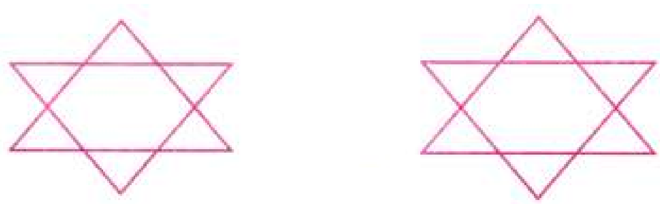
Geometric Twins Class 7 Notes Maths Part 2 Chapter 1

In our day-to-day life, we come across many objects that have the same shape and size. For example, two biscuits from the same packet, two sheets of the same letter pad, two five-rupee coins, two shaving blades of the same brand, etc. Such objects that have the same shape and size are called congruent objects. Congruent objects are a photocopy of each other, or we can say that congruent objects are identical in all respects.

Now take two ten-rupee notes or two stamps of the same shape and size and place one on the top of the other. In each case, we will observe that the two objects coincide with each other exactly. We say that the objects are congruent. This method of examining congruence by placing one object on the other is called the method of superposition.

Congruent Figures

In geometry, any two figures that are exactly of the same (identical) shape and size are called congruent figures. This property of two figures being congruent (identical) is called congruence.
It is denoted by the symbol $\cong$ and is read as 'is congruent to'.
In the figure below, shapes (i) and (ii) are exactly of same shape and size.
So shape (i) $\cong$ shape (ii); that is, shape (i) is congruent to shape (ii).



(i) (ii)

Congruent figures are photocopies of each other. Two plane figures are congruent if one figure is exactly superimposed on the other.

Congruence of Simple Geometrical Shapes

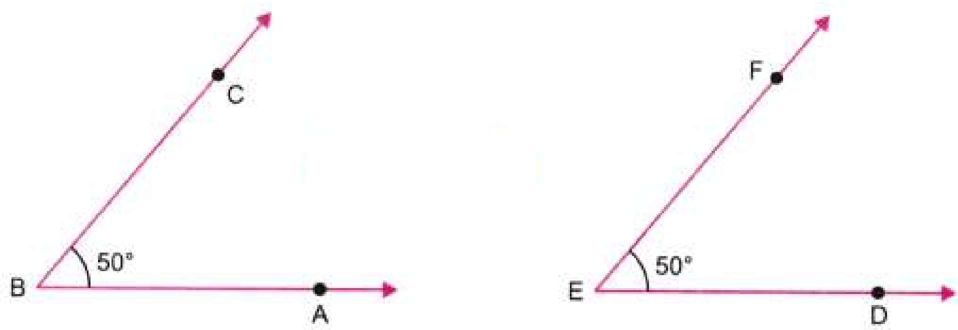
(a) Congruence of two line segments: If two line segments have the same (that is, equal) length, they are said to be congruent.
Let $\overline{AB}$ and $\overline{CD}$ be two line segments, each of 6 cm length.

We write it as $\overline{AB} \cong \overline{CD}$ (read as $\overline{AB}$ is congruent to $\overline{CD}$) because both $\overline{AB}$ and $\overline{CD}$ have the same length (here 6 cm). $\overline{AB} \cong \overline{CD}$ can also be written as:
$\overline{AB} = \overline{CD}$ because equality of lengths of two line segments is the only criterion for two line segments to be congruent.
Let us learn this congruence of line segments $\overline{AB}$ and $\overline{CD}$ by the 'trace-copy' superposition method.

If we trace the line segment AB¯ on a tracing paper and place it on CD¯ such that the point A falls on C and AB¯ falls along CD¯, then we will find that B falls on D that is line segment AB¯ will coincide with (cover) CD¯. Hence AB¯ is congruent to CD¯.

However, if we take segment AB¯ = 5 cm and CD¯ = 7 cm and repeat the above activity of superposition for line segments AB¯ and CD¯, we will find that AB¯ does not coincide with CD¯. Hence, AB¯ is not congruent to CD¯. If two line segments are congruent, they have the same length.

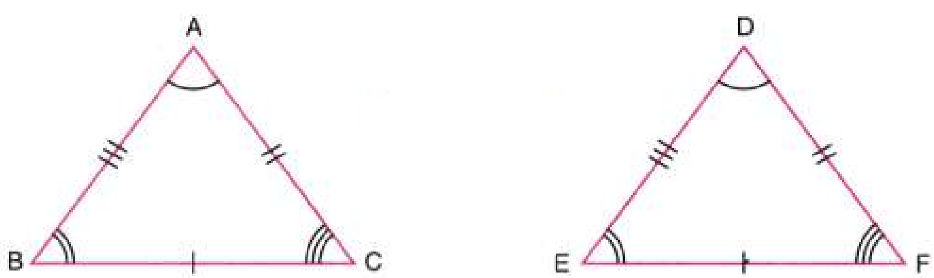
(b) Congruence of two angles: If two angles have the same (that is, equal) measure, they are said to be congruent. Let ∠ABC and ∠DEF be two angles each of measure, say 50°.



