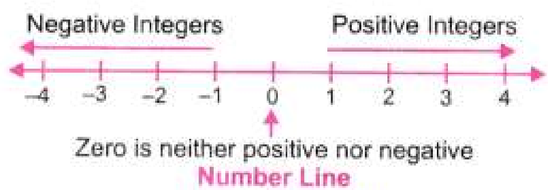
Operations with Integers Class 7 Notes Maths Part 2 Chapter 2

Integers

Integers are a collection of numerical values, including positive numbers, zero, and negative numbers. The numbers …-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5… are called integers. The numbers 1, 2, 3, 4, 5 … are called positive integers, and the numbers …,-5, -4, -3, -2, -1 are called negative integers. Zero (0) is neither positive nor negative.

Number Line

The number line represents both positive and negative numbers, with zero in the middle.
Negative numbers lie to the left of zero, and positive numbers lie to the right of zero.
Example: …, -3, -2, -1, 0, 1, 2, 3, …



Token Model

Positive tokens (representing positive numbers) and negative tokens (representing negative numbers) can be used for performing operations, (+) Positive token, (-) Negative token

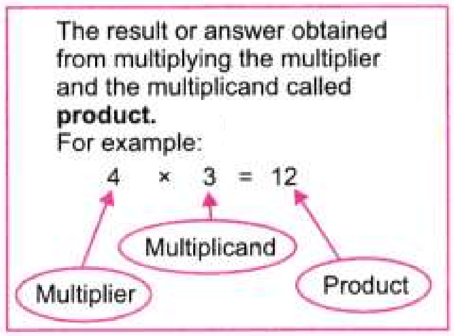
Representing Numbers: One colour (e.g., yellow or green) represents positive tokens (+1 each), and another colour (e.g., red or black) represents negative tokens (-1 each).

Zero Pairs: A pair of one positive token and one negative token cancel each other out, forming a "zero pair" (since +1 + (-1) = 0). These pairs can be added to or removed from a set without changing its total value.

Example: Identify which of the following are integers: 7, -3, 0, 4.5, -2.8, 12 Integers are whole numbers, positive, negative, or zero. 7, -3, 0, 12 are integers. 4.5 and -2.8 are not integers.

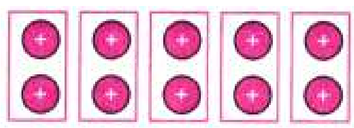
Multiplication of Integers

Multiplication is often treated as repeated addition or a process of "grouping" tokens. A positive token represents +1, a negative token represents -1, and a pair of one positive and one negative token cancel each other out to zero. This model can be used to understand why a positive number multiplied by a negative number is negative, and why two negative numbers multiplied together result in a positive number. The first number in a multiplication expression (the multiplier) indicates how many groups to add or remove. The second number (the multiplicand) indicates the size and sign of the tokens in each group.



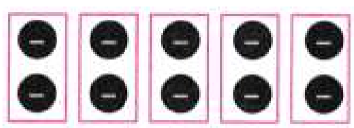
Modeling of Different Multiplication Scenarios

1. Positive Multiplier × Positive Multiplicand (e.g., 5 × 2)
Meaning: Add 5 groups of 2 positive tokens to an empty bag (or starting from zero).
Action: Place 5 groups, each containing 2 positive tokens, into the bag.
Result: We have a total of 10 positive tokens.
Equation: 5 × 2 = 10.



$5 \times 2 = 10$

2. Positive Multiplier × Negative Multiplicand (e.g., 5 × (-2)) Meaning: Add 5 groups of 2 negative tokens to an empty bag. Action: Place 5 groups, each containing 2 negative tokens, into the bag. Result: We have a total of 10 negative tokens. Equation: 5 × (-2) = -10.



$5 \times -2 = -10$

3. Negative Multiplier × Negative Multiplicand (e.g., (-5) × (-2)) Meaning: The negative sign on the multiplier can be interpreted as “the opposite of”. Action: 1. Start by interpreting 5 × (-2) as adding 5 groups of 2 negative tokens, resulting in 10 negative tokens. 2. The initial negative sign in (-5) × (-2) means the “opposite” of this result. 3. The opposite of 10 negative tokens is 10 positive tokens. Result: We have a total of 10 positive tokens. Equation: (-5) × (-2) = 10.

