Finding Common Ground Class 7 Notes Maths Part 2 Chapter 3

Factors

When a number is said to be a factor of any other number, then the first number must divide the second number completely without leaving any remainder. In simple words, if a number (dividend) is exactly divisible by any number (divisor), then the divisor is a factor of that dividend.

- 1 is a factor of every number.
- Every number is a factor of itself.
- Every factor of a number is an exact divisor of that number.
- Every factor of a number is less than or equal to that number.
- The factors of a given number are finite in number.

Common Factors

The factors that are factors of each of the given numbers are called their common factors.
For example:

Factors of 20 are (1), (2), (4), 5, 10 and 20.
Factors of 36 are (1), (2), 3, (4), 6, 9, 12, 18 and 36.

Clearly, the factors common to the factors of 20 and 36 are 1, 2, and 4. These are the common factors of 20 and 36. Common factors are finite in number.

Find the common factors of 18 and 24. First, find all the factors of 18 and 24 and write them in a list in the order of least to greatest to make sure that every factor is covered. 18 – 1, 2, 3, 6, 9, 18 24 – 1, 2, 3, 4, 6, 12, 24 Next, identify the common factors that appear in both lists.

18 — (1), (2), (3), (6), 9, 18
24 — (1), (2), (3), 4, (6), 12, 24

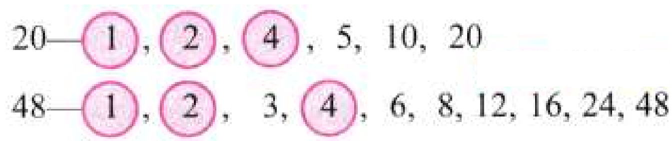
The common factors for 18 and 24 are 1, 2, 3, and 6.

Steps to find Common Factors To find common factors, we follow the following steps:

- Step 1: List down all the factors of the given numbers in separate rows.
- Step 2: Circle all the factors that are common in the given numbers.
- Step 3: Write down all the common factors in a separate row.

Highest Common Factor (H.C.F.)

The highest common factor (H.C.F.) of two or more numbers is a unique number, which is a factor of each of the two numbers, i.e., the common factor of all the numbers, and which is the greatest among the common factors of these numbers. The highest common factor (H.C.F.) is also called the greatest common divisor (G.C.D.) or the greatest common measure (G.C.M.).
Consider two numbers, 20 and 48.
First, find all the factors of 20 and 48 and write them in a list in the order of least to greatest.



Thus, the common factors for 20 and 48 are 1, 2, and 4.
Among these factors, 4 is the greatest factor.
Thus, the H.C.F. of 20 and 48 is 4.
Again, consider two numbers, 24 and 56.

The common factors of 24 and 56 are 1, 2, 4, and 8. The highest of these factors is 8. So, 8 is the HCF of 24 and 56. The H.C.F. of two co-prime numbers is always 1.

Prime Numbers: The numbers having exactly two factors, 1 and the number itself, are called prime numbers. For example, 2, 3, 5, 7, 11, etc., are prime numbers.

Composite Numbers: The numbers having more than two factors are called composite numbers. For example, 4, 6, 8, 9, 10, etc. are composite numbers. The number 1 is neither prime nor composite.

Prime Factorisation

Factorisation: A number when expressed as a product of its factors is said to be factorised. For example: 12 = 3 × 4. We say that 12 has been factorised. This is one of the several factorisations of 12. The others are: 12 = 2 × 6, 12 = 1 × 12.

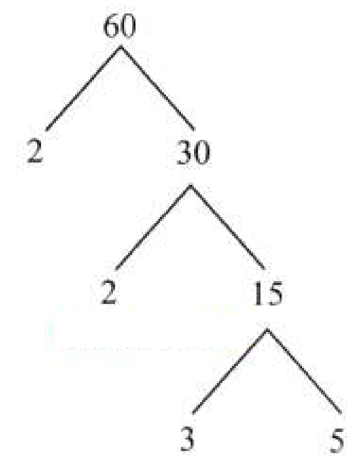
Prime Factorisation: Prime factorisation is the process of breaking down a number into a product of its prime numbers (prime factors). It is the ultimate factorisation of a given number. Moreover, it is unique (exactly one). For example, while factorising 12, we ultimately reach the unique factorisation 2 × 2 × 3. In this factorisation, the only factors 2 and 3 are prime numbers. Such a factorisation of a number is called the prime factorisation of that number. Thus, 2 × 2 × 3 is the only prime factorisation of 12. For example, in this case: 12 = 2 × 6 = 2 × (2 × 3) = 2 × 2 × 3 12 = 3 × 4 = 3 × (2 × 2) = 3 × 2 × 2 12 = 1 × 12 = 1 × (2 × 3 × 2) = 2 × 3 × 2 All above have the same prime factorisation, i.e., 2 × 2 × 3.