We write it as ∠ABC ≅ ∠DEF (read as ∠ABC is congruent to ∠DEF) because both angles have the same measure (= 50°). ∠ABC ≅ ∠DEF can also be written as ∠ABC = ∠DEF because equality of measures of two angles is the only criterion for two angles to be congruent.

Congruence of Triangles

We have learnt that each triangle has six parts – three sides and three angles. So, for the congruence of two triangles, all six parts of one triangle must be congruent (equal) to the six corresponding parts of the other triangle.



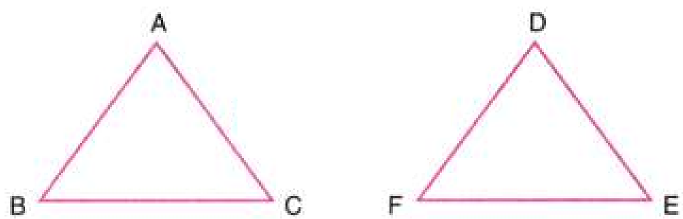
Let us consider two triangles ΔABC and ΔDEF.
These triangles will be congruent, that is ΔABC ≅ ΔDEF
if BC ≅EF ,AC ≅DF ,AB ≅DE That is,
corresponding sides are congruent that is BC = EF, AC = DF, AB = DE …..(1) and ∠A ≅ ∠D, ∠B ≅ ∠E, ∠C ≅ ∠F That is, corresponding angles are congruent that is ∠A = ∠D, ∠B = ∠E, ∠C = ∠F …..(2)
Now, let us learn this congruence of ΔABC and ΔDEF by the ‘trace-copy’ superposition method.

In the above figure if we trace the triangle ABC on a tracing paper and place it on triangle DEF such that vertex A falls on vertex D, B on E, and C on F. Then side AB falls on DE, BC on EF, and CA on FD. Also, ∠A superposes on the corresponding angle D, ∠B on ∠E, and ∠C on ∠F, and equations (1) and (2) written above hold. In the above discussion, we have considered one correspondence (matching) between the vertices of congruent triangles ABC and DEF, namely A ↔ D, B ↔ E, and C ↔ F. ⇒ A matches D and D matches A, B matches E and E matches B, C matches F and F matches C.

In two triangles, in fact, we have the following six matchings or correspondences between their vertices:

- A ↔ D, B ↔ E and C ↔ F written as ABC ↔ DEF
- A ↔ E, B ↔ F and C ↔ D written as ABC ↔ EFD
- A ↔ F, B ↔ D and C ↔ E written as ABC ↔ FDE
- A ↔ D, B ↔ F and C ↔ E written as ABC ↔ DFE
- A ↔ E, B ↔ D and C ↔ F written as ABC ↔ EDF
- A ↔ F, B ↔ E and C ↔ D written as ABC ↔ FED

If ΔABC ≅ ΔDEF, then in one of these six matchings, ΔABC superposes on ΔDEF exactly. Hence, we can conclude that triangles are congruent if and only if there exists all six correspondences (matchings) between the vertices of two triangles listed above in (i) to (vi) such that the corresponding sides and the corresponding angles of the two triangles are congruent (equal).



For example, if ΔABC ≅ ΔDFE and matching (iv) above holds, namely A ↔ D, B ↔ F, and C ↔ E in the adjoining figure, then by equations (1) and (2), $\overline{AB}=\overline{DF},\overline{BC}=\overline{FE},\overline{CA}=\overline{ED}$ and ∠A = ∠D, ∠B = ∠F, ∠C = ∠E. The order of the letters in the names of congruent triangles displays the corresponding relationships as shown in the above example.

Sufficient conditions for the congruence of two triangles We have that for the congruence of two triangles, all six parts of one triangle (three sides and three angles) must be congruent (equal) to the six corresponding parts (three sides and three angles) of the other triangle. Then, on further investigation by mathematicians, it was found that three properly chosen conditions out of the six conditions mentioned and referenced above are also sufficient for two triangles to be congruent. That is, if three properly chosen conditions are satisfied, the remaining three are automatically satisfied that is remaining three corresponding parts of congruent triangles are equal. This property is known as C.P.C.T. (corresponding parts of congruent triangles). There are four such sets of three properly chosen conditions, which we are listing below:

- Side-Side-Side (SSS) (three sides)
- Side-Angle-Side (SAS) (two sides and the included angle)
- Angle-Side-Angle (ASA) (two angles and the included side)
- Right angle-Hypotenuse-Side (RHS)

Measuring the Side lengths
Side-Side-Side (SSS) Congruence Condition (Rule) Two triangles are congruent if three sides of one triangle are respectively equal to the three sides of the other triangle.