$(-5) \times (-2) = 10$

4. Negative Multiplier × Positive Multiplicand (e.g., (-5) × 2) Meaning: “The opposite of’ adding 5 groups of 2 positive tokens. Action: 1. Interpret 5 × 2 as adding 5 groups of 2 positive tokens (10 positive tokens). 2. The initial negative sign means the “opposite” of this result. 3. The opposite of 10 positive tokens is 10 negative tokens. Result: We have a total of 10 positive tokens. Equation: (-5) × 2 = -10.

$(-5) \times 2 = 10$

Brahmagupta’s Rules for Multiplication and Division of Positive and Negative Numbers

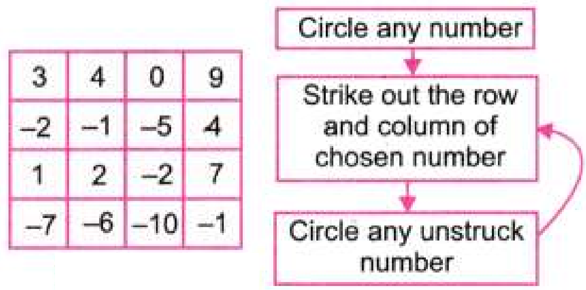
Brahmagupta, in his work Brahmasphutasiddhanta (628 CE), provided foundational rules for the multiplication and division of positive and negative integers, which he termed “fortune” (positive) and “debt” (negative,) respectively. These rules were a landmark development in the history of mathematics and are the same principles used today. Brahmagupta’s rules, from verses 18.30-32, can be summarised as follows:

Operation	Description (using Brahmagupta’s terms)	Modern Equivalent	Result
Multiplication/ Division	The product or quotient of two fortunes	(+/+) or (+ × +)	Fortune (+)
Multiplication/ Division	The product or quotient of two debts	(–/–) or (– × –)	Fortune (+)
Multiplication/ Division	The product or quotient of a debt and a fortune.	(–/+) or (– × +)	Debt (–)
Multiplication/ Division	The product or quotient of a fortune and a debt.	(+/–) or (+ × –)	Debt (–)

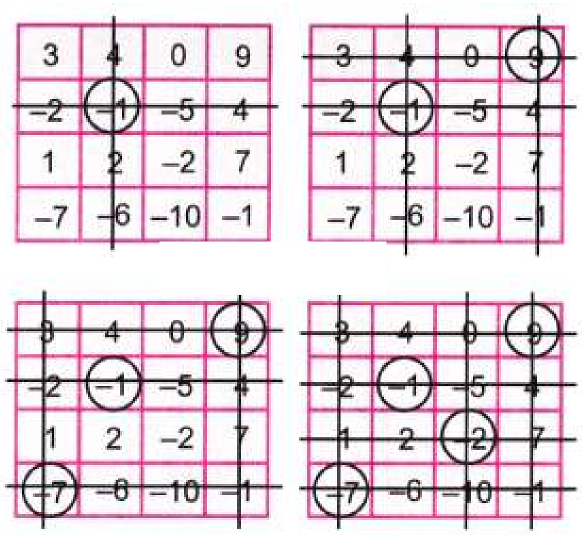
Explorations with Grids

Grid puzzles and number arrangements to explore integer properties, such as finding sums of rows and columns to create specific totals.

An Amazing Grid of Numbers! Below is a grid with some numbers. Follow the steps shown below until no number is left.



Circle any number.
Strike out the row and column of the chosen number.
Circle an unstruck number.
When there are no more unstruck numbers, STOP.
Add the circled numbers.



In the example below, the circled numbers are -1, 9, -7, and -2. When we add them together, we get -1.

Properties of Multiplication of Integers

We know integers are commutative under addition, but are integers commutative under multiplication?
1. The product of two integers is always an integer.

Let us multiply two integers, (-6) and 10, in two different ways.

$(-6) \times 10 = -60$

$10 \times (-6) = -60$

Integers can be multiplied in any order.