Prime Factorisation Property

Every number greater than 1 has exactly one prime factorisation.

Factor Tree:

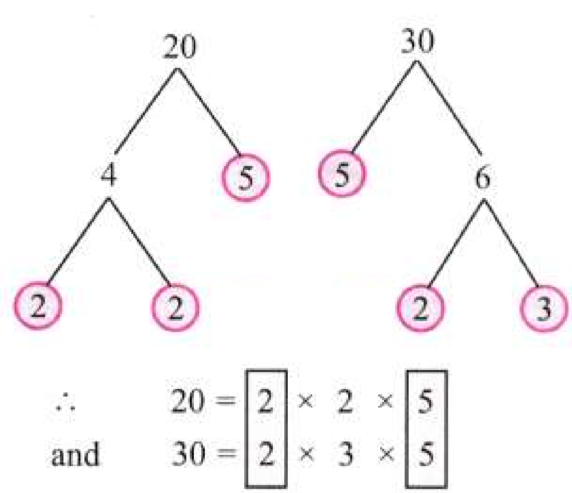
When we go on factorising a number till we reach its ultimate factorisation and write the process as follows, we get the shape of a tree called the Factor Tree.
For example, let us factorise 60.



Thus, 60 = 2 × 2 × 3 × 5.

Finding H.C.F. by the Factor Tree Method (Prime Factorisation)

In this method, we find the prime factorisation of each number using the factor tree method and find the prime factors that are common to those numbers. Then, we find the H.C.F. of those numbers by finding the product of the common prime factors of the given numbers. Let’s understand this method using an example.
Find the H.C.F. of 20 and 30.
Here,



Identify the common factors. The numbers 20 and 30 have the factors 2 and 5 in common. Next, multiply the common factors to find the H.C.F. There is no need to multiply if there is only one common factor. 2 × 5 = 10 Hence, HCF of 20 and 30 = 10

Steps to Find H.C.F. using the Factor Tree Method (Prime Factorisation) To find the H.C.F. of two or more numbers using the factor tree method, we follow the following steps:

- Step 1: Write the prime factorisation of the given numbers using the factor tree method.
- Step 2: Identify common prime factors of the given numbers.
- Step 3: Multiply each common prime factor to find the required H.C.F.

Common Multiples

We learnt that a multiple is a product obtained when one number is multiplied by another number. In this section, we will look at multiples that are common among the multiples of two or more numbers. These multiples are known as common multiples. To find the common multiples of 2 and 4, we list the multiples of 2 and 4 separately and then find the multiples that are common in both lists.

Multiples of 2

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...

Multiples of 4

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...

When we compare the first ten multiples of 2 and 4, we find that they have five common multiples, i.e., 4, 8, 12, 16, and 20. There will be more if we continue to look for more multiples. The common multiples of two or more numbers are the numbers that are completely divisible by each of the given numbers.

Find the common multiples of 3, 4, and 8 First, write down the first few multiples for the numbers and then identify the multiples that the three numbers have in common. Multiples of 3 – 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, … Multiples of 4 – 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, … Multiples of 8 – 8, 16, 24, 32, 40, 48, 56, … Then, identify the common multiples. Multiples of 3 – 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, … Multiples of 4 – 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, … Multiples of 8 – 8, 16, 24, 32, 40, 48, 56, … Some of the common multiples of 3, 4, and 8 are 24, 48,…

Steps to find Common Multiples To find common multiples, we follow the following steps: Step 1: List down some multiples of the given numbers in separate rows. Step 2: Identify all the multiples that are common in the given numbers. Step 3: Write down all the common multiples in a separate row. Every multiple of a number is greater than or equal to that number. The multiples of a given number are infinite in number. Every number is a multiple of itself.

Lowest Common Multiple (L.C.M.)

The smallest number, which is a common multiple of two or more given numbers, is called their lowest common multiple (L.C.M.). In other words, it is the lowest possible number that can be divisible by the given numbers. For example, find the L.C.M. of 3 and 4. Let's look at the common multiples for 3 and 4. Some of the common multiples of 3 and 4 are 12, 24, and 36. The L.C.M. of these two numbers is 12. It is the smallest number that they both have in common.

Finding L.C.M. by the Listing Method

In this method, first, list down some multiples of each given number and find the multiples that are common to those numbers. Then, among the common multiples, we identify the lowest common multiple.

Find the L.C.M. of 4 and 5. First, list down some multiples of 4 and 5. Now, look for the common multiples of 4 and 5. Multiples of 4 – 4, 8, 12, 16, 20, 24, 28, … Multiples of 5 – 5, 10, 15, 20, 25, 30, … 20, 40, … are common multiples, but 20 is the smallest. Hence, LCM(4, 5) = 20.

