …..(ii)

Substitute the value of a in equation (i): – 1r(1 + r + r^2) = 1312
⇒ 1 + r + r^2 = –13r12
⇒ 12 + 12r + $12r^2$ = -13r
⇒ $12r^2$ + 25r + 12 = 0
⇒ (3r + 4)(4r + 3) = 0
⇒ r = –43 or r = −34

Case 1: When r = −43 in equation (ii)
a = −1(−43)
⇒ a = 34
Therefore, the first three term if the GP are 34, -1, 43

Case 2: When r = −34
Substitute r = −34 in equation (ii)
The first three term of the GP are:43,−1,34

Question 12.
If the 4th, 10th and 16th terms of a GP are x, y and z respectively, prove that x, y, z are in GP.

Solution:
Let a be the first term and r be the common ratio of the GP Then nth term is: T_n = a x r^{n-1}
For the 4^{th}, 10^{th} and 16^{th} terms:
x = a × r^3
y = a × r^9
z = a × r^{15}
To prove that x, y, z are in GP, we need to show that the ratio of consecutive terms is constant:

$$\text{i.e., } \frac{y}{x} = \frac{z}{y}$$

$$\frac{y}{x} = \frac{a.r^9}{a.r^3} = r^6$$

$$\text{And } \frac{z}{y} = \frac{a.r^{15}}{a.r^9} = r^6$$

$$\text{Since, } \frac{y}{x} = \frac{z}{y} = r^6.$$

Hence, the terms are x, y and z are in GP.

Question 13.
The sum of the first three terms of a geometric progression is 26, and the sum of their squares is 364. Find the terms of the GP.

Solution:
Let a be the first term and r be the common ratio of GP. First three term of GP be: a, ar, ar2 Given: Sum of first three terms is 26 and sum of the squares of first three terms is 364. ∴ $a + ar + ar^2 = 26$ …(i)
⇒ $a(1 + r + r^2) = 26$
And $a^2 + a^2r^2 + a^2r^4 = 364$ …(ii)
⇒ $a^2(1 + r^2 + r^4) = 364$
Square both sides of equation (i)
$(a + ar + ar^2)^2 = 26^2$
⇒ $(a^2 + a^2r^2 + a^2r^4) + 2 a^2(r + r^2 + r^3) = 676$
⇒ $364 + 2a^2r(1 +r + r^2) = 676$
⇒ $2(ar)(a)(1 + r + r^2) = 676 – 364 = 312$
⇒ $2(ar) × 26 = 312$ [∵ $a(1 + r + r^2) = 26$]
ar = = 6
So, the middle term is 6.
Since ar = 6
⇒ a = 6r
Substitute the value of a in (i)
6r + 6 + 6r = 26
⇒ 6r + 6r = 26 – 6 = 20
⇒ 3r + 3r = 10
⇒ $3 + 3r^2 =10r$
⇒ $3r^2 – 10n + 3 = 0$
⇒ (3r – 1) (r – 3) = 0
⇒ r = 3 or r = 13
If r = 3, the terms are: 6 ÷ 4, 6, 6 × 3 ⇒ 2, 6, 18
If r = 13, the terms are: 6 ÷ 13, 6, 6 × 13 ⇒ 18, 6, 2 Thus, the terms of the GP are 2, 6, and 18.

Question 14.
Suppose $P_1 = 1$, $P_2 = 2$ and for $n > 2$, $P_n = P_1 + P_2 + \ldots + P_{n-1} + 1$. Find the values of $P_1, P_2, \ldots, P_8$. Can you find a simpler recursive formula for P_n? Can you give an explicit formula?

Solution:
Given: $P_n = P_1 + P_2 + \ldots + P_{n-1} + 1$
Using this formula, calculate the values for P_1 through P_8:
$P_1 = 1$ [Given]
$P_2 = 2$ [Given]
$P_3 = P_1 + P_2 + 1 = 1 + 2 + 1 = 4$
$P_4 = P_1 + P_2 + P_3 + 1 = 1 + 2 + 4 + 1 = 8$
$P_5 = P_1 + P_2 + P_3 + P_4 + 1 = 1 + 2 + 4 + 8 + 1 = 16$
$P_6 = P_1 + P_2 + P_3 + P_4 + P_5 + 1 = 1 + 2 + 4 + 8 + 16 + 1 = 32$
$P_7 = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + 1 = 1 + 2 + 4 + 8 + 16 + 32 + 1 = 64$
$P_8 = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + 1 = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 1 = 128$
Thus, the value of P_1 to P_8 are: 1, 2, 4, 8, 16, 32, 64 and 128.
Simpler Recursive Formula: We have,
$P_n = P_1 + P_2 + \ldots\ldots. + P_{n-2} + P_{n-1} + 1$ ……..(i)
$P_{n-1} = (P_1 + P_2 + \ldots\ldots\ldots. + P_{n-2}) + 1$.(ii)
Substitute the (ii) in (i)
$P_n = (P_{n-1} - 1) + P_{n-1} + 1$
$\Rightarrow P_n = 2P_{n-1}$ (for $n > 2$)
This means that starting from P_2, every term is simply double the previous term.
Explicit Formula: Looking at the sequence: 1, 2, 4, 8, 16, 32, 64, 128,…
We can see that these are all powers of 2.
$P_1 = 2^\circ = 1$
$P_2 = 2^1 = 2$
$P_3 = 2^2 = 4$
The explicit formula for any term n is:
$P_n = 2^{n-1}$ for ($n \geq 2$)

Question 15.
Suppose $W_1 = 1$, $W_2 = 2$ and for $n > 2$, $W_n = W_1 + W_2 + \ldots + W_{n-2} + 2$. Find the values of $W_1, W_2, \ldots, W_8$. Do you recognise this sequence?

Solution:
Given: $W_n = W_1 + W_2 + \ldots + W_{n-2} + 2$
Using this formula, calculate the values for Wj through W8:
$W_1 = 1$ [Given]
$W_2 = 2$ [Given]

$W_3 = W_1 + 2 = 1 + 2 = 3$

$W_4 = W_1 + W_2 + 2 = 1 + 2 + 2 = 5$

$W_5 = W_1 + W_2 + W_3 + 2 = 1 + 2 + 3 + 2 = 8$

$W_6 = W_1 + W_2 + W_3 + W_4 + 2 = 1 + 2 + 3 + 5 + 2 = 13$

$W_7 = W_1 + W_2 + W_3 + W_4 + W_5 + 2 = 1 + 2 + 3 + 5 + 8 + 2 = 21$

$W_8 = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + 2 = 1 + 2 + 3 + 5 + 8 + 13 + 2 = 34$

Thus, the values ofW, and W8 are: 1, 2,3, 5, 8, 13,21, 34 The sequence is a variation of the Fibonacci sequence with starting from the second 1; i.e., the sequence is of type 1, 1, 2, 3, 5, 8, 13, 21, 34,…