2. The product of the integers remains the same even though the integers are ordered differently. We can say that integers follow the commutative property of multiplication.

3. Zero (0) is the only integer that is neither negative nor positive!

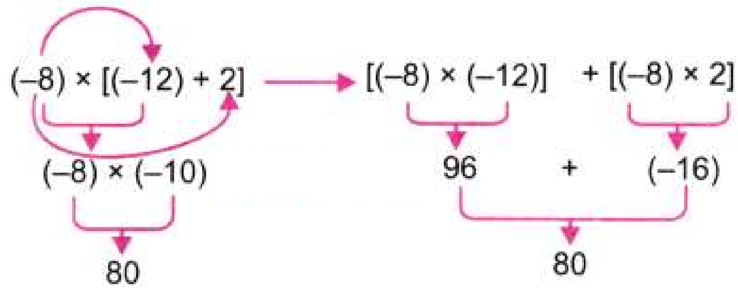
4. The product of any whole number and ‘0’ is ‘0’. The product of any integer and zero is also zero. In general, for any integer ‘a’, a × 0 = 0 × a = 0

5. The product of any integer and 1 is the integer itself. In general, for any integer ‘a’, a × 1 = 1 × a = a Now, to multiply three integers (-a), (-b), and c, which two integers should be grouped to multiply first? Are there any rules?

6. The product of the integers remains the same even though the integers are grouped differently. -a × (-b × c) = (-a × -b) × c = a × b × c We can say that integers follow the associative property of multiplication.

The associative property of multiplication states that when three integers are multiplied, the product will be the same regardless of the grouping. In general, if ‘a’, ‘b’, and ‘c’ are three integers, then (a × b) × c = a × (b × c).

7. Integers follow the distributive property of multiplication over addition. We can verify the distributive property using the integers (-8), (-12), and 2.



Here, the multiplication by (-8) gets distributed over the addition of (-12) and 2. We can say that integers follow the distributive property of multiplication over addition.

8. Integers follow the distributive property of multiplication over subtraction. In general, for three integers, ‘a’, ‘b’ and ‘c’, a × (b – c) = (a × b) – (a × c)

Division of Integers

We know that division can always be connected to multiplication. Instead of directly dividing, we can ask: “What number should be multiplied by the divisor to get the dividend?” For example, consider: 84 ÷ (-12). We can think of it as: “What number multiplied by -12 gives 84?” -12 × ? = 84 We know that -12 × (-7) = 84 So, 84 ÷ (-12) = -7 Similarly, look at: (-72) ÷ 9. This means: “What number multiplied by 9 gives -72?” 9 × ? = -72 We know that, 9 × (-8) = -12 Therefore (-72) ÷ 9 = -8 Now consider dividing two negative numbers: (-45) ÷ (-5) This asks: “What number multiplied by -5 gives -45?” -5 × ? = -45 We know that -5 × 9 = – 45 So, (-45) ÷ (-5) = 9

In short: 1. Division of a positive number by a negative number gives a negative result: a ÷ (-b) = -(a ÷ b) 2. Division of a negative number by a positive number also gives a negative result: (-a) ÷ b = – (a ÷ b) 3. Division of a negative number by a negative number gives a positive result: (-a) ÷ (-b) = a ÷ b

When two integers are multiplied, the product is positive when both the multiplier and multiplicand are positive, or when both are negative. The product is negative if one of them is positive and the other is negative.

−9	−6	−3	3	3	6	9
−6	−4	−2	2	2	4	6
−3	−2	−1	1	1	2	3
−3	−2	−1	×	1	2	3
3	2	1	1	−1	−2	−3
6	4	2	2	−2	−4	−6
9	6	3	3	−3	−6	−9

When two integers are divided, the quotient is positive when both the dividend and divisor are positive, or both are negative. The quotient is negative when one of them is positive, and the other is negative.

Integer multiplication is commutative, i.e., for any two integers a and b, $a \times b = b \times a$

Integer multiplication is associative, i.e., for any three integers, a, b, and c, $a \times (b \times c) = (a \times b) \times c$

Integer multiplication is distributive over addition, i.e., for any three integers, a, b, and c, $a \times (b + c) = (a \times b) + (a \times c)$