Steps to Find L.C.M. using the Listing Method To find the L.C.M. of two or more numbers using lists, we follow the following steps: Step 1: List down some multiples of the given numbers in separate rows. Step 2: Identify all the multiples that are common in the given numbers. Step 3: Identify the smallest number in the list of common multiples (i.e. first common number). This number is the required L.C.M.

Finding L.C.M. by Prime Factorisation Method In this method, we find the prime factorisation of each number using the division method. L.C.M. of the given numbers is the product of all the different prime factors, using each common prime factor occurring the maximum number of times. Let's understand this method using an example.

Find the lowest common multiple of 84 and 90. First, write down the prime factorisation of each number and collect the maximum number of occurrences of each prime factor. Let's find the LCM of 84 and 90. 84 = 2 × 2 × 3 × 7 90 = 2 × 3 × 3 × 5 Here we note that 2 has occurred two times and 3 has occurred two times, 5 one time, and 7 one time. Next, multiply the prime factors by the maximum occurrences in the prime factorisation to find the required L.C.M. Hence, LCM (84, 90) = 2 × 2 × 3 × 3 × 5 × 7 = 1260

Steps to Find L.C.M. using the Prime Factorisation Method To find the L.C.M. of two or more numbers using the prime factorisation method, we follow the following steps: Step 1: Write the prime factorisation of the given numbers using the division method. Step 2: Find the maximum number of occurrences of each prime factor. Step 3: Find the product of the prime factors with maximum occurrences in the prime factorisation to obtain the required L.C.M.

Patterns, Properties, and a Pretty Procedure

A general statement (or a generalisation) is a broad conclusion that describes a pattern or a property observed across all possible instances within a specific set of conditions.

Generalisation is the process of identifying a pattern from specific examples and formulating a general statement or rule based on that observation.

For number pairs where one number is the Highest Common Factor (HCF) of the pair (which means one number is a multiple of the other): (a) If m is one number, the other number could be any multiple of m (or m could be a multiple of the other number). Any number of the form k × m, where k is a positive integer, would form such a pair with m. The HCF of m and k × m would be m. Alternatively, m could be a multiple of the second number. For example, the other number could be a factor of m, such as m/2, m/3, etc.

(b) If 7k is one number, the other number could be any multiple of 7k (or 7k could be a multiple of the other number). Any number of the form j × (7k), where j is a positive integer, would form such a pair with Ik. The HCF of 7k and j × (7k) would be 7k. Alternatively, 7k could be a multiple of the second number. For example, if k allows, the other number could be 7k/2, 7k/3, etc. In both cases, the condition is that one number must be a factor of the other number.

Relationship between L.C.M. and H.C.F.

Consider the numbers 24 and 36.
The prime factorisation of 24 = 2 × 2 × 2 × 3
and that of 36 = 2 × 2 × 3 × 3
HCF (24, 36) = 2 × 2 × 3 = 4 × 3 = 12
LCM (24, 36) = 2 × 2 × 2 × 3 × 3 = 8 × 9 = 72
Verify the relationship:
Product of given numbers = 24 × 36 = 864
Product of HCF (24, 36) and LCM (24, 36) = 12 × 72 = 864
Therefore, we can write it as follows:
H.C.F. of the numbers × L.C.M. of the numbers = The product of the two numbers.
Hence, if a and b are the two given numbers, then the L.C.M. and H.C.F. are related to the given numbers by the following relationship.
a × b = HCF (a, b) × LCM (a, b)
For Example, L.C.M. of two numbers is 378, and their H.C.F. is 9. If one of the numbers is 63, then find the other number.
We know that L.C.M. × H.C.F. = Product of numbers
Let the other number be x.
Then L.C.M × H.C.F = 63 × x
then 378 × 9 = 63 × x
x = 378×963 x = 54 ∴ The other number is 54.

Last year, we looked at common multiples and common factors, and were also introduced to the amazing world of primes! In this chapter, we learnt a method to find the prime factorisation of a number. Finding all the factors of a number from its prime factorisation is easy but quite tedious — we have to list every possible subpart!

The Highest Common Factor (HCF) is the highest among all the common factors of a group of numbers. Every common factor is contained in the prime factorisation of the number. To find the HCF, we include the minimum number of occurrences of each prime across the prime factorisation of all the numbers.

The Lowest Common Multiple (LCM) is the lowest among all the common multiples of a group of numbers. Every common multiple contains the prime factorisation of the numbers. To find the LCM, we include the highest number of occurrences of each prime across the prime factorisations of all the numbers.

We explored more about HCF and LCM; we discovered related properties and patterns when numbers are consecutive, even, co-prime, etc. We learnt a procedure to get both the HCF and the LCM at the same time! We also saw how to make this even quicker! We learned some terms that are used when discussing mathematics, such as ‘conjecture’ and ‘generalisation’.