Measuring the Angles
Two triangles that have the same set of angles need not be congruent.

Measuring two sides and the included angle Two triangles are congruent if two sides and the included angle of one triangle are correspondingly equal to the two sides and the included angle of the other triangle.

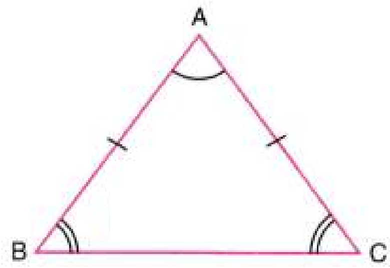
Angle-Side-Angle (ASA) Congruence Condition (Rule)
Two angles and the included side Two triangles are congruent if two angles and the included side of one triangle are correspondingly equal to the two angles and the included side of the other triangle.

AAS congruence rule: Two triangles are congruent if two angles and one side of a triangle are equal to two angles and one side of the other triangle. Proof: By the angle sum property of a triangle, we know that the sum of the three angles of a triangle is 180°. Therefore, if two angles of one triangle are equal to two angles of the other triangle, then the third angle of one triangle is also equal to the third angle of another triangle. ∴ The AAS congruence rule becomes the same as the ASA congruence rule.

Measuring Two Sides in a Right Triangle
Right Angle-Hypotenuse-Side (RHS) Congruence Condition (Rule) Two right-angled triangles are congruent if the hypotenuse and one leg (base or perpendicular) of one triangle are correspondingly equal to the hypotenuse and one leg (base or perpendicular) of the other triangle.

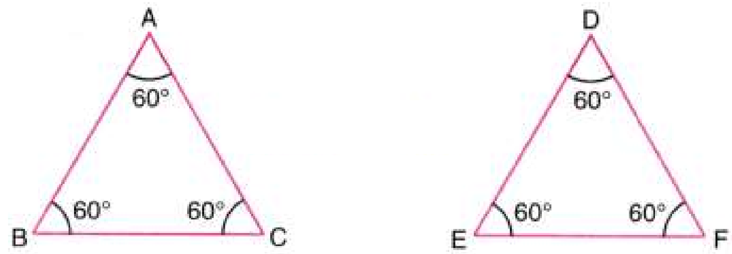
Angles of Isosceles and Equilateral Triangles

If two sides of a triangle are congruent, then the angles opposite to those sides are also congruent. In ΔABC, if $AB \cong AC$, then $\angle ABC \cong \angle ACB$.



Congruence of Equilateral Triangles

Let ΔABC and ΔDEF be two equilateral triangles as shown in the figure below:



We know that the measure of each angle of an equilateral triangle is 60°.
∴ Measure of each of the six angles A, B, C, D, E, F is 60°.
∴ ∠A = ∠D, ∠B = ∠E, and ∠C = ∠F …..(1)
Also, we know that all three sides of an equilateral triangle are equal in length.
∴ All sides of ΔABC are equal in length, and all sides of ΔDEF are also equal in length. …..(2)
So if one side of ΔABC is equal in length to any one side of ΔDEF, say $\overline{AB}=\overline{DE}$.
∴ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$ …..(3) ∴ From (1) and (3), all 3 angles and all 3 sides of the two triangles are congruent. Hence ΔABC ≅ ΔDEF. Hence, we conclude: Two equilateral triangles are congruent if any one side of one triangle is congruent to the side of the other triangle.

Figures that have the same shape and size are said to be congruent. These figures can be superimposed so that one fits exactly over the other. While verifying congruence, a figure can be rotated or flipped to make it fit exactly over the other figure via superimposition.

When two triangles have the same sidelengths, we say that the SSS (Side Side Side) condition is satisfied. The SSS condition guarantees congruence.

When two sides and the included angle of one triangle are equal to the two sides and the included angle of another triangle, we say that the SAS (Side Angle Side) condition is satisfied. The SAS condition also guarantees congruence.

When two angles and the included side of one triangle are equal to the two angles and the included angle of another triangle, we say that the ASA (Angle Side Angle) condition is satisfied. The ASA condition guarantees congruence. Congruence holds even if the side is not included between the angles, the AAS (Angle Angle Side) condition. In a right-angled triangle, the side opposite the right angle is called the hypotenuse.

When a side and a hypotenuse of a right-angled triangle are equal to a side and the hypotenuse of another right-angled triangle, we say that the RHS (Right Hypotenuse Side) condition is satisfied. The RHS condition also guarantees congruence. Two triangles need not be congruent if two sides and a non-included angle are equal. In a triangle, angles opposite to equal sides are equal. The angles in an equilateral triangle are all 60°.